LEAVING CERTIFICATE EXAMINATION, 1988

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 17 JUNE – MORNING, 9.30 – 12.00

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 m/s^2.
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) A particle moving in a straight line with uniform acceleration describes 23 m in the fifth second of its motion and 31 m in the seventh second. Calculate its initial velocity.

   (b) A particle falls freely from rest from a point o, passing three points a, b and c, the distances ab and bc being equal. If the particle takes 3 s to pass from a to b and 2 s from b to c, calculate $|ab|$.

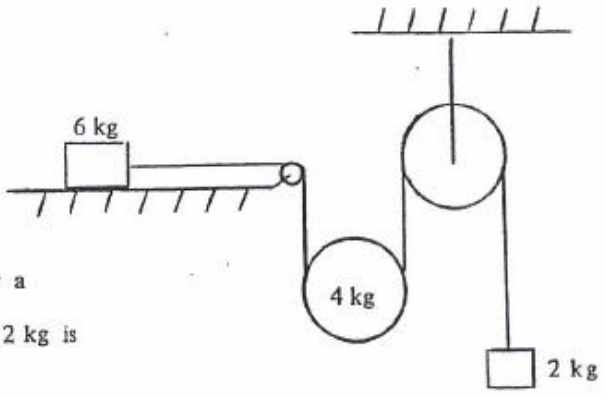
2. (a) Two boats move with constant speed 5 m/s relative to the water and both cross a straight river of width 72 m flowing with constant speed 3 m/s parallel to the banks. One crosses by the shortest path and the other in the shortest time. Show that the difference in the times taken is 3·6 s.

   (b) Two ships A and B move with constant speeds $2u$ and u respectively. At a certain instant, B is 2400 m due east of A and moving northwards. Show that A must move in the direction 30° North of East in order to intercept B and find (in terms of u) the time it takes to intercept B.

3. (a) A particle which is projected with speed u has a horizontal range $\frac{3u^2}{49}$. Calculate the two possible angles of projection.

   (b) A particle is projected up an inclined plane with initial speed $13u$. The line of projection makes an angle $\tan^{-1}(\frac{5}{12})$ with the plane and the plane is inclined at 45° to the horizontal. (The plane of projection is vertical and contains the line of greatest slope.) The particle strikes the plane at a point p.
   If the coefficient of restitution between the particle and the plane is 0·4, show that the particle rises vertically from p and strikes p again on the second bounce.

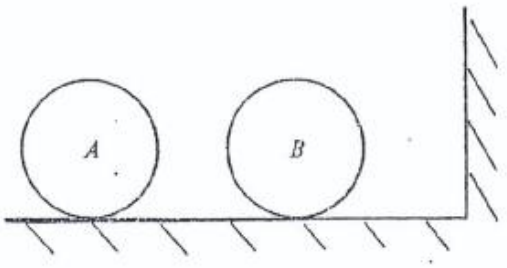
4. One end of a light inextensible string is attached to a mass of 6 kg which rests on a rough horizontal table. The coefficient of friction between the mass and the table is $\frac{1}{6}$. The string passes over a smooth fixed pulley at the edge. Then it passes under a smooth movable pulley of mass 4 kg and over a smooth fixed pulley. A mass of 2 kg is attached to its other end.



   (i) Show on separate diagrams the forces acting on each mass.

   (ii) Calculate the acceleration of each mass and the tension in the string in terms of g, the acceleration due to gravity.

5. Two smooth spheres, A and B, of equal radii, have masses 4 kg and 8 kg respectively. They lie at rest on a smooth horizontal floor so that the line joining their centres is perpendicular to the vertical wall.



A is projected towards B with speed u and collides with B. B then hits the wall, rebounds and collides with A again. This final collision reduces B to rest. If the coefficient of restitution between A and B is $\frac{1}{4}$, calculate

(i) the coefficient of restitution between B and the wall.
(ii) the final velocity of A in terms of u.
(ii) the total loss of energy due to the three collisions.

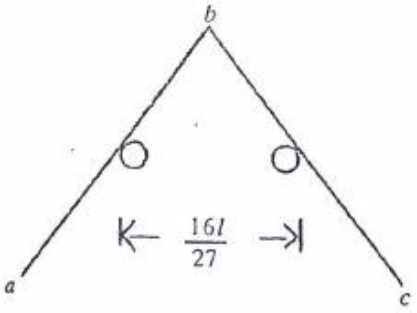
6. A particle of mass 8 kg is describing a circle, with constant speed v, on a smooth horizontal table. It is connected by a light inextensible string of length 3 m to a point which is 1 m vertically above the centre of the circle.

(i) Calculate the tension in the string.

(ii) Show that the particle will remain in contact with the table if $v \leqslant \sqrt{8g}$.

(iii) If the speed of the particle is increased to $\sqrt{9{\cdot}1\,g}$, calculate the height at which the particle rotates above the table.

7. Two equal uniform rods ab and bc each of length $2l$ and weight W, are freely jointed at b and rest in equilibrium, in a vertical plane, across two smooth horizontal pegs at the same horizontal level and distant $\frac{16l}{27}$ apart.



(i) Show in separate diagrams the forces acting on each rod.

(ii) Show that the inclination of each rod to the vertical is $\sin^{-1}\left(\frac{2}{3}\right)$.

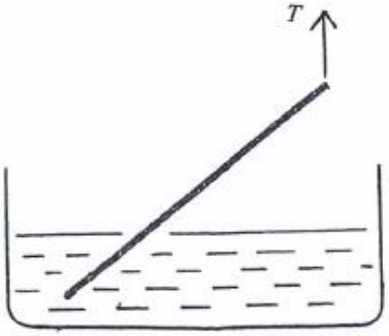
(iii) Determine the magnitude and direction of the reaction at b.

8. Show that the moment of inertia of a uniform rod of mass m and length $2l$, about an axis through its centre of mass perpendicular to the rod is $\frac{1}{3}ml^2$.

Three of these rods are joined together at their ends to form a triangle abc. The triangle is free to rotate about a fixed horizontal axis through a, perpendicular to its plane. Find the period of small oscillations about the equilibrium position.

9. State the Principle of Archimedes.

A uniform rod of weight W and of length $2l$, in equilibrium, is supported at one end by a vertical force T and is immersed in water as shown in the diagram.
The relative density of the rod is $\frac{7}{16}$.



(i) Calculate the length of the immersed part of the rod.

(ii) Show that $T = \frac{3W}{7}$.

10. (a) Solve the differential equation

$$\frac{dx}{dt} = \sqrt{100 - 4x^2}$$

if $x = 5$ when $t = 0$.

(b) A particle of mass m is projected vertically upwards with speed 120 m/s in a medium where there is a resistance of $0{\cdot}098\,v^2$ per unit mass of the particle when v is the speed. Calculate the time taken to reach the highest point.