

### Chapter 3 Exercise 3A

**Q. 1.** (i)  $s_y = 28(3) + \frac{1}{2}(-9.8)(3)^2$   
 $= 39.9 \text{ m}$

(ii) Let  $H =$  maximum height

$$u_y = 28, \quad a_y = -9.8,$$

$$v_y = 0, \quad s = H$$

$$v^2 = u^2 + 2as$$

$$\therefore 0 = (28)^2 + 2(-9.8)H$$

$$\therefore 0 = 784 - 19.6H$$

$$\therefore H = 40$$

(iii)  $s_y = 0$

$$\Rightarrow 28t - 4.9t^2 = 0$$

$$\Rightarrow t = 0 \quad \text{OR} \quad t = \frac{40}{7}$$

$$\begin{aligned} \text{At } t = \frac{40}{7}, s_x &= 21t \\ &= 21\left(\frac{40}{7}\right) \\ &= 120 \text{ m} \end{aligned}$$

**Q. 2.**  $v_x = 56$

$$v_y = 56 - gt$$

$$s_x = 56t$$

$$s_y = 56t - \frac{1}{2}gt^2$$

(i) Let  $H =$  maximum height

$$u_y = 56, \quad a_y = -9.8,$$

$$v_y = 0, \quad s_y = H$$

$$v^2 = u^2 + 2as$$

$$\therefore 0 = (56)^2 + 2(-9.8)H$$

$$\therefore 0 = 3,136 - 19.6H$$

$$\therefore H = 160 \text{ m}$$

(ii) Range:  $s_x$  when  $s_y = 0$

$$s_y = 0$$

$$\Rightarrow 56t - \frac{1}{2}gt^2 = 0$$

$$\Rightarrow 112t - gt^2 = 0$$

$$\Rightarrow t(112 - gt) = 0$$

$$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{112}{g}}_{\text{Time of Flight}}$$

$$\begin{aligned} \Rightarrow \text{Range} &= 56\left[\frac{112}{g}\right] \\ &= 640 \text{ m} \end{aligned}$$

(iii) Velocity after 4 seconds: Find  $v_x$  and  $v_y$  when  $t = 4$

$$v_x = 56$$

$$v_y = 56 - g(4)$$

$$= 16.8$$

$$\Rightarrow \vec{v} = 56\vec{i} + 16.8\vec{j} \text{ m/s}$$

**Q. 3.**  $v_x = 70$

$$v_y = 105 - gt$$

$$s_x = 70t$$

$$s_y = 105t - \frac{1}{2}gt^2$$

(i) Need  $v_x$  and  $v_y$  when  $t = 10$

$$v_x = 70$$

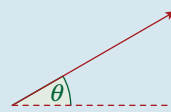
$$v_y = 105 - g(10) = 7$$

$$\Rightarrow \vec{v} = 70\vec{i} + 7\vec{j}$$

**Magnitude**

$$\begin{aligned} |\vec{v}| &= \sqrt{70^2 + 7^2} \\ &= 70.35 \text{ m/s} \end{aligned}$$

**Direction**



$$\tan \theta = \frac{7}{70} = \frac{1}{10}$$

$$\Rightarrow \theta = \tan^{-1}\left[\frac{1}{10}\right] = 5.71^\circ$$

(ii) Range:  $s_x$  when  $s_y = 0$

$$s_y = 0$$

$$\Rightarrow 105t - \frac{1}{2}gt^2 = 0$$

$$\Rightarrow 210t - gt^2 = 0$$

$$\Rightarrow t(210 - gt) = 0$$

$$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{210}{g}}_{\text{Time of Flight}}$$

$$\begin{aligned} \Rightarrow \text{Range} &= 70\left[\frac{210}{g}\right] \\ &= 1,500 \text{ m} \end{aligned}$$

**Q. 4.** (i)  $s_y = 21t - 4.9t^2$   
 $= 22.4$   
 $\Rightarrow 7t^2 - 30t + 32 = 0$   
 $\Rightarrow (7t - 16)(t - 2) = 0$   
 $t = \frac{16}{7}$  **OR**  $t = 2$

(ii) When it is at its greatest height,  
 $v_y = 0$  and  $v_x = 14$ , as always.  
 $\therefore \vec{v} = 14\vec{i} + 0\vec{j}$

**Q. 5.** (i)  $u_y = 49$ ,  $a_y = -9.8$ ,  
 $v_y = 0$ ,  $s_y = H$   
 $v^2 = u^2 + 2as$   
 $\therefore 0 = (49)^2 + 2(-9.8)H$   
 $\therefore H = 122.5$  m

(ii)  $s_y = 0$   
 $\Rightarrow 105t - 4.9t^2 = 0$   
 $t = 0$  **OR**  $t = \frac{105}{4.9} = \frac{150}{7}$   
 At  $t = \frac{150}{7}$ ,  $s_x = 70t$   
 $= 70\left(\frac{150}{7}\right)$   
 $= 1,500$  m

