LEAVING CERTIFICATE EXAMINATION, 1976

APPLIED MATHEMATICS – HIGHER LEVEL

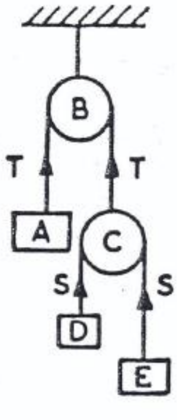
FRIDAY, 25 JUNE – MORNING, 9.30 to 12

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be $9 \cdot 8$ metres/second^2. $\vec{i}$ and $\vec{j}$ are perpendicular unit vectors.

1. Show that, if a particle is moving in a straight line with constant acceleration k and initial speed u, the distance travelled in time t is given by $s = ut + \frac{1}{2}kt^2$. Two points a and b are a distance l apart. A particle starts from a and moves towards b in a straight line with initial velocity u and constant acceleration k. A second particle starts at the same time from b and moves towards a with initial velocity $2u$ and constant deceleration k. Find the time in terms of u, l at which the particles collide, and the condition satisfied by u, k, l if this occurs before the second particle returns to b.

2. A particle is projected upwards with a speed of 35 m/s from a point 0 on a plane inclined at 45° to the horizontal. The plane of projection meets the inclined plane in a line of greatest slope and the angle of projection, measured to the inclined plane, is ϕ. Write down the velocity of the particle and its displacement from 0, in terms of $\vec{i}$ and $\vec{j}$, after time t seconds. If the particle is moving horizontally when it strikes the plane at q prove that $\cot\phi = 3$ and calculate $|oq|$.

3. The diagram shows a light inelastic string, passing over a fixed pulley B, connecting a particle A of mass 3M to a light movable pulley C. Over this pulley passes a second light inelastic string to the ends of which are attached particles D, E of masses 2M, M respectively. Show in separate diagrams the forces acting on A, D and E. Write down the three equations of motion involving the tensions T, S in the strings, the acceleration of A and the common acceleration of D, E *relative* to C. Show that T = 2S = 48Mg/17.



4. A light smooth ring of mass M is threaded on a smooth fixed vertical wire and is connected by a light inelastic string, passing over a fixed smooth peg at a distance l from the wire, to a particle of mass 2M hanging freely. The system is released from rest when the string is horizontal. Explain why the conservation of energy can be applied to the system. If the ring descends a distance of x while the particle rises through a distance y show that

$$x^2 = y^2 + 2ly \text{ and } (l + y)\dot{y} = x\dot{x}$$

where $\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$ are the speeds of the ring and particle respectively. Find $\dot{x}$ when (i) $x = l$ and (ii) when $x = 4l/3$.

5. State the laws governing the oblique collision of elastic spheres.
A sphere of mass M moving with speed u collides obliquely with a second smooth sphere at rest. The direction of motion of the moving sphere is inclined at 45° to the line of centres at impact, and the coefficient of restitution is $\frac{1}{2}$. After impact the directions of motion of the spheres are at right angles.

Find the mass of the second sphere in terms of M, and the velocities of the two spheres after impact in terms of u. Hence show that one quarter of the kinetic energy is lost.

6. Two uniform rods ab, bc of lengths $2l$, $2r$ and of weights 2W, 3W respectively are smoothly hinged together at b. They stand in equilibrium in a vertical plane with the end a resting on rough horizontal ground and the end c resting against a smooth vertical wall. The point a is farther from the wall than b and the rods ab, bc are inclined at angles α, 45° respectively to the horizontal where $\alpha > 45^\circ$. Show in separate diagrams the forces acting on each rod. By considering separately the equilibrium of the system abc and of the rod bc, find the coefficient of friction at a and show that $\tan\alpha = 8/3$.

7. Define simple harmonic motion.

A particle of mass 2 kg is attached to the ends of two light elastic strings, each of natural length 1 m and elastic constant 49 N/m. The other ends of the two strings are attached to two fixed points a and b in the same vertical line, where a is 4 m above b. The particle is released from rest from the midpoint of ab. By considering the forces acting on the particle when it is x metres from a, where $2 < x < 2{\cdot}4$, show that it is moving with simple harmonic motion. Find the least time taken for the particle to reach the point $x = 2{\cdot}3$, and find its speed there.

8. A pendulum of a clock consists of a thin uniform rod ab of mass M and length $6l$ to which is rigidly attached a uniform circular disc of mass 4M and radius l with the centre of the disc being at the point c on ab where $bc = l$. Using the parallel axes theorem for the disc, show that the moment of inertia of the pendulum about an axis at a perpendicular to the plane of the disc is $114Ml^2$.

The pendulum is free to oscillate in a vertical plane about such a fixed horizontal axis at a. It is released from rest with ab horizontal. Find the speed of b when ab is vertical.

9. An atomic nucleus of mass M is repelled from a fixed point o by a force Mk^2x^{-5}, where x is the distance of the nucleus from o and k is a constant. It is projected directly towards o with speed $\dfrac{2k\sqrt{3}}{d^2}$ from a point a where $|oa| = d$. Find the speed of the nucleus when it reaches the midpoint of oa and find how near it gets to o.

10. (i) Using Taylor's theorem (Mathematics Tables, p.42) find the first two terms in the Taylor series for e^{x^2} in the neighbourhood of $x = 0$, i.e. the Maclaurin series for e^{x^2}.

(ii) State Archimedes principle for a body wholly or partly immersed in a liquid.

A uniform thin rod is of length $2a$, of weight 4W and specific gravity $4/9$. The rod rests in equilibrium in an inclined position partly immersed in water with its lower end freely pivoted to a fixed point at a depth $\dfrac{2a}{3}$ below the surface of the water. Show in a diagram the forces acting on the rod and calculate the inclination of the rod to the vertical.