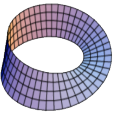
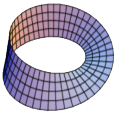


Ordinary Differential Equations – First Order

1. * Solve the differential equation $y \frac{dy}{dx} = x + xy^2$ given that $y = 0$ when $x = 0$.
2. * If $x^2 \frac{dy}{dx} - xy = 7y$ and $y = 1$ when $x = 1$, find the value of y when $x = 2$.
3. * Solve the differential equation $\frac{dy}{dx} = \frac{1}{xy} + \frac{y}{x}$ given that $y = \sqrt{3}$ when $x = 1$.
4. * If $x^2 y \frac{dy}{dx} + y \frac{dy}{dx} = 1$ and $y = 0$ when $x = 0$, find the value of x when $y = \sqrt{\frac{\pi}{2}}$.
5. * Solve the differential equation $\frac{dy}{dx} = y^2 \sin x$ given that $y = 1$ when $x = \frac{\pi}{2}$.
6. * Solve the differential equation $x \frac{dy}{dx} - xy - y = 0$ given that $y = 1$ when $x = 1$.
7. * Solve the differential equation $x^2 \frac{dy}{dx} - y + 4 = 0$ given that $y = 5$ when $x = 1$.
8. * Solve the differential equation $\frac{dy}{dx} = \frac{xy}{2x^2 - 3}$ given that $y = 1$ when $x = \sqrt{2}$.
9. * Solve the differential equation $\frac{dy}{dx} = e^{x-y}$ given that $y = \ln 4$ when $x = 0$.
10. * If $x \frac{dy}{dx} + xy \frac{dy}{dx} - 1 = 0$ and $y = 2$ when $x = e$, find,
correct to two places of decimals, the positive value of y when $x = e^2$.
11. * Consider the substitution $u = 1 + x$.
 - (a) If $u = 1 + x$ then what is the value of $\frac{du}{dx}$? Hence or otherwise find du in terms of x .
 - (b) Calculate $\int \frac{x}{1+x} dx$.
 - (c) Hence, solve the differential equation $\frac{dy}{dx} = \frac{xy}{1+x}$ given that $y = e$ when $x = 0$.
12. * Find $\frac{d}{dx} \left(\frac{y}{x} \right)$. Hence, or otherwise,
solve the differential equation $\frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = \frac{1}{x}$ given that $y = 1$ when $x = 1$.



Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.

Summary:

- Separation of Variable: $f\left(x, y, \frac{dy}{dx}\right) = 0 \Rightarrow \int g(x) dx = \int h(y) dy$
- $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$

Answers:

1. $y = \sqrt{e^{x^2} - 1}$

2. 66.23

3. $y = \sqrt{4x^2 - 1}$

4. $x = 1$

5. $y = \frac{1}{1 + \cos x}$

6. $y = xe^{x-1}$

7. $y = e^{1-\frac{1}{x}} + 4$

8. $y = (2x^2 - 3)^{\frac{1}{4}}$

9. $y = \ln(e^x + 3)$

10. 2.32

11. (a) $\frac{du}{dx} = 1, du = dx$

(b) $1 + x - \ln(1 + x) + C$

(c) $y = \frac{e^{x+1}}{x+1}$

12. $\frac{d}{dx} \left(\frac{y}{x} \right) = \frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2}$ and $y = x(1 + \ln x)$