(iii)  $v_x = 10$   
 $v_y = 49 - (9.8)(6)$   
 $= -9.8$   
 $\therefore \vec{v} = 10\vec{i} - 9.8\vec{j}$  m/s

**Q. 6.**  $v_x = 70$   
 $v_y = 140 - gt$   
 $s_x = 70t$   
 $s_y = 140t - \frac{1}{2}gt^2$

(i) Need to find  $v_x$  and  $v_y$  when  $t = 5$   
 $v_x = 70$   
 $v_y = 140 - g(5) = 91$   
 $\Rightarrow \vec{v} = 70\vec{i} + 91\vec{j}$

**Magnitude**  
 $|\vec{v}| = \sqrt{70^2 + 91^2}$   
 $= 114.8$  m/s

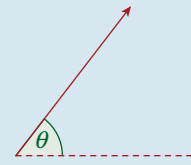
**Direction**

$$\tan \theta = \frac{91}{70}$$

$$= \frac{13}{10}$$

$$\Rightarrow \theta = \tan^{-1}\left[\frac{13}{10}\right]$$

$$= 52.43^\circ$$



(ii) Range:  $s_x$  when  $s_y = 0$   
 $s_y = 0$   
 $\Rightarrow 140t - \frac{1}{2}gt^2 = 0$   
 $\Rightarrow 280t - gt^2 = 0$   
 $\Rightarrow t(280 - gt) = 0$   
 $\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \quad \quad \quad \underbrace{t = \frac{280}{g}}_{\text{Time of Flight}}$   
 Range =  $70\left[\frac{280}{g}\right]$   
 $= 2,000$  m

**Q. 7.**  $v_x = 20$   
 $v_y = 28 - gt$   
 $s_x = 20t$   
 $s_y = 28t - \frac{1}{2}gt^2$

(i) Need  $s_y$  when  $t = 3$   
 $= 28(3) - \frac{1}{2}g(9)$   
 $= 39.9$  m

(ii) Need  $v_x$  and  $v_y$  when  $t = 4$   
 $v_x = 20$   
 $v_y = 28 - g(4)$   
 $= -11.2$   
 $\Rightarrow \vec{v} = 20\vec{i} - 11.2\vec{j}$   
 $\Rightarrow \text{Speed} = \sqrt{20^2 + (-11.2)^2}$   
 $= 22.92$  m/s

**Q. 8.** (i)  $v_y = 0$   
 $\Rightarrow 35 - 9.8t = 0$   
 $\Rightarrow t = \frac{35}{9.8}$   
 $= \frac{25}{7}$

$$\begin{aligned} \text{(ii) } v_y &= 10.5 \\ \Rightarrow 35 - 9.8t &= 10.5 \\ \Rightarrow t &= \frac{24.5}{9.8} \\ &= \frac{5}{2} \end{aligned}$$

$$\begin{aligned} \text{(iii) } v_y &= -10.5 \\ \Rightarrow 35 - 9.8t &= -10.5 \\ \Rightarrow t &= \frac{45.5}{9.8} \\ &= \frac{65}{14} \end{aligned}$$

(iv) Midway between (ii) and (iii) is

$$\frac{\frac{5}{2} + \frac{65}{14}}{2} = \frac{25}{7} = \text{(i) QED}$$

$$\begin{aligned} \text{Q. 9. } s_y &= -490 \\ \Rightarrow -4.9t^2 &= -490 \\ \Rightarrow t &= 10 \end{aligned}$$

$$\begin{aligned} \text{At } t = 10, s_x &= 200t \\ &= 200(10) \\ &= 2,000 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Q. 10. (i) } v_y &= 0 \\ \Rightarrow 98 - 9.8t &= 0 \\ \Rightarrow t &= 10 \\ \text{(ii) At } t = 10, s_x &= 10t \\ &= 10(10) \\ &= 100 \text{ m} \\ s_y &= 98t - 4.9t^2 \\ &= 98(10) - 4.9(100) \\ &= 490 \text{ m} \\ \therefore \vec{r} &= 100\vec{i} + 490\vec{j} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Q. 11. } v_x &= 100 \\ v_y &= 98 - gt \\ s_x &= 100t \\ s_y &= 98t - \frac{1}{2}gt^2 \end{aligned}$$

$$\begin{aligned} \text{(i) Need to find } t \text{ when } s_y &= 470.4 \\ \Rightarrow 98t - \frac{1}{2}gt^2 &= 470.4 \\ \Rightarrow 98t - 4.9t^2 &= 470.4 \\ \Rightarrow 4.9t^2 - 98t + 470.4 &= 0 \\ \Rightarrow t^2 - 20t + 96 &= 0 \\ \Rightarrow (t - 8)(t - 12) &= 0 \\ \Rightarrow t = 8, t = 12 \end{aligned}$$

$$\begin{aligned} \text{(ii) Find } v_x \text{ and } v_y \text{ when } t &= 8 \\ v_x &= 100 \\ v_y &= 98 - g(8) \\ &= 19.6 \\ \Rightarrow \vec{v} &= 100\vec{i} + 19.6\vec{j} \\ \Rightarrow \text{Speed} &= \sqrt{100^2 + 19.6^2} \\ &= 102 \text{ m/s} \end{aligned}$$

