Simple Harmonic and Circular Motion – Further Simple Harmonic Motion

1. * A particle of mass 5 kg is suspended from a fixed point by a light elastic string which hangs vertically. The elastic constant of the string is 500 N/m. The mass is pulled down a vertical distance of 20 cm from the equilibrium position and is then released from rest.

   (i) Show that the motion is simple harmonic.

   (ii) Find the speed and acceleration of the mass 0.1 s after it is released from rest.

2. * A particle of mass m kg is suspended from a fixed point p by a light elastic string. The extension of the string is d when the particle is in equilibrium. The particle is then displaced vertically from the equilibrium position a distance not greater than d and is then released from rest.

   (i) Show that the motion is simple harmonic.

   (ii) Find, in terms of d, the period of the motion.

3. * A light elastic string of natural length a and elastic constant k is fixed at one end to a point o on a smooth horizontal table. A particle of mass m is attached to the other end of the string. Initially the particle is held at rest on the table at a distance $2a$ from o, and is then released. Show that the time taken for the particle to reach o is $\sqrt{\frac{m}{k}}\left(1+\frac{\pi}{2}\right)$.

4. * A mass m kg is suspended from a fixed point p by a light elastic string of natural length l and elastic constant $\frac{4mg}{l}$.

   (i) Find the distance of the equilibrium position from the point p, in terms of l.

   (ii) The particle is pulled down until it is a distance $\frac{7}{4}l$ vertically below p and is then released from rest. Find the time taken, in terms of l for the string to go slack.

5. * A particle of mass 0.3 kg is attached to the midpoint of a light elastic string of natural length 1 m and elastic constant k. The string is then stretched between two points a and b. The point a is 2 m vertically above b. Find

   (i) the extensions of the two parts of the string, in terms of k, when the system is in equilibrium.

   (ii) the minimum value of k which will ensure that the lower part of the string is taut

   (iii) the period of small oscillations, in terms of k, when the particle is displaced vertically. (Assume both parts of the string remain taut.)

6. * A particle of mass 0.5 kg at rest on a smooth horizontal table is attached to two points p and q, which are 1.2 m apart, by two light elastic strings. The string attached to p has a natural length 0.4 m and elastic constant 75 N/m. The string attached to q has a natural length 0.6 m and elastic constant 50 N/m.

   (i) Find the equilibrium position.

   (ii) Prove that if the particle is displaced in the direction pq, through such distance that neither string goes slack and is then released, it moves with simple harmonic motion.

7. * A light perfectly elastic string of natural length a and elastic constant k is fastened at one end p to a fixed point of a smooth horizontal table, and a particle of mass m is attached to the other end. The particle is held on the table at a distant $2a$ from p and then released. Prove that the particle executes simple harmonic motion while the string is taut and also prove that the particle reaches p after $\left(\frac{\pi}{2}+1\right)\sqrt{\frac{m}{k}}$ seconds.

8. * A heavy particle is hung from two points on the same horizontal line and a distance $2d$ apart by means of two light elastic strings of natural length l_1, l_2 and elastic constant k_1, k_2 respectively. In the equilibrium position the two strings make equal angles θ with the vertical. Prove that $\sin\theta = \dfrac{d\left(k_1-k_2\right)}{k_1 l_1 - k_2 l_2}$

9. * A particle of mass 5 kg is connected by a light elastic string of natural length 2 m and elastic constant 140 N/m to a fixed point q on a rough horizontal surface where the coefficient of friction is 1. The particle is released from rest at a point 3 m from q. Prove that the motion is Simple Harmonic Motion when it is $2.35+x$ m from q. Find the position of the centre of Simple Harmonic Motion and also find the amplitude.

10. * A table moves in a horizontal plane with simple harmonic motion. The table completes N oscillations per minute. Find, in terms of μ and N, the greatest allowable amplitude of the motion if an object placed on the table is not to slip, where μ is the coefficient of friction.

11. * A rectangular block of wood of mass 20 kg and height 2 m floats in a liquid. The block experiences an upward force of $400d$ N, where d is the depth, in metres, of the bottom of the block below the surface. Find

    (a) value of d when the block is in equilibrium

    (b) the period of the motion of the block if it is pushed down 0.3 m

Source(s):

- http://www. Maths Grinds .ie/
- http://www.examinations.ie/

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.

Answers:

1. (i) N/A
   (ii) 1.68 m/s and 10.8 m/s^2
2. (i) N/A
   (ii) $2\pi\sqrt{\frac{d}{g}}$
3. N/A
4. (i) $\frac{5}{4}l$
   (ii) $\frac{\pi}{3}\sqrt{\frac{l}{g}}$
5. (i) $x_1 = \frac{k+0.3g}{2k}$ and $x_2 = \frac{k-0.3g}{2k}$
   (ii) $0.3g$
   (iii) $\pi\sqrt{\frac{0.6}{k}}$
6. (i) 0.48 m from p
   (ii) N/A
7. N/A
8. N/A
9. The Centre is 2.35 m from q and the amplitude is 0.65 m
10. $\frac{900\mu g}{\pi^2 N^2}$
11. (a) 0.49
    (b) $\frac{\pi}{\sqrt{5}}$ s