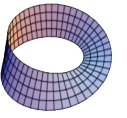
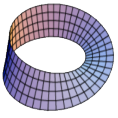


Ordinary Differential Equations – Second Order Practical

- A one kg mass is subjected to a constant force of two Newtons. If the mass starts at rest and at the origin then what is the distance s travelled after t seconds?
- * A mass m is fired horizontally through a block of resistive gel. The resistance to motion is mkv^2 N (where v m/s is the speed). The particle enters the gel at a speed of 1000 m/s and $\frac{1}{100}$ seconds later exits the gel at a speed of 10 m/s.
 - Show that $k = \frac{99}{10}$.
 - Show that length of the block is $\frac{99}{10} \ln 100$ m
- * A car of mass 490 kg moves along a straight level horizontal road against a resistance of $70v$ N, where v m/s is the speed of the car. The engine exerts a constant power of 63 kW.
 - Show that the equation of motion is $7\frac{dv}{dt} = \frac{900 - v^2}{v}$
 - Calculate, correct to two decimal places, the time it takes the car to increase its speed from 10 m/s to 20 m/s.
- * A particle starts from rest moves in a horizontal line. Its speed v at time t is given by the equation $\frac{dv}{dt} = 100 - v$. Find the time taken for the speed of the particle to increase from 25 m/s to 75 m/s.
- * A car of mass m kg is travelling along a level road. The resistance to motion is mkv^2 N, where v m/s is the speed. When the car is travelling at 14 m/s, the engine cuts out. Ten seconds after the engine cuts out, the speed of the car is 7 m/s.
 - Show that $k = \frac{1}{140}$.
 - The car travels a distance s metres in the first T seconds after the engine cuts out. Show that $s = 140 \ln \left(1 + \frac{T}{10}\right)$.
- * The deceleration of a particle moving in a straight line with speed v m/s has magnitude $4e^{\frac{v}{6}}$ m/s². The particle has an initial speed of 6 m/s.
 - Find the time t_1 for the speed to decrease to 3 m/s.
 - Find the time t_2 for the particle to come to rest.
 - Deduce that $\frac{t_2 - t_1}{t_1} = \sqrt{e}$.
- * The rocket engine of a 12 tonne missile produces a thrust of 180.1 kN. The missile is launched in a vertical direction. The air resistance is v^2 N where v is the speed of the missile.
 - Find the speed of the missile after 30 seconds.
 - Find the percentage error in this speed if air resistance is ignored.
- * A mass m is dropped from rest a height. The mass experience two forces, downward gravity and an upward resistive force that is proportional to its speed.
 - How long until the mass is travelling a speed of $\frac{g}{2k}$?
 - What is the terminal velocity of the mass?
- * A 1000 kg car of power 75 kW experience a constant friction force of 15000 while it in motion.
 - What is the equation of motion for the car? Hint: Recall Power = Force time Velocity
 - If the car starts at rest than how long would it take to reach a speed of 25 m/s?



Source(s):

- <http://www.MathsGrinds.ie/>
- <http://www.examinations.ie/>

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.

Answers:

1. $s = t^2$
2. N/A
3. (a) N/A
(b) 1.65 s
4. 1.1 s
5. N/A
6. (a) $\frac{3}{2} \left(\frac{1}{\sqrt{e-\frac{1}{e}}} \right)$
(b) $\frac{3}{2} \left(\frac{1}{1-\frac{1}{e}} \right)$
7. (a) 138.64 m/s
(b) 12.7 %
8. (a) $\frac{\ln 2}{k}$?
(b) $\frac{g}{k}$
9. (a) $\frac{75}{v} - 15 = \frac{dv}{dt}$
(b) 6.44 seconds