Find  $v_x$  and  $v_y$  when  $t = 12$

$$\begin{aligned} v_x &= 100 \\ v_y &= 98 - g(12) \\ &= -19.6 \\ \Rightarrow \vec{v} &= 100\vec{i} - 19.6\vec{j} \\ \Rightarrow \text{Speed} &= \sqrt{100^2 + (-19.6)^2} \\ &= 102 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{Q. 12. } v_x &= 8 \\ v_y &= 28 - gt \\ \sqrt{8^2 + (28 - gt)^2} &= 10 \\ \Rightarrow 64 + (784 - 56gt + g^2t^2) &= 100 \\ \Rightarrow g^2t^2 - 56gt + 748 &= 0 \\ \Rightarrow \frac{2,401}{25}t^2 - \frac{2,744}{5}t + 748 &= 0 \\ \Rightarrow 2,401t^2 - 13,720t + 18,700 &= 0 \end{aligned}$$

$$t = \frac{13,720 \pm \sqrt{13,720^2 - 4(2,401)(18,700)}}{2(2,401)}$$

$$t_1 = \frac{110}{49} \text{ s} \quad t_2 = \frac{170}{49} \text{ s}$$

### Exercise 3B

**Q. 1.** (i)  $\cos \alpha = \frac{2}{\sqrt{5}}, \sin \alpha = \frac{1}{\sqrt{5}}$   
 $\vec{u} = 7\sqrt{5} \cos \alpha \vec{i} + 7\sqrt{5} \sin \alpha \vec{j}$   
 $= 14\vec{i} + 7\vec{j}$

(ii)  $s_y = 0$   
 $\Rightarrow 7t - 4.9t^2 = 0$   
 $\Rightarrow t = 0$  OR  $t = \frac{7}{4.9}$   
 $= \left(\frac{10}{7}\right)$

At  $t = \frac{10}{7}, s_x = 14t$   
 $= 14\left(\frac{10}{7}\right)$   
 $= 20 \text{ m}$

**Q. 2.**  $\vec{u} = 35 \cos \theta \vec{i} + 35 \sin \theta \vec{j}$   
 $= 35\left(\frac{4}{5}\right)\vec{i} + 35\left(\frac{3}{5}\right)\vec{j}$   
 $= 28\vec{i} + 21\vec{j}$

$s_y = 10$   
 $\Rightarrow 21t - 4.9t^2 = 10$   
 $\Rightarrow 49t^2 - 210t + 100 = 0$   
 $\Rightarrow t = 0.546$  (OR 3.740)

At  $t = 0.546,$   
 $v_x = 28$  and  $v_y = 21 - 9.8t$   
 $= 21 - 9.8(0.546)$   
 $= 15.65$

$\therefore \vec{v} = 28\vec{i} + 15.65\vec{j}$   
 $\Rightarrow |\vec{v}| = \sqrt{28^2 + (15.65)^2}$   
 $= 32.08 \text{ m/s}$

**Q. 3.**  $s_y = 0$   
 $\Rightarrow 7t - 4.9t^2 = 0$   
 $\Rightarrow t = 0$  OR  $t = \frac{7}{4.9} = \frac{10}{7}$

At  $t = \frac{10}{7}, s_x = 10t$   
 $= 10\left(\frac{10}{7}\right)$   
 $= \frac{100}{7} \text{ m}$   
 $= R, \text{ the range}$

$$\frac{3}{4}R = \frac{75}{7} \text{ m}$$

$$s_x = \frac{75}{7}$$

$$\Rightarrow t = \frac{75}{70}$$

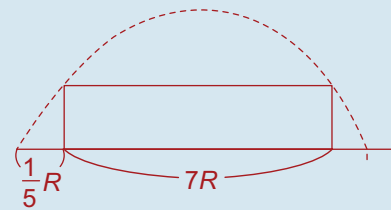
$$= \frac{15}{14}$$

At  $t = \frac{15}{14}, s_y = 7\left(\frac{15}{14}\right) - 4.9\left(\frac{225}{196}\right)$   
 $= 7.5 - 5.625$   
 $= 1.875 \text{ m}$

**Q. 4.**  $s_y = 0$   
 $\Rightarrow 4t - 4.9t^2 = 0$   
 $\Rightarrow t = 0$  OR  $t = \frac{4}{4.9} = \frac{40}{49}$   
 At  $t = \frac{40}{49}, s_x = 3t$   
 $= 3\left(\frac{40}{49}\right)$   
 $= \frac{120}{49} = R, \text{ the range}$

$\therefore \frac{1}{5}R = \frac{24}{49} \text{ m}$   
 When is  $s_x = \frac{24}{49}$ ? When  $3t = \frac{24}{49}$   
 $\Rightarrow t = \frac{8}{49}$

