

## APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 24 JUNE – MORNING, 9:30 to 12:00

Six questions to be answered. All questions carry equal marks.  
Mathematics Tables may be obtained from the Superintendent.  
Take the value of  $g$  to be  $9.8 \text{ m/s}^2$ .

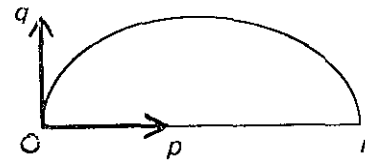
**Marks may be lost if necessary work is not shown or you do not indicate where a calculator has been used.**

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1. (a) A lift, in a continuous descent, had uniform acceleration of  $0.6 \text{ m/s}^2$  for the first part of its descent and a retardation of  $0.8 \text{ m/s}^2$  for the remainder. The time, from rest to rest, was 14 seconds.
- Draw a time-velocity graph and hence, or otherwise, find the distance descended.
- (b) In a lift, moving upwards with acceleration  $f$ , a spring balance indicates an object to have a weight of 98 N. When the lift is moving downwards with acceleration  $2f$  the weight appears to be 68.6 N.
- Calculate
- the actual weight
  - the downward acceleration of the lift.
2. A cyclist A is pedalling at 3 m/s due east along a straight road. A second cyclist B is pedalling at 4 m/s due north along another straight road intersecting the first at a junction  $p$ .
- (a) If A is 80 m and B is 40 m from  $p$  at a given moment, calculate
- the velocity of B relative to A.
  - how far each cyclist is from  $p$  when they are nearest together.
- (b) If when A and B are 80 and 40 m from  $p$ , respectively, then A immediately accelerates at  $0.1 \text{ m/s}^2$  and B decelerates at  $q \text{ m/s}^2$ .
- Find the velocity of B relative to A in terms of time  $t$ .
  - Determine the value of  $q$  which causes them to arrive at  $p$  together.

3.

- (a) A ball is kicked from level ground. The first bounce occurs at the point  $r$ , 45m from the kicking point  $O$  and the greatest height reached was 22.5m. If the horizontal and vertical components of the initial velocity are taken as  $p$  and  $q$  as in the diagram, calculate



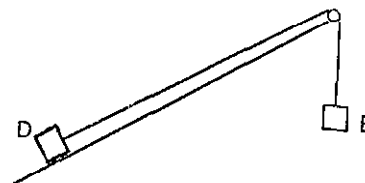
- (i) the value of  $p$  and the value of  $q$ .
  - (ii) the farthest distance from  $r$  that a person running at  $7 \text{ m/s}$  can be, so that starting when the ball was kicked, the person can be at  $r$  just as the ball lands.
- (b) A dart-player stood 3 m from a dart-board hanging on a vertical wall. The dart is thrown horizontally from a point 1.8 m above the ground. It strikes the board at a point 1.5 m above the ground. Calculate:

- (i) the initial speed of the dart.
- (ii) the speed of the dart on striking the board.

4.

A particle D, of mass  $m$ , placed on a rough plane inclined at an angle of  $\tan^{-1} \frac{5}{12}$  to the horizontal, is attached to one end of an inextensible string. The string passes over a small smooth pulley at the top of the plane. An identical particle E hangs freely from the other end of the string. The particles are released from rest. The coefficient of friction,  $\mu$ , between D and the plane is  $\frac{1}{3}$ .

- (i) On separate diagrams show the forces acting on each particle and on the pulley.
- (ii) Find the tension in the string.
- (iii) The string broke after two seconds. Find the total distance travelled by D before coming to rest for the first time.



5.

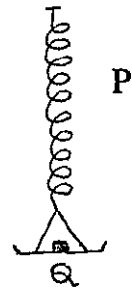
A small smooth sphere moves on a smooth horizontal table and strikes an identical sphere lying at rest on the table at a distance of 1 m from a vertical wall, the impact being along the line of centres and perpendicular to the wall. Prove that the next impact between the two sphere will take place at a distance

$$\frac{2e^2}{1 + e^2} \text{ metres}$$

from the wall, where  $e$  is the coefficient of restitution for all impacts involved.

6.

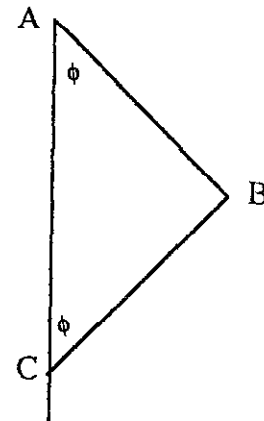
A scale pan is suspended from a fixed point P by a light elastic spring. A particle Q of mass 0.2 kg is attached to the pan with glue. The pan is pulled down from its equilibrium position and set in motion. Given that the motion of Q is simple harmonic, with period  $\frac{\pi}{6}$  seconds and that the maximum and minimum distances of Q below P are 1.5 m and 0.9 m, respectively, calculate



- (i) the maximum speed of Q.
- (ii) the maximum force that the glue has to exert on Q.
- (iii) the length of the spring, when, in the absence of glue, Q would leave the pan.

7.

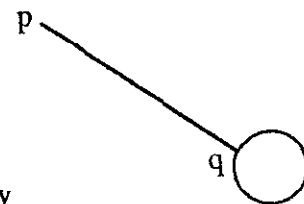
Two uniform rods AB and BC each of length  $l$  and weight  $W$  are smoothly jointed at B. The end A is fixed by a smooth hinge to a rough vertical wall. The system rests in equilibrium in a vertical plane perpendicular to the wall with C in contact with the wall and each rod inclined at an angle  $\phi$  to the vertical.



- (i) Find the frictional force and the normal reaction at C.
- (ii) A force  $W$ , applied vertically downwards at C, is just sufficient to cause slipping. Show that the coefficient of friction between C and the wall is  $\frac{4}{\tan \phi}$ .

8.

A pendulum consists of a rod pq of mass  $m$  and length  $3r$  attached to the rim of a disc of mass  $2m$  and radius  $r$ , as shown. The compound body is set in motion about an axis through p, which is perpendicular to the plane of the rod and the disc.



- (i) Find the period of small oscillations.
- (ii) If the pendulum is released from rest when pq makes an angle  $\theta$  with the downward vertical, show that the angular speed acquired is given, by  $\frac{19g}{36r} (\cos \alpha - \cos \theta)$  when the angle made by pq with the downward vertical is  $\alpha$ .

9. (a) A submarine of mass 200 tonnes is completely immersed in sea-water of density  $1030 \text{ kg/m}^3$ .
- (i) What is the volume of water displaced ?
- (ii) What weight of water must be pumped from the submarine's ballast tanks if it is to just float in fresh water ?
- (b) A liquid of density  $p$  rests on another liquid of density  $q$  without mixing. A solid of density  $d$  floats with its surface totally covered by liquid and with part of its volume immersed in the lower liquid. Show that the fraction of the volume of the solid immersed in the lower liquid is

$$\frac{d-p}{q-p}$$

10. (a) Solve the differential equation

$$\frac{dy}{dx} = \frac{y - xy}{1 + x}$$

if  $y = 1$  when  $x = 0$ .

- (b) A car of mass 1000 kg moves with velocity  $v \text{ m/s}$  along a horizontal road against a constant resistance of 1500 N. The engine is working at a constant rate of 75 kW.
- (i) Show that the acceleration of the car is  $\frac{150-3v}{2v} \text{ m/s}^2$ .
- (ii) Calculate, correct to two decimal places, the time taken by the car to increase its speed from 0 m/s to 25 m/s.