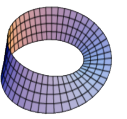
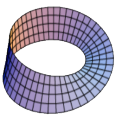


## Moment of Inertia – Planes and Wheels

1. \* An annulus is created when a central hole of radius  $b$  is removed from a uniform circular disc of radius  $a$ . The mass of the annulus (shaded area) is  $M$ .
  - (a) Show that the moment of inertia of the annulus about an axis through its centre and perpendicular to its plane is  $\frac{1}{2}M(a^2 + b^2)$ .
  - (b) The annulus rolls, from rest, down an incline of  $30^\circ$ . Find its angular velocity, in terms of  $g$ ,  $a$  and  $b$ , when it has rolled a distance  $\frac{a}{2}$ .
2. \* Consider a wheel of inner and outer radii of  $6a$  and  $8a$  respectively. The wheel is supported by four spokes each of mass  $m$  and of course of length  $6a$ . The total mass of the wheel and four spokes is  $18m$ .
  - (a) Show that the mass per unit area of the wheel is  $\frac{m}{2\pi a^2}$ .
  - (b) Show that the total moment of inertia of the wheel and four spokes about an axis through the centre and perpendicular to the plane of the wheel is  $748ma^2$ .
  - (c) If  $m = 100$  grammes and  $a = 10$  cm, how much work is done in bringing this wheel and spokes to rest from 6000 revolutions per minute?
3. \* A uniform disc of mass  $m$  and radius  $r$  rolls from rest, without sliding, 30 m down a plane inclined at an angle of  $30^\circ$  to the horizontal.
  - (a) Find the linear speed of the disc after rolling 30 m down the plane.
  - (b) Find the time taken for the disc to roll 30 m down the plane, correct to two places of decimals.
  - (c) The disc is now replaced by a hoop of mass  $m$  and radius  $r$ . The hoop rolls from rest, without sliding, 30 m down the plane. Show that the ratio of the acceleration down the plane of the hoop to that of the disc is  $\frac{3}{4}$ .
4. \*\* A rough plane is inclined at an angle  $\tan^{-1} \frac{3}{4}$ . A disc of mass  $m$  and radius  $r$  rolls straight down the plane without slipping.
  - (a) Find its acceleration in terms of  $g$ .
  - (b) Assuming that it is actually on the point of slipping, find the coefficient of friction between the disc and the plane.
5. \*\* A disc of radius 2 m has a disc of radius 1 m removed from its center. The hoop thus formed then rolls straight down a rough incline at an angle  $45^\circ$  to the horizontal.
  - (a) If the hoop does not slip, find its acceleration.
  - (b) Find the minimum value of the coefficient of friction.



## Source(s):

- <http://www.examinations.ie/>
- <http://www.MathsGrinds.ie/>

## Further Information:

- Questions marked with an asterisk \* are past Leaving Cert Exam questions.
- Questions marked with a double asterisk \*\* are taken from Oliver Murphy's Book "Fundamental Applied Maths".

## Answers:

- (a) N/A

(b)  $\sqrt{\frac{ga}{3a^2 + b^2}}$
- (a) N/A

(b) N/A

(c)  $14960\pi^2 \text{ J}$
- (a) 14 m/s

(b) 4.29 s
- (c)  $\frac{3}{4}$

(a)  $\frac{2}{5}g$

(b)  $\frac{1}{4}$

(a)  $\frac{2g}{3\sqrt{2}}$

(b)  $\frac{5}{13}$