At  $t = \frac{8}{49}, s_y = 4\left(\frac{8}{49}\right) - 4.9\left(\frac{64}{2,401}\right)$   
 $= \frac{32}{49} - \frac{64}{490}$   
 $= \frac{128}{245}$



By symmetry, it will reach the same height when  $s_x = \frac{4}{5}R$ . The time will be  $\frac{4}{5}$  of the time of flight  $= \frac{4}{5}\left(\frac{40}{49}\right) = \frac{32}{49} \text{ s}$

**Q. 5.** (a)  $(7t + 50)(t - 10)$   
 (b)  $s_y = -350$   
 $\Rightarrow 14t - 4.9t^2 = -350$   
 $\Rightarrow 7t^2 - 20t - 50 = 0$   
 $\Rightarrow (7t + 50)(t - 10) = 0$   
 $\Rightarrow t = 10$  ( $t = -\frac{50}{7}$  is rejected)

$$\begin{aligned} \text{At } t = 10, s_x &= 10t \\ &= 10(10) \\ &= 100 \text{ m} \end{aligned}$$

**Q. 6.**  $s_y = 49t - \frac{1}{2}(9.8)t^2 = 0$   
 $\therefore 49t - 4.9t^2 = 0$   
 $\therefore t(49 - 4.9t) = 0$   
 $\therefore t = 0$  **OR**  $t = 10$

at  $t = 10, s_x = 50t$   
 $= 50(10)$   
 $= 500 \text{ m}$

**Q. 7.** (i)  $u_y = 14, a_y = -9.8,$   
 $s_y = H, v_y = 0$   
 $v^2 = u^2 + 2as$   
 $\therefore 0 = (14)^2 + 2(-9.8)H$   
 $\therefore H = 10 \text{ m}$

(ii)  $\frac{3}{4}H = \frac{3}{4}H(10)$   
 $= 7.5$   
 $s_y = 7.5$   
 $\Rightarrow 14t - 4.9t^2 = 7.5$   
 $\Rightarrow 49t^2 - 140t + 75 = 0$   
 $\Rightarrow (7t - 5)(7t - 15) = 0$   
 $\Rightarrow t = \frac{5}{7}$  **OR**  $\frac{15}{7}$

**Q. 8.**  $s_y = -82.5$   
 $\Rightarrow 8t - 4.9t^2 = -82.5$   
 $\Rightarrow 49t^2 - 80t - 825 = 0$   
 $\Rightarrow t = 5$  **OR**  $-\frac{330}{98}$   
 At  $t = 5, s_x = 12t$   
 $= 12(5)$   
 $= 60 \text{ m}$

**Q. 9.** (i)  $s_y = -78.4$   
 $\Rightarrow -4.9t^2 = -78.4$   
 $\Rightarrow t^2 = 16$   
 $\Rightarrow t = 4$

(ii)  $\vec{u} = 98 \cos 30^\circ \vec{i} + 98 \sin 30^\circ \vec{j}$   
 $= 84.868 \vec{i} + 49 \vec{j}$   
 $s_y = -78.4$   
 $\Rightarrow 49t - 4.9t^2 = -78.4$

$$\begin{aligned} \Rightarrow 49t^2 - 490t - 784 &= 0 \\ \Rightarrow t^2 - 10t - 16 &= 0 \\ \Rightarrow t = 11.4 \quad (-1.4 \text{ is rejected}) \end{aligned}$$

**Q. 10.**  $u_x = 50 \cos \theta$   
 $= 50\left(\frac{3}{5}\right)$   
 $= 30$   
 $u_y = 50 \sin \theta$   
 $= 50\left(\frac{4}{5}\right)$   
 $= 40$   
 $v_x = 30$   
 $v_y = 40 - gt$   
 $s_x = 30t$   
 $s_y = 40t - \frac{1}{2}gt^2$

(i)  $\vec{r} = 30t\vec{i} + \left[40t - \frac{1}{2}gt^2\right]\vec{j}$   
 $= 30t\vec{i} + (40 - 4.9t^2)\vec{j} \text{ m}$

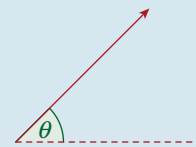
(ii)  $v_x = 30$   
 $v_y = 40 - g(1) = 30.2$   
 $\Rightarrow \vec{v} = 30\vec{i} + 30.2\vec{j} \text{ m/s}$

**Magnitude**

$$\begin{aligned} |\vec{v}| &= \sqrt{30^2 + 30.2^2} \\ &= 43 \text{ m/s} \end{aligned}$$

**Direction**

$$\begin{aligned} \tan \theta &= \frac{30.2}{30} \\ \Rightarrow \theta &= \tan^{-1} \frac{30.2}{30} \\ &= 45^\circ \end{aligned}$$



(iii) Range:  $s_x$  when  $s_y = 0$

$$\begin{aligned} s_y &= 0 \\ \Rightarrow 40t - \frac{1}{2}gt^2 &= 0 \\ \Rightarrow 80t - gt^2 &= 0 \\ \Rightarrow t(80 - gt) &= 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow t &= 0 & t &= \frac{80}{g} \\ \text{Point of} & & \text{Time of} & \\ \text{Projection} & & \text{Flight} & \end{aligned}$$

$$\begin{aligned} \text{Range} &= 30\left(\frac{80}{g}\right) \\ &= 245 \text{ m} \end{aligned}$$

