Ordinary Differential Equations – Second Order Practical

1. A one kg mass is subjected to a constant force of two Newtons. If the mass starts at rest and at the origin then what is the distance s travelled after t seconds?

2. * A mass m is fired horizontally through a block of resistive gel. The resistance to motion is mkv^2 N (where v m/s is the speed). The particle enters the gel at a speed of 1000 m/s and $\frac{1}{100}$ seconds later exits the gel at a speed of 10 m/s.

   (a) Show that $k = \frac{99}{10}$.

   (b) Show that length of the block is $\frac{99}{10}\ln 100$ m

3. * A car of mass 490 kg moves along a straight level horizontal road against a resistance of $70v$ N, where v m/s is the speed of the car. The engine exerts a constant power of 63 kW.

   (a) Show that the equation of motion is $7\frac{dv}{dt} = \frac{900 - v^2}{v}$

   (b) Calculate, correct to two decimal places, the time it takes the car to increase its speed from 10 m/s to 20 m/s.

4. * A particle starts from rest moves in a horizontal line. Its speed v at time t is given by the equation $\frac{dv}{dt} = 100 - v$. Find the time taken for the speed of the particle to increase from 25 m/s to 75 m/s.

5. * A car of mass m kg is travelling along a level road. The resistance to motion is mkv^2 N, where v m/s is the speed. When the car is travelling at 14 m/s, the engine cuts out. Ten seconds after the engine cuts out, the speed of the car is 7 m/s.

   (a) Show that $k = \frac{1}{140}$.

   (b) The car travels a distance s metres in the first T seconds after the engine cuts out. Show that $s = 140\ln\left(1 + \frac{T}{10}\right)$.

6. * The deceleration of a particle moving in a straight line with speed v m/s has magnitude $4e^{\frac{v}{6}}$ m/s^2. The particle has an initial speed of 6 m/s.

   (a) Find the time t_1 for the speed to decrease to 3 m/s.

   (b) Find the time t_2 for the particle to come to rest.

   (c) Deduce that $\frac{t_2 - t_1}{t_1} = \sqrt{e}$.

7. * The rocket engine of a 12 tonne missile produces a thrust of 180.1 kN. The missile is launched in a vertical direction. The air resistance is v^2 N where v is the speed of the missile.

   (a) Find the speed of the missile after 30 seconds.

   (b) Find the percentage error in this speed if air resistance is ignored.

8. * A mass m is dropped from rest a height. The mass experience two forces, downward gravity and an upward resistive force that is proportional to its speed.

   (a) How long until the mass is travelling a speed of $\frac{g}{2k}$?

   (b) What is the terminal velocity of the mass?

9. * A 1000 kg car of power 75 kW experience a constant friction force of 15000 while it in motion.

   (a) What is the equation of motion for the car? Hint: Recall Power = Force time Velocity

   (b) If the car starts at rest than how long would it take to reach a speed of 25 m/s?

Source(s):

- http://www. Maths Grinds .ie/
- http://www.examinations.ie/

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.

Answers:

1. $s = t^2$
2. N/A
3. (a) N/A
   (b) 1.65 s
4. 1.1 s
5. N/A
6. (a) $\frac{3}{2}\left(\frac{1}{\sqrt{e}-\frac{1}{e}}\right)$
   (b) $\frac{3}{2}\left(\frac{1}{1-\frac{1}{e}}\right)$
   (c) N/A
7. (a) 138.64 m/s
   (b) 12.7 %
8. (a) $\frac{\ln 2}{k}$?
   (b) $\frac{g}{k}$
9. (a) $\frac{75}{v} - 15 = \frac{dv}{dt}$
   (b) 6.44 seconds