

APPLIED MATHEMATICS – HIGHER LEVEL

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

Take the value of g to be 9.8 m/s^2 .

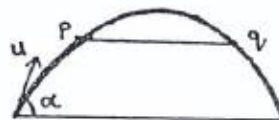
Marks may be lost if all your work is not shown or you do not indicate where a calculator has been used.

1. (a) The maximum acceleration of a body is 4 m/s^2 and its maximum retardation is 8 m/s^2 . What is the shortest time in which the body can travel a distance of 1200 m from rest to rest?
30 secs
- (b) A car A starts from a point p with initial velocity of 8 m/s and then travels with a uniform acceleration of 4 m/s^2 . Two seconds later a second car B starts from p with an initial velocity of 30 m/s and then moves with a uniform acceleration of 3 m/s^2 .
Show that after passing A , B will never be ahead by more than 74 m.

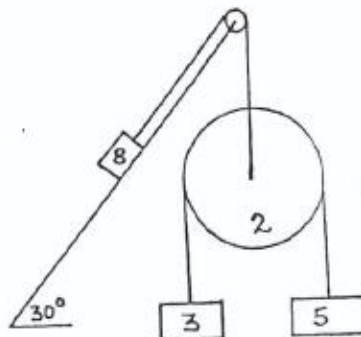
2. At a certain instant a ship H is 37.5 km due West of a ship K . Ship H is travelling South-East at 25 km/h and ship K is travelling South at 15 km/h .
- (i) Draw a diagram to show the velocity of K relative to H and calculate the magnitude and direction of the relative velocity.
- (ii) If H and K can exchange signals when they are not more than 20 km apart, calculate when they can begin to exchange signals and for how long they can continue to exchange signals.

3. (a) *1987* A particle is projected up a plane, which is inclined at an angle $\tan^{-1} \frac{1}{4}$ to the horizontal. The direction of projection makes an angle α with the inclined plane. (The plane of projection is vertical and contains the line of greatest slope). If the particle strikes the inclined plane at right angles, show that $\tan \alpha = 2$.

- (b) A particle is projected with speed u at an angle α to the horizontal. The particle takes 4 s to travel between two points p and q which are on the same horizontal line. Show that the greatest height the particle reaches above this line is 19.6 m.



4. A particle of mass 8 kg rests on a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between the particle and the plane is $\frac{1}{\sqrt{3}}$. The 8 kg mass is connected, by a light inextensible string passing over a smooth fixed pulley at the top of the plane, to a pulley of mass 2 kg hanging freely. Over this pulley which is also smooth, a second light inextensible string is passed having particles of mass 3 kg and 5 kg respectively, attached.



- (i) Show in a diagram the forces acting on each mass when the system is released from rest.
- (ii) Calculate the acceleration of the 8 kg mass.

5. State the laws governing the oblique collision of smooth elastic spheres.

Two smooth elastic spheres A and B of mass 4 kg and 8 kg respectively, collide obliquely. The coefficient of restitution is 0.4. Before collision the velocity of A is $(3\vec{i} + 4\vec{j})$ m/s and that of B is $(-4\frac{1}{2}\vec{i} - p\vec{j})$ m/s where \vec{i} and \vec{j} are unit vectors along and perpendicular to the line of centres at the moment of impact.

- Find the velocity of each sphere after impact.
- Show that the loss of kinetic energy, as a result of the impact, is 63 J.
- If after impact the spheres are moving at right angles to each other, calculate the value of p .

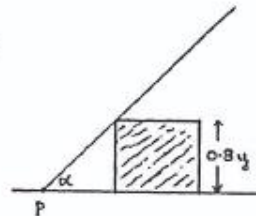
6. Define simple harmonic motion.

A particle of mass m is suspended from a fixed point p by a light extensible string of natural length d and elastic constant $\frac{49m}{d}$. It is pulled down a distance $\frac{8d}{5}$ below p and is then released from rest.

- Show that the particle moves with simple harmonic motion as long as the string remains taut.
- Find in terms of d , when the string becomes slack for the first time.

7. Define limiting friction and coefficient of friction.

A uniform rod of mass 2 kg and of length $6y$ metres, leans against the smooth edge of a rectangular block of mass 6 kg and height $0.8y$ metres. The rod is smoothly hinged at p to a rough horizontal floor and the block also rests on the floor (see diagram). The block is on the point of slipping when the rod makes an angle α with the horizontal, where $\tan \alpha = \frac{4}{3}$.



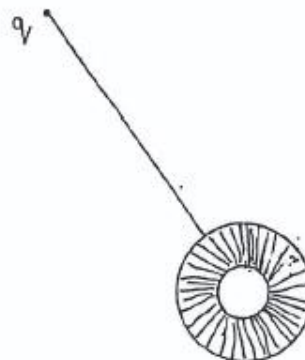
- Show in separate diagrams the forces acting on the rod and on the block.
- Show that the coefficient of friction between the block and floor is $\frac{6}{17}$.
- Find, correct to the nearest Newton, the magnitude of the reaction at the hinge.

8. Prove that the moment of inertia of a uniform annulus of internal diameter p , external diameter $3p$ and mass $4m$, about an axis through its centre perpendicular to its plane is $5mp^2$. (see Tables P.40)

A uniform rod of mass m and length $6p$ is attached to the rim of this annulus so that the rod and the annulus are in the same plane and the rod is collinear with a diameter of the annulus (see diagram).

If the compound body is set in motion about an axis through q which is perpendicular to the plane of the rod and the annulus,

- find the period of small oscillations
- show that the length of the equivalent simple pendulum is $\frac{22p}{3}$.



9. (a) A wooden cube of side 10 cm, and relative density 0.8, is floating horizontally in water. What mass of aluminium, whose relative density is 2.8, must be attached to
- the upper face, so that the cube will just be completely immersed horizontally with the aluminium above water?
 - the lower face, so that the cube is just immersed and horizontal?

- (b) A uniform rod in equilibrium is inclined to the horizontal with one fifth of its length immersed in a liquid and its upper end supported by a vertical force P .
- Show in a diagram the forces acting on the rod.
 - If the relative density of the rod is 0.72, calculate the relative density of the liquid.

10. (a) Solve the differential equation

$$2x(1 + y) \frac{dx}{dy} = 8 + x^2$$

if $x = 2$ when $y = 3$.

- (b) The resistance to the motion of a train of mass m is constant and equal to $60 N$ per tonne. When moving with constant speed 16 m/s on a level line the train begins to ascend an incline of 1 in 98, i.e. $\sin^{-1}(1/98)$. Assuming that the engine continues to work at the same rate (i.e. power is constant) and the $v \text{ m/s}$ is the speed of the train up the incline t seconds after the train has begun climb, show the equation of motion is

$$\left(\frac{v}{v - 6} \right) \frac{dv}{dt} + \frac{4}{25} = 0$$

Calculate the time which elapses before the velocity falls to 12 m/s .
[Tables P.29 may be needed.]