Ordinary Differential Equations – Further Second Order Practical

1. A particle of mass m is subjected to a resistive force that is proportional to its position s. That is, a force of size ks where k is a positive constant. The mass starts at rest at a position of A m from the origin. What is the distance s travelled after t seconds?

2. * A particle travelling in a straight line has a deceleration of $\frac{v^2}{400} + 16$ m/s^2 where v is its speed at any time t. If its initial speed is 40 m/s, find

   (a) the distance travelled before it comes to rest

   (b) the average speed of the particle during the motion.

3. * The acceleration of a cyclist freewheeling down a slight hill is $0.12 - 0.0006v^2$ m/s^2 where the velocity v is in metres per second. The cyclist starts from rest at the top of the hill. Find

   (a) the speed of the cyclist after travelling 120 m down the hill

   (b) the time taken by the cyclist to travel the 120 m if his average speed is 2.65 m/s.

4. * A particle of mass m is projected vertically upwards with speed u. The air resistance is kv^2 per unit mass when the speed is v. The maximum height reached by the particle is $\frac{\ln 4}{2k}$. Find the value of

   (a) u in terms of k

   (b) k if the time to reach the greatest height is $\frac{\pi}{3}$ seconds.

5. * A train of mass 200 tonnes moves along a straight level track against a resistance of $400v^2$, where v m/s is the speed of the train. The engine exerts a constant power of P kW. The acceleration of the train is $\frac{8000 - v^3}{500v}$.

   (a) Find the value of P.

   (b) The train travels a distance 69.07 m while its speed increases from 10 m/s to v_1 m/s. Find the value of v_1.

6. * The acceleration of a racing car at a speed of v m/s is $\left(1 - \frac{v^2}{3200}\right)$ m/s^2. The car starts from rest. Calculate correct to two decimal places the

   (a) speed when it has travelled 1500 m from rest

   (b) maximum speed.

7. * The acceleration of a particle moving horizontally in a straight line is $\frac{1}{x^3}$ away from a fixed point o, where x is its distance from o. The particle starts from rest at $x = 1$. Calculate the velocity when $x = \frac{4}{3}$

8. * A mass of 9 kg is suspended at the lower end of a light vertical rope. Initially the mass is at rest. The mass is pulled up vertically with an initial pull on the rope of 137.2 N. The pull diminishes uniformly at the rate of 1 N for each metre through which the mass is raised.

   (a) Show that the resultant upward force on the mass when it is x metres above its initial position is $49 - x$.

   (b) Find the speed of the mass when it has been raised 15 metres.

   (c) Find the work done by the pull on the rope when the mass has been raised by 15 m.

9. * A particle is projected vertically upwards with an initial speed of $2g$ m/s in a medium in which there is a resistance kv^2 N per unit mass where v is the speed of the particle and k is a constant, where $k > 0$.

   (a) Prove that the maximum height reached is $\frac{ln\,(1 + 4kg)}{2k}$

   (b) If the speed of the particle is g m/s when it has reached half its maximum height, find the value of k.

10. * A particle starts from rest moves in a horizontal line. Its speed v at time t is given by the equation $\frac{dv}{dt} = 100 - v$. How far does the particle travel in going from rest to a speed of 75 m/s?

11. * A particle moves in a straight line. The initial speed is u and the retardation is kv^3, where v is the speed at the time t. If s is the distance travelled in time t, prove $v = \frac{u}{1 + ksu}$

Source(s):

- http://www. Maths Grinds .ie/
- http://www.examinations.ie/

Further Information:

- Questions marked with an asterisk * are past Leaving Cert Exam questions.

Summary:

- The substitution $a = v\frac{dv}{ds}$ can be used in ODEs with no time variable stated or needed.

Answers:

1. $s = A\cos\left(\sqrt{\frac{k}{m}}t\right)$
2. (a) 44.63 m
   (b) 19.24 m/s
3. (a) 5.18 m/s
   (b) 45.3 s
4. (a) $\sqrt{\frac{3g}{k}}$ (b) $\frac{1}{g}$
5. (a) 3200
   (b) 15 m/s
6. (a) 44.12 m/s (b) 56.57 m/s
7. 0.66 m/s
8. (a) N/A
   (b) 11.76 m/s
   (c) 622.5
9. (a) N/A
   (b) $\frac{2}{g}$
10. 63.63 m
11. N/A