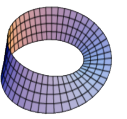
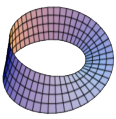


## Collisions – Oblique Collisions

- \* A smooth sphere P collides with an identical smooth sphere Q which is at rest. The velocity of P before impact makes an angle  $\alpha$  with the line of centres at impact, where  $\theta^\circ \leq \alpha < 90^\circ$ . The velocity of P is deflected through an angle  $\theta$  by the collision. The coefficient of restitution between the spheres is  $\frac{1}{3}$ . Show that  $\tan \theta = \frac{2 \tan \alpha}{1 + 3 \tan^2 \alpha}$
- \* A smooth sphere A, of mass  $m$ , moving with speed  $u$ , collides with an identical smooth sphere B which is at rest. The direction of motion of A before and after impact makes angles  $\alpha$  and  $\beta$  respectively with the line of centres at the instant of impact. The coefficient of restitution between the spheres is  $e$ .
  - If  $\tan \alpha = k \tan \beta$ , find  $k$ , in terms of  $e$ .
  - If the magnitude of the impulse imparted to each sphere due to the collision is  $\frac{7}{8}mu \cos \alpha$ , find the value of  $e$ .
- \* A smooth sphere, of mass  $m$ , moving with velocity  $6\vec{i} + 2\vec{j}$  collides with a smooth sphere, of mass  $km$ , moving with velocity  $2\vec{i} + 4\vec{j}$  on a smooth horizontal table. After the collision the spheres move in parallel directions. The coefficient of restitution between the spheres is  $e$ .
  - Find  $e$  in terms of  $k$ .
  - Prove that  $k \geq \frac{1}{3}$ .
- \* A smooth sphere A moving with speed  $u$ , collides with an identical smooth sphere B which is at rest. The direction of motion of A, before impact, makes an angle of  $45^\circ$  with the line of centres at the instant of impact. The coefficient of restitution between the spheres is  $e$ . Show that the direction of motion of A is deflected through an angle  $\alpha$  where  $\tan \alpha = \frac{1+e}{3-e}$ .
- \* A smooth sphere A, of mass 4 kg, moving with speed  $u$ , collides with a stationary smooth sphere B of mass 8 kg. The direction of motion of A, before impact, makes an angle  $\alpha$  with the line of centres at impact. The coefficient of restitution between the spheres is  $\frac{1}{2}$ . Find in terms of  $u$  and  $\alpha$ 
  - the speed of each sphere after the collision
  - the angle through which the 4 kg sphere is deflected as a result of the collision
  - the loss in kinetic energy due to the collision.
- \* A smooth sphere A moving with speed  $u$ , collides with an identical smooth sphere B which is moving in a perpendicular direction with the same speed  $u$ . The line of centres at the instant of impact is perpendicular to the direction of motion of sphere B. The coefficient of restitution between the spheres is  $e$ .
  - Find, in terms of  $e$ , the speed of each sphere after impact and hence, or otherwise, show that it is not possible for the two spheres to have the same speed after impact.
  - Prove that  $\tan \theta = \frac{1+e}{2}$  where  $\theta$  is the angle through which sphere B is turned as a result of the impact.
- \* A smooth sphere A, of mass  $m$ , moving with speed  $u$ , collides with an identical smooth sphere B moving with speed  $u$ . The direction of motion of A, before impact, makes an angle  $45^\circ$  with the line of centres at impact. The direction of motion of B, before impact, makes an angle  $45^\circ$  with the line of centres at impact. The coefficient of restitution between the spheres is  $e$ .
  - Find, in terms of  $e$  and  $u$ , the speed of each sphere after the collision.
  - If  $e = \frac{1}{2}$ , show that after the collision the angle between the directions of motion of the two spheres is  $\tan^{-1} \frac{4}{3}$ .

**Please turn over – more questions on the other side**



8. \* A smooth sphere A, of mass  $m$ , moving with speed  $u$ , collides with an identical smooth sphere B which is at rest. The direction of motion of A, before impact, makes an angle  $30^\circ$  with the line of centres at impact. After impact the direction of A makes an angle  $\theta$  with the line of centres, where  $0^\circ \leq \theta < 90^\circ$ . The coefficient of restitution between the spheres is  $e$ . The speeds of A and B immediately after impact are equal.
- (i) Calculate the value of  $\theta$ . (ii) Find  $e$ .
9. \* A smooth sphere A, of mass  $m$ , moving with speed  $u$ , collides with a smooth sphere B, of mass  $2m$ , which is at rest. The direction of motion of A, before impact, makes an angle  $\alpha$  with the line of centres at impact, where  $0^\circ \leq \alpha < 90^\circ$ . As a result of the collision, the direction of A is deflected through an angle of  $90^\circ$ . The coefficient of restitution between the spheres is  $e$ .
- (i) Show that  $\tan \alpha = \sqrt{\frac{2e-1}{3}}$ .  
(ii) Find  $e$ , if the magnitude of the impulse exerted by A on B is  $mu \cos \alpha$ .
10. \* A smooth sphere A, of mass  $m$  kg, moving with speed  $u$ , collides with a stationary identical sphere B. The direction of motion of A, before impact, makes an angle  $N\alpha W$  with the line of centres (which runs NS) at impact. The direction of motion for A is also a tangent to B just before impact. The coefficient of restitution between the spheres is  $\frac{4}{5}$ .
- (i) Show that  $\alpha = 30^\circ$   
(ii) Find the direction in which each sphere travels after the collision.  
(iii) Find the percentage loss in kinetic energy due to the collision.

## Source(s):

- <http://www.MathsGrinds.ie/>
- <http://www.examinations.ie/>

## Further Information:

- Questions marked with an asterisk \* are past Leaving Cert Exam questions.

## Answers:

- |   |   |   |
|---|---|---|
| 1. N/A  | (ii) $90^\circ - \alpha$                                    | (ii) $\frac{1}{3}$  |
| 2. (i) $\frac{1-e}{2}$                                | (iii) $u^2 \cos^2 \alpha$                                   |   |
| (ii) $e = \frac{3}{4}$                                | 6. (i) $\frac{1}{2}u(1-e)$ and $\frac{u}{2}\sqrt{e^2+2e+5}$ | 9. (i) N/A  |
| 3. (i) $\frac{3+k}{2+4k}$                             | (ii) N/A  | (ii) $\frac{1}{2}$  |
| (ii) N/A  | 7. (i) $-\frac{eu}{\sqrt{2}}$ and $\frac{eu}{\sqrt{2}}$     | 10. (i) Show that $\alpha = 30^\circ$   |
| 4. N/A  | (ii) N/A  | (ii) $-\frac{u}{2}\vec{i} + \frac{u\sqrt{3}}{20}\vec{j}$ and $\frac{9u\sqrt{3}}{20}\vec{j}$ |
| 5. (i) $u \sin \alpha$ and $\frac{1}{2}u \cos \alpha$ | 8. (i) $60^\circ$   | (iii) 13.5%   |