LEAVING CERTIFICATE EXAMINATION, 1982

APPLIED MATHEMATICS - HIGHER LEVEL

MONDAY, 28 JUNE - AFTERNOON, 2.00 to 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 metres/second^2.

1. (a) A car A passes a point p on a straight road at a constant speed of 10 m/s. At the same time another car B starts from rest at p with uniform acceleration 2·5 m/s^2.

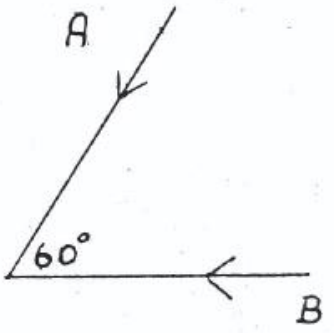
(i) When and how far from p will B overtake A ?

(ii) If B ceases to accelerate on overtaking, what time elapses between the two cars passing a point q three kilometres from p ?

(b) A particle of mass 3 grammes falls from rest from a height of 0·4 m on to a soft material into which it sinks 0·0245 m. Neglecting air resistance, calculate the constant resistance of the material.

2. Two straight roads intersect at an angle of 60°. Cyclists A and B move towards the point of intersection at 30 km/h and 40 km/h, respectively. Calculate the velocity of A relative to B.

If A is 3·5 km and B is 2 km from the intersection at a given moment, calculate the shortest distance between them in their subsequent motion.



3. A particle is projected with a speed of 10 m/s at an angle $\alpha°$ to the horizontal up a plane inclined at 30° to the horizontal. If the particle strikes the plane at right angles, show that the time of flight can be represented by the two expressions

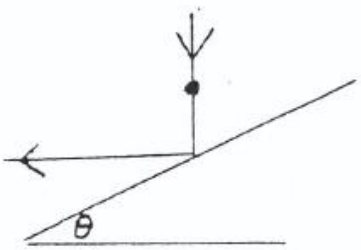
$$\frac{10\cos(\alpha - 30)}{g \sin 30} \quad \text{and} \quad \frac{20\sin(\alpha - 30)}{g \cos 30}$$

Hence deduce a value for $\tan(\alpha - 30)$.

Calculate the range of the particle along the plane.

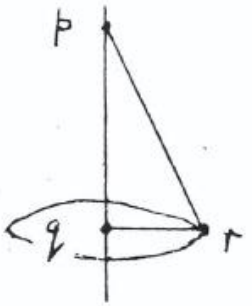
4. (a) A smooth sphere of mass 10 kg moving at 10 m/s impinges directly on another smooth sphere of mass 50 kg moving in the opposite direction at 5 m/s. If the coefficient of restitution is $\frac{1}{2}$, calculate the speeds after impact and the magnitude of the impulse during impact.

(b) A smooth metal sphere falls vertically and strikes a fixed smooth plane inclined at an angle of θ to the horizontal. If the coefficient of restitution is $\frac{2}{3}$ and the sphere rebounds horizontally, calculate the fraction of kinetic energy lost during impact.



5. (a) The diagram shows a string prq which is fixed at p and q where q is vertically below p. r is a small ring threaded on the string which is made to rotate at an angular velocity, ω rad/s, in a horizontal circle, centre q, the string being taut.

If $|pq| = 0{\cdot}12$ m, $|pr| + |rq| = 0{\cdot}18$ m, show that $\omega = \sqrt{294}$ rad/s.



(b) A small bead of mass m is threaded on a smooth circular wire of radius a, fixed with its plane vertical. The bead is projected from the lowest point of the wire with speed u. Show that the reaction between the bead and the wire, when the radius to the bead makes an angle of 60° with the downward vertical, is

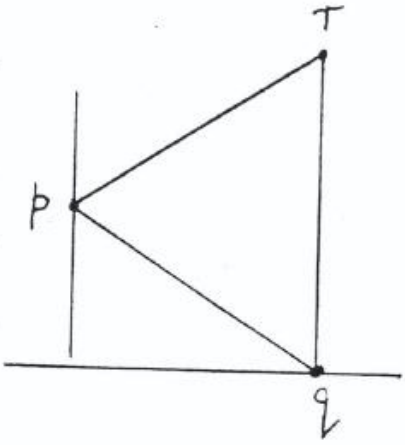
$$m\left[\frac{u^2}{a} - \frac{g}{2}\right]$$

