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LEAVING CERTIFICATE EXAMINATION, 1990

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 22 JUNE – MORNING, 9.30 – 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9{\cdot}8$ m/s^2.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) A particle is projected vertically upwards with velocity u m/s and is at a height h after t_1 and t_2 seconds respectively. Prove that

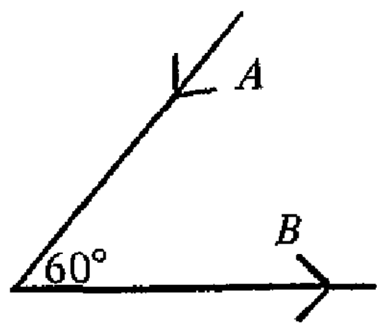
$$t_1 \cdot t_2 = \frac{2h}{g}$$

(b) A car accelerates uniformly from rest to a speed v m/s. It continues at this constant speed for t seconds and then decelerates uniformly to rest. The average speed for the journey is $\frac{3v}{4}$.

(i) Draw a speed-time graph and hence, or otherwise, prove that the time for the journey is $2t$ seconds.

(ii) If the car-driver had observed the speed limit of $\frac{1}{2}v$, find the least time the journey would have taken, assuming the same acceleration and deceleration as in (i).

2. Two straight roads intersect at an angle of $60°$. Car A moves towards the junction with uniform speed 16 m/s, while car B moves away from the junction with uniform speed 20 m/s.



(a) Calculate the velocity of A relative to B.

(b) If A is 450 m and B is 200 m from the intersection at a given moment, calculate the time interval in seconds until the cars

(i) are nearest to each other

(ii) are equidistant from the intersection.

3. A particle is projected from a point p, up a plane inclined at an angle $\tan^{-1} \frac{1}{6}$ to the horizontal. The direction of projection makes an angle α with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope.)

(i) If the particle were to strike the inclined plane horizontally at a point q, show that $\tan \alpha = \frac{3}{19}$.

(ii) If the particle were to be projected from p with the same speed but at an angle $\tan^{-1} 3$ to the inclined plane, show that it would strike the plane at right angles at q.

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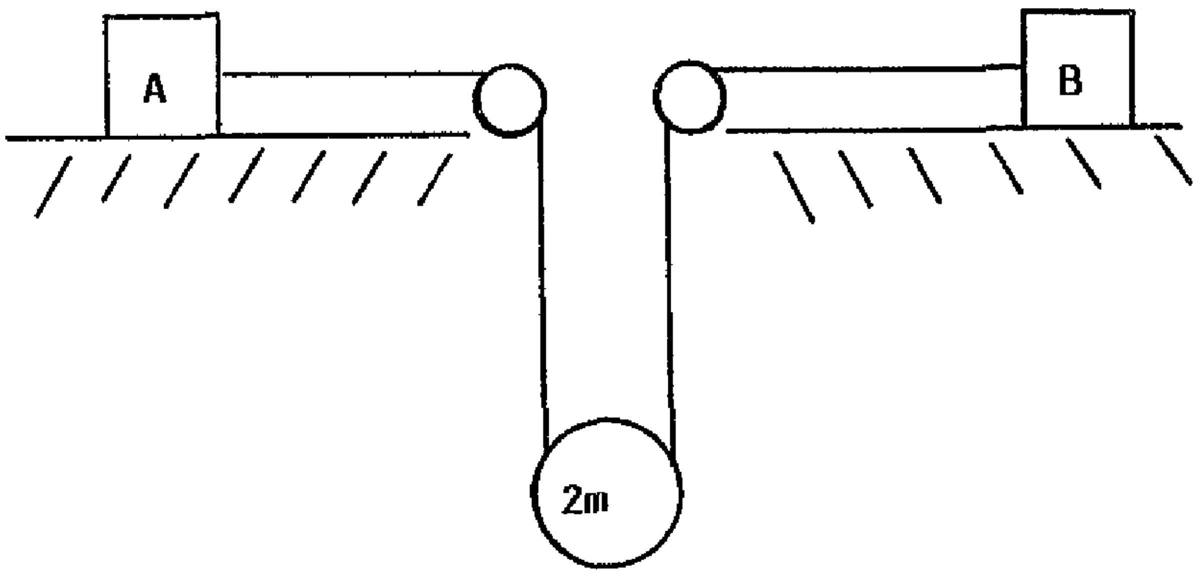
4. Two blocks A and B each of mass m kg, lie at rest on horizontal rough tables. The coefficient of friction between A and the table is μ, and between B and its table is $\frac{1}{4}$. The blocks are connected by a light inextensible string which passes under a smooth movable pulley of mass $2m$ kg.



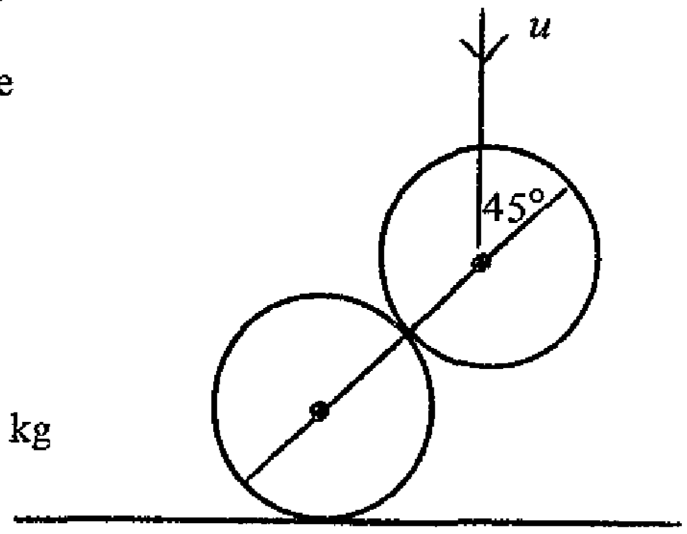
(i) Show in a diagram the forces acting on each mass when the system is released from rest.