**Q. 11.**  $s_x = 12t$

$$s_y = kt - \frac{1}{2}gt^2$$

$$s_x = 30 \text{ when } s_y = 9.375$$

$$\Rightarrow 12t = 30$$

$$\Rightarrow t = \frac{5}{2}$$

$$\text{when } kt - 4.9t^2 = 9.375$$

$$\Rightarrow k\left(\frac{5}{2}\right) - 4.9\left(\frac{5}{2}\right)^2 = 9.375$$

$$\Rightarrow \frac{5k}{2} - \frac{245}{8} = \frac{75}{8}$$

$$\Rightarrow 20k - 245 = 75$$

$$\Rightarrow 20k = 320 \Rightarrow k = 16$$

**Q. 12.**  $u_x = u \cos \alpha$        $u_y = u \sin \alpha$

$$v_x = u \cos \alpha$$

$$v_y = u \sin \alpha - gt$$

$$s_x = ut \cos \alpha$$

$$s_y = ut \sin \alpha - \frac{1}{2}gt^2$$

(i) Greatest height:  $s_y$  when  $v_y = 0$

$$v_y = 0$$

$$\Rightarrow u \sin \alpha - gt = 0$$

$$\Rightarrow t = \frac{u \sin \alpha}{g}$$

$$\Rightarrow \text{Greatest height} = u\left(\frac{u \sin \alpha}{g}\right) \sin \alpha - \frac{1}{2}g\left(\frac{u \sin \alpha}{g}\right)^2$$

$$\Rightarrow \text{Greatest height} = \frac{u^2 \sin^2 \alpha}{g} - \frac{u^2 \sin^2 \alpha}{2g}$$

$$\Rightarrow \text{Greatest height} = \frac{2u^2 \sin^2 \alpha - u^2 \sin^2 \alpha}{2g}$$

$$\Rightarrow \text{Greatest height} = \frac{u^2 \sin^2 \alpha}{2g}$$

**OR**

$$v_y^2 = u_y^2 + 2a_y s_y$$

$$0 = (u \sin \alpha)^2 + 2(-g)H$$

$$\therefore H = \frac{u^2 \sin^2 \alpha}{2g}$$

(ii) Range:  $s_x$  when  $s_y = 0$

$$s_y = 0$$

$$\Rightarrow ut \sin \alpha - \frac{1}{2}gt^2 = 0$$

$$\Rightarrow 2ut \sin \alpha - gt^2 = 0$$

$$\Rightarrow t(2u \sin \alpha - gt) = 0$$

$$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \quad \quad \quad t = \frac{2u \sin \alpha}{g}$$

Time of Flight

$$\Rightarrow \text{Range} = u\left(\frac{2u \sin \alpha}{g}\right) \cos \alpha$$

$$\Rightarrow \text{Range} = \frac{2u^2 \sin \alpha \cos \alpha}{g}$$

(iii)  $u = 70$  and Greatest height = 125

$$\Rightarrow \frac{4,900 \sin^2 \alpha}{2g} = 125$$

$$\Rightarrow 250 \sin^2 \alpha = 125$$

$$\Rightarrow \sin^2 \alpha = \frac{1}{2}$$

$$\Rightarrow \sin \alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \alpha = 45^\circ$$

$$\Rightarrow \text{Range} = \frac{9,800 \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{2}}\right)}{9.8}$$

$$= 500 \text{ m}$$

Q. 13.  $\frac{u^2 \sin^2 \alpha}{2g} = \frac{2u^2 \sin \alpha \cos \alpha}{g}$

$\Rightarrow \tan \alpha = 4$

$\Rightarrow \alpha = 76^\circ$

Q. 14. Let the speed of projection =  $u$  and the angle of projection =  $\alpha$

$u_x = u \cos \alpha \quad u_y = u \sin \alpha$

$v_x = u \cos \alpha$

$v_y = u \sin \alpha - gt$

$s_x = ut \cos \alpha$

$s_y = ut \sin \alpha - \frac{1}{2}gt^2$

Maximum height:  $s_y$  when  $v_y = 0$

$v_y = 0$

$\Rightarrow u \sin \alpha - gt = 0$

$\Rightarrow t = \frac{u \sin \alpha}{g}$

$\Rightarrow$  Maximum height =  $u \left( \frac{u \sin \alpha}{g} \right) \sin \alpha - \frac{1}{2}g \left( \frac{u \sin \alpha}{g} \right)^2$

$\Rightarrow$  Maximum height =  $\frac{u^2 \sin^2 \alpha}{g} - \frac{u^2 \sin^2 \alpha}{2g}$

$\Rightarrow$  Maximum height =  $\frac{2u^2 \sin^2 \alpha - u^2 \sin^2 \alpha}{2g}$

