Moment of Inertia – Pendulums

1. * A uniform circular lamina, of mass 8m and radius r, can turn freely about a horizontal axis through P perpendicular to the plane of the lamina. Particles each of mass m are fixed at four points which are on the circumference of the lamina and which are the vertices of square PQRS. The compound body is set in motion. Find

   (a) the period of small oscillations of the compound pendulum

   (b) the length of the equivalent simple pendulum.

2. * A square lamina PQRS, of side 60 cm and mass m, can turn freely about a horizontal axis through P perpendicular to the plane of the lamina. The lamina is released from rest when PS is horizontal.

   (a) Find the angular velocity of the lamina when PR is vertical.

   (b) A mass m is attached to the lamina at R. The compound pendulum is set in motion. Find the period of small oscillations of the compound pendulum and hence, or otherwise, find the length of the equivalent simple pendulum.

3. * Three equal uniform rods, each of length $2l$ and mass m, form the sides of an equilateral triangle abc.

   (a) Find the moment of inertia of the frame abc about an axis through a perpendicular to the plane of the triangle.

   (b) The triangular frame abc is attached to a smooth hinge at a about which it can rotate in a vertical plane. The frame is held with ab horizontal, and c below ab, and then released from rest. Find the maximum angular velocity of the triangle in the subsequent motion.

4. * A uniform square lamina abcd of side $2r$ oscillates in its own plane about a horizontal axis through a, perpendicular to its plane.

   (a) If the period of small oscillations is $2\pi\sqrt{\dfrac{8}{3g}}$ find the value of r.

   (b) If the lamina is released from rest when ab is vertical, find the maximum velocity of corner c in the subsequent motion.

5. * A uniform rod of mass 3 m and length 1.2 metres can turn freely in a vertical plane about a horizontal axis through one end. The rod oscillates through an angle of $120°$, as shown in the diagram.

   (a) Find the angular velocity of the rod when the rod is vertical.

   (b) Find, in terms of m , the vertical thrust on the axis when the rod is vertical.

6. * A uniform rod [pq], of mass 9m and length 2l, has a particle of mass 2m attached at q. The system is free to rotate about a smooth horizontal axis through p. The rod is held in a horizontal position and is then given an initial angular velocity $\sqrt{\dfrac{3g}{2l}}$ downwards. The diagram shows the rod [pq] when it makes an angle θ with the horizontal.

   (a) Show that when the rod makes an angle below its initial horizontal position, its angular velocity is $\sqrt{\dfrac{g\,(15+13\sin\theta)}{10l}}$

   (b) Hence, or otherwise, show that the rod performs complete revolutions about p.

7. * Three rods, each of mass m and length $2l$, are jointed together at their ends to form a triangle pqr. The triangle is free to rotate about a fixed horizontal axis through p perpendicular to its plane.

   (a) Find, in terms of l, the period of small oscillations.

   (b) Find, in terms of l, the length of the equivalent simple pendulum.

8. * State the Parallel and Perpendicular Axes Theorems. Hence, or otherwise, find the moment of inertia of a uniform disc of mass m and radius r about an axis tangential to its circumference and lying in the plane of the disc. The disc may rotate smoothly about a fixed horizontal axis tangential to its circumference and lying in the plane of the disc. The disc is held in the horizontal plane and then released from rest. Find the angular speed of the disc when it has rotated through an angle θ, in terms of r and θ. Find, in terms of r, the maximum angular speed in the subsequent motion.

Please turn over – more questions on the other side

9. * A uniform rod of mass m is free to rotate in a vertical plane about an axis which is perpendicular to the rot and 0.32 m from its centre of gravity. For small oscillations the rod has the same period as simple pendulum of length 0.5 m.
   (a) Find the length of the rod.
   (b) For what other distance between the axis and centre of gravity will the period be the same?
   (c) Where must the axis be located to give a minimum period?
10. * A uniform rod [ab], of mass m and length $2l$, is free to rotate in a vertical plane about a fixed horizontal axis at a, with a particle of mass $3m$ attached to the rod at b. The system is released from rest with the rod vertical and the end b above a.
   (a) Show that the angular velocity of the rod when next is is vertical is $\sqrt{\frac{21g}{10l}}$.
   (b) If at this point the mass falls off, find the height to which the end b subsequently rises.
11. * A thin uniform rod AB of mass m, and length $2a$ can turn freely in a vertical plane, about a fixed horizontal axis through A. A uniform circular disc of mass $24m$ and radius $\frac{a}{3}$ has its centre C clamped to the rod so that the length $AC = x$ and the plane of the disc passes through the axis of rotation.
   (a) Show that the moment of inertia of the system about the axis is $2m\left(a^2 + 12x^2\right)$.
   (b) The system makes small oscillations. Find the period and show that the period is a minimum when $x = \frac{a}{4}$.
12. * A uniform circular disc of radius r can move freely about a smooth pivot at a point a on its circumference. When its plane is vertical and the diameter [ab] is horizontal the point b is given a velocity p vertically downwards. Find
   (a) the angular velocity of the disc when b is vertically below a
   (b) the value of p, in terms of r, if b just reaches the point where it is vertically above a.

Source(s):

- http://www.examinations.ie/
- http://www. Maths Grinds .ie/

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.
- Questions marked with a double asterisk ** are taken from Oliver Murphy's Book "Fundamental Applied Maths".

Answers:

1. (a) $2\pi\sqrt{\frac{5r}{3g}}$
   (b) $\frac{5r}{3}$
2. (a) 3.19 rad/s
   (b) 0.75 m
3. (a) $6ml^2$
   (b) $\sqrt{\frac{g\sqrt{3}}{3l}}$
4. (a) $sqrt2$
   (b) 5.87 m/s
5. (a) 3.5 rad/s
   (b) $5.25mg$ N
6. (a) N/A
   (b) N/A
7. (a) $2\pi\sqrt{\frac{\sqrt{3}l}{g}}$
   (b) $\sqrt{3}l$
8. $\frac{5}{4}mr^2$, $\sqrt{\frac{8g\sin\theta}{5r}}$ and $\sqrt{\frac{8g}{5r}}$
9. (a) 0.83 m
   (b) 18 cm
   (c) 24 cm
10. (a) N/A
   (b) $\frac{14l}{5}$
11. (a) N/A
   (b) $2\pi\sqrt{\frac{2\left(a^2+12x^2\right)}{24gx+ga}}$
12. (a) $\sqrt{\frac{16gr+4p}{12r}}$
   (b) $4\sqrt{\frac{3g}{r}}$

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