6. Define (i) limiting friction, (ii) coefficient of friction.

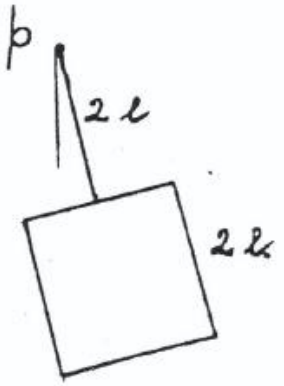
A lamina of weight W in the shape of a thin equilateral triangle pqr is positioned vertically with the vertex p against a smooth vertical wall, and q on a rough horizontal floor. $[qr]$ is parallel to the wall.

Find in terms of W, the horizontal and vertical reactions at q.

Find the least value of μ, the coefficient of friction, so that slipping will not occur.



7. (i) Show that the moment of inertia of a uniform square lamina of side $2l$ and mass m about an axis perpendicular to the lamina through its centre of mass is $\frac{2\ ml^2}{3}$.



(ii) A thin uniform rod of length $2l$ and of mass m is attached to the mid point of the rim of the square. Find the moment of inertia of the system about an axis through p perpendicular to the common plane of the lamina and rod. [See Tables, P.40].

(iii) When this system makes small oscillations in a vertical plane about the axis through p, show that the period of the oscillations is

$$2\pi\sqrt{\frac{31\ l}{7g}}$$

8. Define simple harmonic motion.

The distance, x, of a particle from a fixed point, o, is given by

$$x = a\cos(\omega t + \alpha)$$

where a, ω, α are positive constants.

Show that the particle is describing simple harmonic motion about o and calculate ω and α if the velocity $v = -2a$ and $x = \frac{3a}{5}$ when $t = o$.

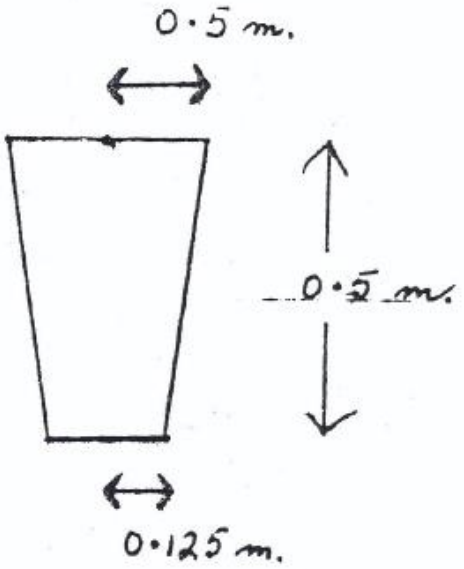
After how many seconds from the start of the motion is $x = o$ for the first time ? (See Tables P.8. Take $\pi = 3{\cdot}142$).

9. (a) A bucket has the form of a frustum of a right circular cone. When it is completely filled with water, find

(i) the pressure at a point on the base

(ii) the thrust, T, on the base

(iii) the ratio $\dfrac{\text{weight of water}}{T}$.



(b) A cubical block of wood of mass 50 kg floats in water with three quarters of its volume immersed.

In oil, when a mass of 10 kg is placed on the same block, it floats just totally immersed , the 10 kg mass being above the oil.

Find the specific gravity of the oil.

10. (a) Find the solution of the differential equation

$$(1 + x^3)\frac{dy}{dx} = x^2 y$$

when $y = 2$ at $x = 1$.

(b) Find the solution of the differential equation

$$\frac{d^2s}{dt^2} = -\left(\frac{ds}{dt}\right)^2$$

when $\dfrac{ds}{dt} = 1$ at $t = 0$

and $s = 0$ at $t = 0$.

A particle moves in a straight line with acceleration equal to minus the square of its velocity. If its initial velocity is 1 m/s, calculate the distance travelled one second later.