$\Rightarrow$  Maximum height =  $\frac{u^2 \sin^2 \alpha}{2g}$

Range:  $s_x$  when  $s_y = 0$

$s_y = 0$

$\Rightarrow ut \sin \alpha - \frac{1}{2}gt^2 = 0$

$\Rightarrow 2ut \sin \alpha - gt^2 = 0$

$\Rightarrow t(2u \sin \alpha - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \quad \quad \quad \underbrace{t = \frac{2u \sin \alpha}{g}}_{\text{Time of Flight}}$

$\Rightarrow$  Range =  $u \left( \frac{2u \sin \alpha}{g} \right) \cos \alpha$

$\Rightarrow$  Range =  $\frac{2u^2 \sin \alpha \cos \alpha}{g}$

Maximum Height = 2(Range)

$\Rightarrow \frac{u^2 \sin^2 \alpha}{2g} = 2 \left( \frac{2u^2 \sin \alpha \cos \alpha}{g} \right)$

$\Rightarrow \sin^2 \alpha = 8 \sin \alpha \cos \alpha$

$\Rightarrow \sin \alpha (\sin \alpha - 8 \cos \alpha) = 0$

$\Rightarrow \cancel{\sin \alpha} = 0 \quad \sin \alpha - 8 \cos \alpha = 0 \quad \dots \text{divide by } \cos \alpha$

$\Rightarrow \tan \alpha - 8 = 0$

$\Rightarrow \tan \alpha = 8$

$\Rightarrow \alpha = \tan^{-1} 8$

$= 83^\circ$

OR  $v_y^2 = u_y^2 + 2a_y s_y$   
 $\therefore 0 = (u \sin \alpha)^2 + 2(-g)H$   
 $\therefore H = \frac{u^2 \sin^2 \alpha}{2g}$

**Q. 15.** (i)  $\frac{u^2 \sin^2 \alpha}{2g} = 3.6 \dots \textcircled{1}$   
 $\frac{2u^2 \sin \alpha \cos \alpha}{g} = 19.2 \dots \textcircled{2}$

Dividing  $\textcircled{1}$  by  $\textcircled{2}$

$$\Rightarrow \frac{\tan \alpha}{4} = \frac{3}{16}$$

$$\Rightarrow \tan \alpha = \frac{3}{4}$$

$$\Rightarrow \sin \alpha = \frac{3}{5}$$

Putting this result into  $\textcircled{1}$  gives

$$\frac{u^2 \left(\frac{9}{25}\right)}{19.6} = 3.6$$

$$\Rightarrow u^2 = 196$$

$$\Rightarrow u = 14 \text{ m/s}$$

(ii)  $R_{\max} = \frac{u^2}{g}$   
 $= \frac{196}{9.8} = 20 \text{ m}$

**Q. 16.** (i)  $30^\circ, 150^\circ$

(ii)  $R = \frac{u^2 \sin 2\alpha}{g}$   
 $= 40$

$$\Rightarrow \frac{784 \sin 2\alpha}{9.8} = 40$$

$$\Rightarrow \sin 2\alpha = \frac{1}{2}$$

$$\Rightarrow 2\alpha = 30^\circ \quad \text{OR} \quad 150^\circ$$

$$\Rightarrow \alpha = 15^\circ \quad \text{OR} \quad 75^\circ$$

**Q. 17.**  $H = \frac{u^2 \sin^2 \alpha}{2g}$

$$\Rightarrow 2.5 = \frac{100 \sin^2 \alpha}{19.6}$$

$$\Rightarrow \sin^2 \alpha = 0.49$$

$$\Rightarrow \sin \alpha = 0.7$$

**Q. 18.** As in Q. 18(iii),  $\alpha = 45^\circ$

$$H = \frac{u^2 \sin^2 45^\circ}{2g}$$

$$= \frac{u^2 \left(\frac{1}{2}\right)}{2g}$$

$$= \frac{u^2}{4g}$$

$$R = \frac{u^2 \sin 2(45^\circ)}{g}$$

$$= \frac{u^2}{g}$$

Therefore,  $H : R = 1 : 4$

**Q. 19.** (i) Let  $T =$  time of flight

$$\therefore PT = 60 \dots \textcircled{I} \quad \text{and} \quad QT - \frac{1}{2}(9.8)T^2 = 0 \dots \textcircled{II}$$

