Moment of Inertia – Pulleys

1. * A string is wrapped around a smooth pulley wheel of radius r. A particle of mass m is attached to the string. The axis of rotation of the wheel is horizontal, perpendicular to the wheel, and passes through the centre of the wheel. The moment of inertia of the wheel about the axis is I. The particle is released from rest and moves vertically downwards.

   (a) Find, in terms of I, m and r, the tension in the string.

   (b) If the acceleration of the particle is $\frac{g}{5}$ find the mass of the pulley wheel in terms of m.

2. * Masses of 4 kg and 6 kg are suspended from the ends of a light inextensible string which passes over a pulley. The axis of rotation of the pulley is horizontal, perpendicular to the pulley, and passes through the centre of the pulley. The moment of inertia of the pulley is 0.08 kg m ^{2} and its radius is 20 cm. The particles are released from rest and move vertically. When each mass has acquired a speed of 1 m/s, find

   (a) the common acceleration of the masses

   (b) the tensions in the vertical portions of the string.

3. * A smooth pulley wheel has a mass of 3 kg and a radius of 0.3 m. One end of a light inextensible rope is attached to a point p on the rim of the wheel. A particle of mass 0.2 kg attached to the other end of the rope hangs freely. The axis of rotation of the wheel is horizontal, perpendicular to the wheel, and passes through the centre of the wheel. The particle is released from rest and moves vertically downwards. When the particle has acquired a speed of 1.2 m/s, find

   (a) the kinetic energy gained by the wheel

   (b) the distance descended by the particle, correct to two decimal places.

4. ** A fixed pulley is made of a disc of mass m and radius r which is free to rotate without friction about an axis through its centre, perpendicular to its plane. One end of a light inextensible string is attached to a point on the rim of the disc. The string is then would around the rim and a particle of mass $2m$ hangs freely from the other end.

   (a) Find the tension in the string.

   (b) Find the acceleration of the $2m$ mass.

5. ** A fixed pulley of mass m and radius r turns on a frictionless axis. Masses m and $4m$ hand freely from the ends of a light inextensible string which passes over the pulley. Find the common acceleration of the particles.

6. ** A hoop is formed by removing a disc of radius r from the middle of a disc of radius R. A light inextensible string is wrapped around the outer rim of this hoop. The free end is then fixed to a point on a ceiling, so that the hoop hands freely under gravity. If the hoop and the string remain vertical,

   (a) find the tension in the string

   (b) and the acceleration of the hoop.

Source(s):

- http://www.examinations.ie/
- http://www. Maths Grinds .ie/

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.
- Questions marked with a double asterisk ** are taken from Oliver Murphy's Book "Fundamental Applied Maths".

Answers:

1. (a) $\frac{mgI}{I+mr^2}$
   (b) $8M$
2. (a) $\frac{g}{6}$
   (b) 49 N and 45.7 N
3. (a) 1.08
   (b) 0.62
4. (a) $\frac{2}{5}mg$
   (b) $\frac{4}{5}g$
5. $\frac{6}{11}g$
6. (a) $\frac{Mg\left(R^2+r^2\right)}{3R^2+r^2}$
   (b) $\frac{2R^2g}{3R^2+r^2}$