Simple Harmonic and Circular Motion – Further Circular Motion

1. * A particle is placed on a horizontal rotating turntable, 10 cm from the centre of rotation. There is a coefficient of friction of 0.4 between the particle and the turntable. If the speed of the turntable is gradually increased, at what angular speed will the particle begin to slide?

2. * A and B are two fixed pegs. A is 4 m vertically above B. A mass m kg, connected to A and B by two light inextensible strings of equal length, l , is describing a horizontal circle with uniform angular velocity ω. Find the value of ω if the ratio of the tensions in the two strings is $11 : 9$.

3. * A and B are two fixed pegs, A is 4 m vertically above B. A mass m kg, connected to A and B by two light inextensible strings of equal length, is describing a horizontal circle with uniform angular velocity ω. For what value of ω will the tension in the upper string be double the tension in the lower string?

4. * A bead slides on a smooth fixed circular hoop, of radius r, in a vertical plane. The bead is projected with speed $\sqrt{10gr}$ from the highest point c. It impinges upon and coalesces with another bead of equal mass at d. cd is the vertical diameter of the hoop. Show that the combined mass will not reach the point c in the subsequent motion.
(Hint: You only need to use Conservation of Energy and Conservation of Momentum.)

5. * A hollow cone with its vertex downwards and its axis vertical, revolves about its axis with a constant angular velocity of 4π rad/s. A particle of mass m is placed on the inside rough surface of the cone. The particle remains at rest relative to the cone. The coefficient of friction between the particle and the cone is $\frac{1}{4}$. The semi-vertical angle of the cone is $30°$ and the particle is a distance l m from the vertex of the cone. Find the maximum value of l, correct to 2dp.

6. * A conical pendulum consists of a light inelastic string [pq], fixed at the end p, with a particle attached to the other end q. The particle moves uniformly in a horizontal circle whose centre o is vertically below p. If po $= h$, find the period of the motion in terms of h

7. * A particle can move on the smooth outer surface of a fixed sphere of radius r. The particle is released from rest on the smooth surface of the sphere at a height $\frac{4}{5}r$ above the horizontal plane through the centre o of the sphere. Find, in terms of r, the height above this plane at which the particle leaves the sphere.

8. * A particle of mass m is held at a point p on the surface of a fixed smooth sphere, centre o and radius r. op makes an angle α with the upward vertical. The particle is released from rest. When the particle reaches an arbitrary point q, its speed is v. oq makes an angle β with the upward vertical.

   (i) Show that $v^2 = 2gr\,(\cos\alpha - \cos\beta)$.

   (ii) If $\cos\alpha = \frac{2}{3}$ and if q is the point at which the particle leaves the surface, find the value of β.

9. * A particle of mass m kg lies on the top of a smooth sphere of radius $2m$. The sphere is fixed on a horizontal table at P. The particle slides down the sphere. The particle leaves the sphere at B and strikes the table at Q.

   (i) Find the speed of the particle at B

   (ii) Find the speed of the particle on striking the table at Q.

10. * A particle of mass m kg lies on the top of a smooth fixed sphere of radius 30 cm. The particle is slightly displaced and slides down the sphere. The particle leaves the sphere at B.

    (i) Find the speed of the particle at B.

    (ii) The horizontal distance, in metres, of the particle from the centre of the sphere t seconds after it has left the surface of the sphere is $\frac{\sqrt{5}}{10} + kt$. Find the value of k correct to two places of decimals.

11. * A smooth uniform vertical hoop of radius r and mass M kg stands in a vertical plane on a horizontal surface. The hoop threads two small rings, each of mass m kg. The rings are released from rest at the top of the hoop.

    (i) When the two rings have each fallen through an angle of θ on opposite sides of the hoop, show that the normal force of reaction exerted by the hoop on each ring is $mg\,(3\cos\theta - 2)$ N.

    (ii) Show that the hoop will rise from the table if $m > \frac{3M}{2}$.

12. * 1kg mass is swung around in a 1m radius vertical circle (constant speed, inelastic string). Find the Tension difference.

13. If the Earth (on average) is observed to be 149,600,000 km from the Sun then estimate the Sun's Mass. You will require Newton's Law of Gravitational Attraction. And note that one year $\approx$ 365.25 days.

Source(s):

- http://www. Maths Grinds .ie/
- http://www.examinations.ie/

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.

Answers:

1. $2\sqrt{g}$
2. 7 rad/s
3. $\sqrt{\frac{3g}{2}}$
4. N/A
5. 0.43 m
6. $2\pi\sqrt{\frac{h}{g}}$
7. $\frac{8r}{15}$
8. (i) N/A
   (ii) 63.6°
9. (i) $\sqrt{\frac{4g}{3}}$
   (ii) $\sqrt{8g}$
10. (i) 1.4 m/s
    (ii) $\frac{14}{15}$
11. (i) N/A
    (ii) N/A
12. The Tension at the bottom is $6g$ N more than at the top.
13. $\approx 2 \times 10^{30}$ kg