When it is at greatest height

$$v_y^2 = u_y^2 + 2a_y s_y$$

$$0 = Q^2 + 2(-9.8)(5.625)$$

$$\therefore 110.25 = Q^2$$

$$\therefore Q = 10.5$$

Put this into  $\textcircled{II}$ :  $10.5T - 4.9T^2 = 0$

$$\therefore T = \frac{15}{7}$$

Put this into  $\textcircled{I}$ :  $P\left(\frac{15}{7}\right) = 60$

$$\therefore P = 28$$

(ii) The greatest distance  $= 7 \times \frac{15}{7}$   
 $= 15 \text{ m}$



**Q. 20.** Let the angle of projection =  $\alpha$

$$u_x = u \cos \alpha \quad u_y = u \sin \alpha$$

$$v_x = u \cos \alpha$$

$$v_y = u \sin \alpha - gt$$

$$s_x = ut \cos \alpha$$

$$s_y = ut \sin \alpha - \frac{1}{2}gt^2$$

Greatest height:  $s_y$  when  $v_y = 0$

$$v_y = 0$$

$$\Rightarrow u \sin \alpha - gt = 0$$

$$\Rightarrow t = \frac{u \sin \alpha}{g}$$

$$\Rightarrow \text{Greatest height} = u \left( \frac{u \sin \alpha}{g} \right) \sin \alpha - \frac{1}{2}g \left( \frac{u \sin \alpha}{g} \right)^2$$

$$\Rightarrow \text{Greatest height} = \frac{u^2 \sin^2 \alpha}{g} - \frac{u^2 \sin^2 \alpha}{2g}$$

$$\Rightarrow \text{Greatest height} = \frac{2u^2 \sin^2 \alpha - u^2 \sin^2 \alpha}{2g}$$

$$\Rightarrow \text{Greatest height} = \frac{u^2 \sin^2 \alpha}{2g}$$

Range:  $s_x$  when  $s_y = 0$

$$s_y = 0$$

$$\Rightarrow ut \sin \alpha - \frac{1}{2}gt^2 = 0$$

$$\Rightarrow 2ut \sin \alpha - gt^2 = 0$$

$$\Rightarrow t(2u \sin \alpha - gt) = 0$$

$$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \quad \underbrace{t = \frac{2u \sin \alpha}{g}}_{\text{Time of Flight}}$$

$$\Rightarrow \text{Range} = u \left( \frac{2u \sin \alpha}{g} \right) \cos \alpha$$

$$\Rightarrow \text{Range} = \frac{2u^2 \sin \alpha \cos \alpha}{g}$$

$$\frac{\text{Greatest height}}{\text{Range}} = \frac{2}{7}$$

$$\Rightarrow 2(\text{Range}) = 7(\text{Greatest height})$$

$$\Rightarrow 2 \left( \frac{2u^2 \sin \alpha \cos \alpha}{g} \right) = 7 \left( \frac{u^2 \sin^2 \alpha}{2g} \right) \quad \dots \text{ multiply by } \frac{2g}{u^2}$$

$$\Rightarrow 8 \sin \alpha \cos \alpha = 7 \sin^2 \alpha$$

$$\Rightarrow \sin \alpha (8 \cos \alpha - 7 \sin \alpha) = 0$$

$$\Rightarrow \cancel{\sin \alpha} < 0 \quad 8 \cos \alpha - 7 \sin \alpha = 0 \quad \dots \text{ divide by } \cos \alpha$$

$$\Rightarrow 8 - 7 \tan \alpha = 0$$

$$\Rightarrow \tan \alpha = \frac{8}{7}$$

$$\Rightarrow \alpha = \tan^{-1} \left( \frac{8}{7} \right)$$

$$= 49^\circ$$

Exercise 3C

Q. 1.  $u_x = \sqrt{24g} \cos A$        $u_y = \sqrt{24g} \sin A$

$v_x = \sqrt{24g} \cos A$

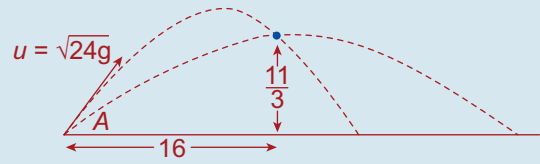
$v_y = \sqrt{24g} \sin A - gt$

$s_x = \sqrt{24g} t \cos A$

$s_y = \sqrt{24g} t \sin A - \frac{1}{2}gt^2$

$s_x = 16$  when  $\vec{s}_y = \frac{11}{3}$

$\Rightarrow \sqrt{24g} t \cos A = 16$       when       $\sqrt{24g} t \sin A - \frac{1}{2}gt^2 = \frac{11}{3}$   
 $\Rightarrow t = \frac{16}{\sqrt{24g} \cos A}$



$\frac{\sin A}{\cos A} = \tan A$

$\frac{1}{\cos^2 A} = 1 + \tan^2 A$

$\Rightarrow \sqrt{24g} \left[ \frac{16}{\sqrt{24g} \cos A} \right] \sin A - \frac{1}{2g} \left[ \frac{256}{24g \cos^2 A} \right] = \frac{11}{3}$

