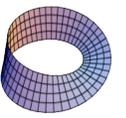
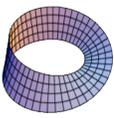


Simple Harmonic and Circular Motion – Further Simple Harmonic Motion

- * A particle of mass 5 kg is suspended from a fixed point by a light elastic string which hangs vertically. The elastic constant of the string is 500 N/m. The mass is pulled down a vertical distance of 20 cm from the equilibrium position and is then released from rest.
 - Show that the motion is simple harmonic.
 - Find the speed and acceleration of the mass 0.1 s after it is released from rest.
- * A particle of mass m kg is suspended from a fixed point p by a light elastic string. The extension of the string is d when the particle is in equilibrium. The particle is then displaced vertically from the equilibrium position a distance not greater than d and is then released from rest.
 - Show that the motion is simple harmonic.
 - Find, in terms of d , the period of the motion.
- * A light elastic string of natural length a and elastic constant k is fixed at one end to a point o on a smooth horizontal table. A particle of mass m is attached to the other end of the string. Initially the particle is held at rest on the table at a distance $2a$ from o, and is then released. Show that the time taken for the particle to reach o is $\sqrt{\frac{m}{k}} \left(1 + \frac{\pi}{2}\right)$.
- * A mass m kg is suspended from a fixed point p by a light elastic string of natural length l and elastic constant $\frac{4mg}{l}$.
 - Find the distance of the equilibrium position from the point p, in terms of l .
 - The particle is pulled down until it is a distance $\frac{7}{4}l$ vertically below p and is then released from rest. Find the time taken, in terms of l for the string to go slack.
- * A particle of mass 0.3 kg is attached to the midpoint of a light elastic string of natural length 1 m and elastic constant k . The string is then stretched between two points a and b . The point a is 2 m vertically above b . Find
 - the extensions of the two parts of the string, in terms of k , when the system is in equilibrium.
 - the minimum value of k which will ensure that the lower part of the string is taut
 - the period of small oscillations, in terms of k , when the particle is displaced vertically. (Assume both parts of the string remain taut.)
- * A particle of mass 0.5 kg at rest on a smooth horizontal table is attached to two points p and q, which are 1.2 m apart, by two light elastic strings. The string attached to p has a natural length 0.4 m and elastic constant 75 N/m. The string attached to q has a natural length 0.6 m and elastic constant 50 N/m.
 - Find the equilibrium position.
 - Prove that if the particle is displaced in the direction pq, through such distance that neither string goes slack and is then released, it moves with simple harmonic motion.
- * A light perfectly elastic string of natural length a and elastic constant k is fastened at one end p to a fixed point of a smooth horizontal table, and a particle of mass m is attached to the other end. The particle is held on the table at a distant $2a$ from p and then released. Prove that the particle executes simple harmonic motion while the string is taut and also prove that the particle reaches p after $\left(\frac{\pi}{2} + 1\right) \sqrt{\frac{m}{k}}$ seconds.
- * A heavy particle is hung from two points on the same horizontal line and a distance $2d$ apart by means of two light elastic strings of natural length l_1, l_2 and elastic constant k_1, k_2 respectively. In the equilibrium position the two strings make equal angles θ with the vertical. Prove that $\sin \theta = \frac{d(k_1 - k_2)}{k_1 l_1 - k_2 l_2}$
- * A particle of mass 5 kg is connected by a light elastic string of natural length 2 m and elastic constant 140 N/m to a fixed point q on a rough horizontal surface where the coefficient of friction is 1. The particle is released from rest at a point 3 m from q. Prove that the motion is Simple Harmonic Motion when it is $2.35 + x$ m from q. Find the position of the centre of Simple Harmonic Motion and also find the amplitude.
- * A table moves in a horizontal plane with simple harmonic motion. The table completes N oscillations per minute. Find, in terms of μ and N , the greatest allowable amplitude of the motion if an object placed on the table is not to slip, where μ is the coefficient of friction.
- * A rectangular block of wood of mass 20 kg and height 2 m floats in a liquid. The block experiences an upward force of $400d$ N, where d is the depth, in metres, of the bottom of the block below the surface. Find
 - value of d when the block is in equilibrium
 - the period of the motion of the block if it is pushed down 0.3 m



Source(s):

- <http://www.MathsGrinds.ie/>
- <http://www.examinations.ie/>

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.

Answers:

- (i) N/A
(ii) 1.68 m/s and 10.8 m/s²
- (i) N/A
(ii) $2\pi\sqrt{\frac{d}{g}}$
- N/A
- (i) $\frac{5}{4}l$
(ii) $\frac{\pi}{3}\sqrt{\frac{l}{g}}$
- (i) $x_1 = \frac{k+0.3g}{2k}$ and $x_2 = \frac{k-0.3g}{2k}$
(ii) 0.3g
(iii) $\pi\sqrt{\frac{0.6}{k}}$
- (i) 0.48 m from p
(ii) N/A
- N/A
- N/A
- The Centre is 2.35 m from q and the amplitude is 0.65 m
- $\frac{900\mu g}{\pi^2 N^2}$
- (a) 0.49
(b) $\frac{\pi}{\sqrt{5}}$ s