LEAVING CERTIFICATE EXAMINATION, 1984

APPLIED MATHEMATICS - HIGHER LEVEL

WEDNESDAY, 27 JUNE - AFTERNOON, 2.00 - 4.30

Six questions to be answered. All questions carry equal marks.
Mathematics Tables may be obtained from the Superintendent.
Take the value of g to be 9·8 m/s^2.

1. The driver of a car travelling at 20 m/s sees a second car 120 m in front, travelling in the same direction at a uniform speed of 8 m/s.

   (a) What is the least uniform retardation that must be applied to the faster car so as to avoid a collision ?

   (b) If the actual retardation is 1 m/s^2, calculate

      (i) the time interval, in seconds, for the faster car to reach a point 66 m behind the slower.

      (ii) the shortest distance between the cars.

2. A ship B is travelling in a direction 41° East of North at 15 m/s. A second ship C is travelling 41° South of East at 20 m/s.

   Calculate: (i) the velocity of B relative to C;
   (ii) the shortest distance between the ships if C is 3 km east of B at a particular moment ;
   (iii) the time interval during which the ships remain in visual contact, if visibility is limited to 3 km.

3. A plane is inclined at an angle $\tan^{-1}(\frac{1}{2})$ to the horizontal. A particle is projected up the plane with velocity u at an angle θ to the plane. (The plane of projection is vertical and contains the line of greatest slope.) The particle strikes the plane parallel to the horizontal.

   1984

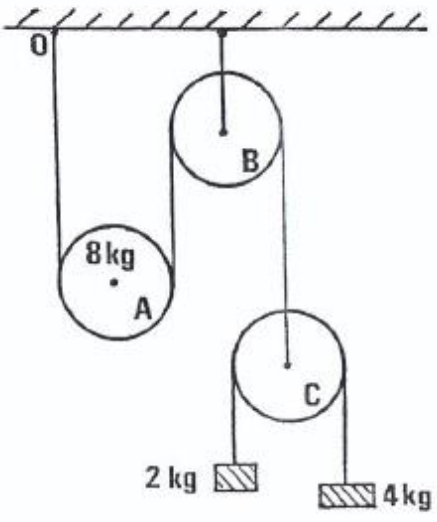
   Express t, the time of flight, in terms of u and θ.

   Hence, or otherwise, establish that

   $$\tan\theta = \frac{1}{3}.$$

   Calculate the range along the plane.

4. The diagram shows a light inextensible string having one end fixed at O, passing under a movable pulley A of mass 8 kg and then over a fixed light pulley B. The other end of the string is attached to a light pulley C, of negligible mass. Over pulley C, a second light inextensible string is passed having particles of mass 2 and 4 kg respectively, attached. All pulleys are smooth.



   (i) Show in a diagram the forces acting on each pulley when the system is released from rest.

   (ii) Find the acceleration of
   pulley A
   pulley C
   each particle.

5. (a) A smooth sphere of mass 3 kg and velocity u_1 collides directly with another smooth sphere of mass 4 kg and velocity u_2 both moving in the same direction. Show that

$$7v_1 = u_1(3 - 4e) + 4u_2(1 + e)$$

where v_1 is the velocity of the 3 kg sphere after the collision.
Hence, show that the impulse which each sphere receives is

$$\frac{12}{7}(1 + e)(u_2 - u_1).$$

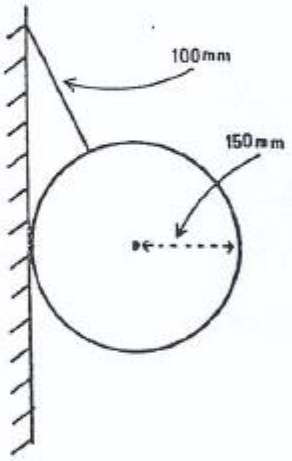
(b) A smooth sphere of mass 4 kg collides obliquely with another smooth sphere of mass m which is at rest. After impact the two spheres move at right angles to each other. If the coefficient of restitution was $\frac{4}{7}$, calculate the value of m.