(ii) If $\mu < \frac{3}{4}$, prove that the tension in the string is $\frac{mg(9 + 4\mu)}{16}$

(iii) Prove that A will not move if $\mu > \frac{3}{4}$.

5. State the laws governing the oblique collision of two smooth elastic spheres.

A smooth elastic sphere of mass 6 kg rests on a smooth horizontal table. A second smooth elastic sphere of mass 4 kg falls vertically on it. At the moment of impact the line of centres makes an angle of 45° with the vertical, and the velocity of the falling sphere is u. The 6 kg sphere moves horizontally after the collision.



(i) Explain why the principle of conservation of momentum may be applied horizontally.

(ii) Hence, or otherwise, prove that the speed of the 6 kg mass after impact is $\frac{u(1 + e)}{4}$ where e is the coefficient of restitution between the two spheres.

(iii) If $e = \frac{1}{3}$, prove that the loss of kinetic energy due to the impact is $\frac{2u^2}{3}$.

6. (a) A particle starts from rest, and moves with simple harmonic motion of period $6n$ seconds. Show that the particle moves from the position of maximum velocity to the position in which the velocity is half the maximum in n seconds.

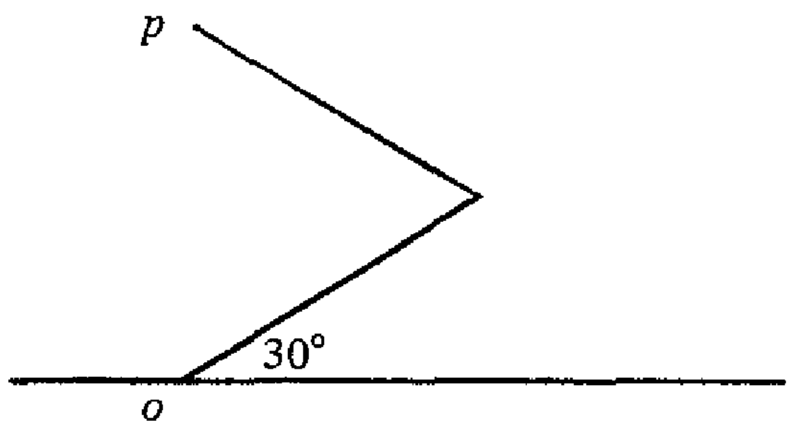
(b) The depth of water in a harbour is assumed to rise and fall with time in simple harmonic motion. On a certain day the low tide had a height of 13 m at 12.58 p.m. and the following high tide had a height of 18 m at 6.58 p.m.

If a ship requires a depth of 16·5 m of water before it can leave the harbour, find the latest time on that day that the ship can leave the harbour.

7. (a) Define
(i) limiting friction,
(ii) coefficient of friction,
(iii) angle of friction.

(b) A uniform rod of length $2a$ rests on a rough horizontal plane at point o, and is held inclined at an angle of 30° to the horizontal by a string tied at its top end and to a fixed point p distant $2a$ vertically above o.

If the rod is on the point of slipping, calculate the coefficient of friction between the rod and the plane.



8. (a) Prove that the moment of inertia of a uniform square lamina, of mass m and side $2a$, about an axis through its centre parallel to one of its sides is $\frac{1}{3}ma^2$.

(b) A square lamina p, q, r, s can turn freely about a horizontal axis through p perpendicular to the plane of the lamina.

(i) If the lamina is released from rest when diagonal pr is horizontal, find its angular velocity when pr is vertical.

(ii) What mass must be attached to the lamina at r so that the combined body will oscillate with period (of small oscillations)

$$2\pi\sqrt{\frac{8a}{3g}}$$

9. (a) State Archimedes' Principle.
A plastic block of volume 330 cm^3 has air bubbles in it. The block floats in water with 80% of its volume immersed. If the relative density of the plastic is 1·2, calculate the volume of the air bubbles.

(b) A cubical block of wood of side 10 cm and of relative density 0·6 floats horizontally in a container of water. Oil of relative density 0·8 is poured on the water until the top of the oil layer is 3 cm below the top of the block.

(i) How deep is the layer of oil ?
(ii) What is the pressure on the lower face of the block ?

10. (a) Solve the differential equation

$$x\frac{dy}{dx} = y(1 + y)$$

if $x = 1$ when $y = 1$.

(b) A particle of mass 8 kg starts from rest and is acted on by a force which increases uniformly in $10s$ from zero to $16N$.

(i) Prove that t seconds after the particle begins to move, its acceleration is $\frac{t}{5}$ m/s^2.

(ii) Prove that, when the particle has moved x m, its speed is v m/s, where $10v^3 = 9x^2$.