Moment of Inertia – Planes and Wheels

1. * An annulus is created when a central hole of radius b is removed from a uniform circular disc of radius a. The mass of the annulus (shaded area) is M.
   (a) Show that the moment of inertia of the annulus about an axis through its centre and perpendicular to its plane is $\frac{1}{2}M\left(a^2+b^2\right)$.
   (b) The annulus rolls, from rest, down an incline of 30° Find its angular velocity, in terms of g, a and b, when it has rolled a distance $\frac{a}{2}$.
2. * Consider a wheel of inner and outer radii of $6a$ and $8a$ respectively. The wheel is supported by four spokes each of mass m and of course of length $6a$. The total mass of the wheel and four spokes is $18m$.
   (a) Show that the mass per unit area of the wheel $\frac{m}{2\pi a^2}$.
   (b) Show that the total moment of inertia of the wheel and four spokes about an axis through the centre and perpendicular to the plane of the wheel is $748ma^2$.
   (c) If $m=100$ grammes and $a=10$ cm, how much work is done in bringing this wheel and spokes to rest from 6000 revolutions per minute?
3. * A uniform disc of mass m and radius r rolls from rest, without sliding, 30 m down a plane inclined at an angle of 30° to the horizontal.
   (a) Find the linear speed of the disc after rolling 30 m down the plane.
   (b) Find the time taken for the disc to roll 30 m down the plane, correct to two places of decimals.
   (c) The disc is now replaced by a hoop of mass m and radius r. The hoop rolls from rest, without sliding, 30 m down the plane. Show that the ratio of the acceleration down the plane of the hoop to that of the disc is $\frac{3}{4}$
4. ** A rough plane is inclined at an angle $\tan^{-1}\frac{3}{4}$. A disc of mass m and radius r rolls straight down the plane without slipping.
   (a) Find its acceleration in terms of g.
   (b) Assuming that it is actually on the point of slipping, find the coefficient of friction between the disc and the plane.
5. ** A disc of radius 2 m has a disc or radius 1 m remove from its center. The hoop thus formed then rolls straight down a rough inclined at an angle 45° to the horizontal.
   (a) If the hoop does not slip, find its acceleration.
   (b) Find the minimum value of the coefficient of friction.

Source(s):

- http://www.examinations.ie/
- http://www. Maths Grinds .ie/

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.
- Questions marked with a double asterisk ** are taken from Oliver Murphy's Book "Fundamental Applied Maths".

Answers:

1. (a) N/A
   (b) $\sqrt{\frac{ga}{3a^2+b^2}}$
2. (a) N/A
   (b) N/A
   (c) $14960\pi^2$ J
3. (a) 14 m/s
   (b) 4.29 s
   (c) $\frac{3}{4}$
4. (a) $\frac{2}{5}g$
   (b) $\frac{1}{4}$
5. (a) $\frac{2g}{3\sqrt{2}}$
   (b) $\frac{5}{13}$