Collisions – Oblique Collisions

1. * A smooth sphere P collides with an identical smooth sphere Q which is at rest. The velocity of P before impact makes an angle α with the line of centres at impact, where $\theta^\circ \leq \alpha < 90^\circ$. The velocity of P is deflected through an angle θ by the collision. The coefficient of restitution between the spheres is $\frac{1}{3}$. Show that $\tan\theta = \frac{2\tan\alpha}{1+3\tan^2\alpha}$

2. * A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B which is at rest. The direction of motion of A before and after impact makes angles α and β respectively with the line of centres at the instant of impact. The coefficient of restitution between the spheres is e.

   (i) If $\tan\alpha = k\tan\beta$, find k, in terms of e.

   (ii) If the magnitude of the impulse imparted to each sphere due to the collision is $\frac{7}{8}mu\cos\alpha$, find the value of e.

3. * A smooth sphere, of mass m, moving with velocity $6\vec{i}+2\vec{j}$ collides with a smooth sphere, of mass km, moving with velocity $2\vec{i}+4\vec{j}$ on a smooth horizontal table. After the collision the spheres move in parallel directions. The coefficient of restitution between the spheres is e.

   (i) Find e in terms of k.

   (ii) Prove that $k \geq \frac{1}{3}$.

4. * A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is at rest. The direction of motion of A, before impact, makes an angle of 45° with the line of centres at the instant of impact. The coefficient of restitution between the spheres is e. Show that the direction of motion of A is deflected through an angle α where $\tan\alpha = \frac{1+e}{3-e}$.

5. * A smooth sphere A, of mass 4 kg, moving with speed u, collides with a stationary smooth sphere B of mass 8 kg. The direction of motion of A, before impact, makes an angle α with the line of centres at impact. The coefficient of restitution between the spheres is $\frac{1}{2}$. Find in terms of u and α

   (i) the speed of each sphere after the collision

   (ii) the angle through which the 4 kg sphere is deflected as a result of the collision

   (iii) the loss in kinetic energy due to the collision.

6. * A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is moving in a perpendicular direction with the same speed u. The line of centres at the instant of impact is perpendicular to the direction of motion of sphere B. The coefficient of restitution between the spheres is e.

   (i) Find, in terms of e, the speed of each sphere after impact and hence, or otherwise, show that it is not possible for the two spheres to have the same speed after impact.

   (ii) Prove that $\tan\theta = \frac{1+e}{2}$ where θ is the angle through which sphere B is turned as a result of the impact.

7. * A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B moving with speed u. The direction of motion of A, before impact, makes an angle $45\circ$ with the line of centres at impact. The direction of motion of B, before impact, makes an angle 45° with the line of centres at impact. The coefficient of restitution between the spheres is e.

   (i) Find, in terms of e and u, the speed of each sphere after the collision.

   (ii) If $e = \frac{1}{2}$, show that after the collision the angle between the directions of motion of the two spheres is $\tan^{-1}\frac{4}{3}$.

Please turn over – more questions on the other side

8. * A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B which is at rest. The direction of motion of A, before impact, makes an angle 30° with the line of centres at impact. After impact the direction of A makes an angle θ with the line of centres, where $0^\circ \leq \theta < 90^\circ$. The coefficient of restitution between the spheres is e. The speeds of A and B immediately after impact are equal.

   (i) Calculate the value of θ.

   (ii) Find e.

9. * A smooth sphere A, of mass m, moving with speed u, collides with a smooth sphere B, of mass $2m$, which is at rest. The direction of motion of A, before impact, makes an angle α with the line of centres at impact, where $0^\circ \leq \alpha < 90^\circ$. As a result of the collision, the direction of A is deflected through an angle of 90°. The coefficient of restitution between the spheres is e.

   (i) Show that $\tan\alpha = \sqrt{\frac{2e-1}{3}}$.

   (ii) Find e, if the magnitude of the impulse exerted by A on B is $mu\cos\alpha$.

10. * A smooth sphere A, of mass m kg, moving with speed u, collides with a stationary identical sphere B. The direction of motion of A, before impact, makes an angle NαW with the line of centres (which runs NS) at impact. The direction of motion for A is also a tangent to B just before impact. The coefficient of restitution between the spheres is $\frac{4}{5}$.

    (i) Show that $\alpha = 30^\circ$

    (ii) Find the direction in which each sphere travels after the collision.

    (iii) Find the percentage loss in kinetic energy due to the collision.

Source(s):

- http://www. Maths Grinds .ie/
- http://www.examinations.ie/

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.

Answers:

1. N/A
2. (i) $\frac{1-e}{2}$
   (ii) $e = \frac{3}{4}$
3. (i) $\frac{3+k}{2+4k}$
   (ii) N/A
4. N/A
5. (i) $u\sin\alpha$ and $\frac{1}{2}u\cos\alpha$
   (ii) $90^\circ - \alpha$
   (iii) $u^2\cos^2\alpha$
6. (i) $\frac{1}{2}u(1-e)$ and $\frac{u}{2}\sqrt{e^2+2e+5}$
   (ii) N/A
7. (i) $-\frac{eu}{\sqrt{2}}$ and $\frac{eu}{\sqrt{2}}$
   (ii) N/A
8. (i) 60°
   (ii) $\frac{1}{3}$
9. (i) N/A
   (ii) $\frac{1}{2}$
10. (i) Show that $\alpha = 30^\circ$
    (ii) $-\frac{u}{2}\vec{i} + \frac{u\sqrt{3}}{20}\vec{j}$ and $\frac{9u\sqrt{3}}{20}\vec{j}$
    (iii) 13.5%