6. (a) A particle moving on the inside smooth surface of a fixed hollow sphere of internal radius $\sqrt{2}$ m describes a horizontal circle of radius 1 m. Calculate the angular velocity of the particle.

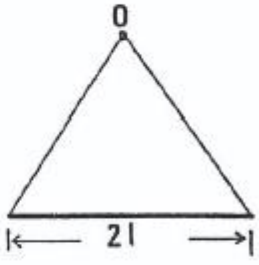
(b) Two particles of equal mass attached by a taut inextensible string of length $2y$ rests on a horizontal circular table. The particles are respectively y and $3y$ from the centre of the table so that centre and particles are collinear.
The table rotates about its centre with angular velocity ω and the coefficient of friction is $\frac{y}{2}$. If both particles are on the point of slipping,

(i) show on a diagram, all the forces of the string/particle system;
(ii) calculate ω.

7. (a) A sphere of mass 3 kg and radius 150 mm is suspended by a string 100 mm long, the string joining a point on the surface with a point on a smooth vertical wall.
Find the tension in the string in terms of g.



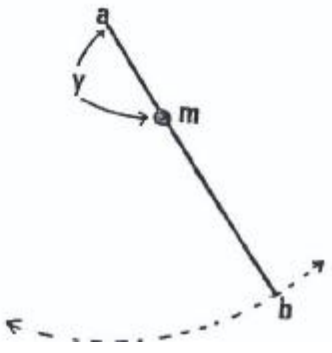
(b) A heavy uniform rod of mass m and length $2l$ is suspended from a point, o, by two equal inelastic strings. Each string is fixed to o and to an end point of the rod so that the rod hangs horizontally.
If, then, a mass $\frac{m}{2}$ is suspended half-way between the centre and one end of the rod so that the rod is no longer horizontal, calculate the ratio $T_1 : T_2$, where T_1 is the tension in one of the strings and T_2, the tension in the other.



8. A uniform rod $[ab]$ of length $2p$ and of mass $3m$ has a mass m attached to it at a distance y from a. Prove that the moment of inertia of this system about a smooth horizontal axis through a is

$$4mp^2 + my^2.$$

The system oscillates in a vertical plane about a. If the length of the equivalent simple pendulum is $\frac{40}{33}p$, show that y is either $\frac{2}{3}p$ or $\frac{6}{11}p$.



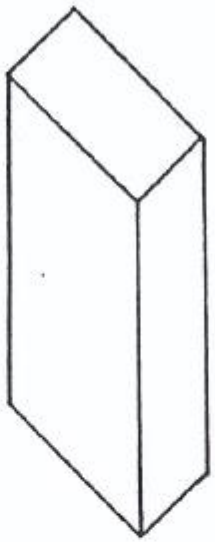
9. (a) A body of mass 1·5 kg weighs 2·1 N in water and 3·36 N in a mixture of another liquid A and water. If there was no reduction in volume when the mixture was made, calculate

(i) the relative density of the body

(ii) the relative density of the mixture

(iii) the volume of liquid A, of relative density 0·82, which must be added to 100 ml of water to form the mixture.

(b) A uniform rectangular block of wood 200 mm × 100 mm × 8 mm and of relative density d floats in water with its longest edge vertical. If the block is depressed vertically a further small distance x and released, verify that it will perform simple harmonic motion. Calculate the periodic time of the motion.



10. (a) Find the general solution to

$$\frac{dv}{dt} = g - kv$$

where g and k are constants.

Show that $\lim_{t \to \infty} v = \frac{g}{k}$.

(b) A car, free-wheeling on a straight road, experiences a retardation which is proportional to the square of its speed. Its speed is reduced from 20 m/s to 10 m/s in a distance of 100 m. Calculate the time taken to travel the 100 m.