$\Rightarrow 16 \tan A - \frac{16}{3}(1 + \tan^2 A) = \frac{11}{3}$

$\Rightarrow 48 \tan A - 16 - 16 \tan^2 A = 11$

$\Rightarrow 16 \tan^2 A - 48 \tan A + 27 = 0$  ... let  $x = \tan A$

$\Rightarrow 16x^2 - 48x + 27 = 0$

$\Rightarrow (4x - 9)(4x - 3) = 0$

$\Rightarrow x = \frac{9}{4}$       OR       $x = \frac{3}{4}$

$\Rightarrow \tan A = \frac{9}{4}$       OR       $\tan A = \frac{3}{4}$

$\Rightarrow A = 66^\circ$       OR       $A = 37^\circ$

Q. 2.  $u_x = \sqrt{g} \cos A$        $u_y = \sqrt{g} \sin A$

$v_x = \sqrt{g} \cos A$

$v_y = \sqrt{g} \sin A - gt$

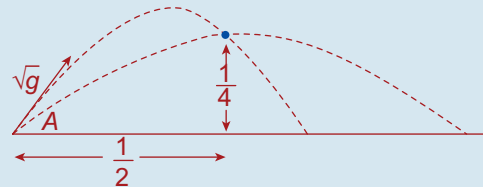
$s_x = t\sqrt{g} \cos A$

$s_y = t\sqrt{g} \sin A - \frac{1}{2}gt^2$

$s_x = \frac{1}{2}$       when       $s_y = \frac{1}{4}$

$\Rightarrow t\sqrt{g} \cos A = \frac{1}{2}$       when       $t\sqrt{g} \sin A - \frac{1}{2}gt^2 = \frac{1}{4}$

$\Rightarrow t = \frac{1}{2\sqrt{g} \cos A}$



$\frac{\sin A}{\cos A} = \tan A$

$\frac{1}{\cos^2 A} = 1 + \tan^2 A$

$\Rightarrow \left[ \frac{1}{2\sqrt{g} \cos A} \right] \sqrt{g} \sin A - \frac{1}{2g} \left[ \frac{1}{4g \cos^2 A} \right] = \frac{1}{4}$

$\Rightarrow \frac{1}{2} \tan A - \frac{1}{8}(1 + \tan^2 A) = \frac{1}{4}$  ... multiply by 8.

$\Rightarrow 4 \tan A - 1 - \tan^2 A = 2$

$$\Rightarrow \tan^2 A - 4 \tan A + 3 = 0 \quad \dots \text{ let } x = \tan A$$

$$\Rightarrow x^2 - 4x + 3 = 0$$

$$\Rightarrow (x - 3)(x - 1) = 0$$

$$\Rightarrow x = 3 \quad \text{OR} \quad x = 1$$

$$\Rightarrow \tan A = 3 \quad \text{OR} \quad \tan A = 1$$

$$\Rightarrow A = 72^\circ \quad \text{OR} \quad A = 45^\circ$$

**Q. 3.**  $u_x = 4\sqrt{2g} \cos A$        $u_y = 4\sqrt{2g} \sin A$

$$v_x = 4\sqrt{2g} \cos A$$

$$v_y = 4\sqrt{2g} \sin A - gt$$

$$s_x = 4t\sqrt{2g} \cos A$$

$$s_y = 4t\sqrt{2g} \sin A - \frac{1}{2}gt^2$$

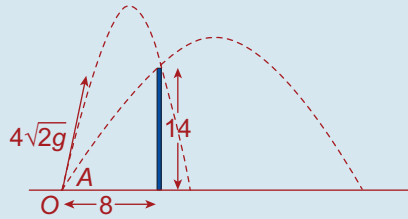
$$s_x = 8$$

$$\text{when } s_y = 14$$

$$\Rightarrow 4t\sqrt{2g} \cos A = 8 \quad \text{when} \quad 4t\sqrt{2g} \sin A - \frac{1}{2}gt^2 = 14$$

$$\Rightarrow t\sqrt{2g} \cos A = 2$$

$$\Rightarrow t = \frac{2}{\sqrt{2g} \cos A}$$



$$\frac{\sin A}{\cos A} = \tan A$$

$$\frac{1}{\cos^2 A} = 1 + \tan^2 A$$

$$\Rightarrow 4 \left[ \frac{2}{\sqrt{2g} \cos A} \right] \sqrt{2g} \sin A - \frac{1}{2}g \left[ \frac{2}{g \cos^2 A} \right] = 14$$

$$\Rightarrow 8 \tan A - (1 + \tan^2 A) = 14$$

$$\Rightarrow \tan^2 A - 8 \tan A + 15 = 0 \quad \dots \text{ let } x = \tan A$$

$$\Rightarrow x^2 - 8x + 15 = 0$$

$$\Rightarrow (x - 5)(x - 3) = 0$$

$$\Rightarrow x = 5 \quad \text{OR} \quad x = 3$$

$$\Rightarrow \tan A = 5 \quad \text{OR} \quad \tan A = 3$$

$$\Rightarrow A = 79^\circ \quad \text{OR} \quad A = 72^\circ$$

We get the time of flight by letting  $s_x = 8$  (Flight ends when it hits the target)

Firstly, look at the particle fired at an angle  $A$  where  $\tan A = 5 \Rightarrow \cos A = \frac{1}{\sqrt{26}}$

$$\Rightarrow 4t\sqrt{2g} \cos A = 8$$

$$\Rightarrow t\sqrt{2g} \cos A = 2$$

$$\Rightarrow t = \frac{2}{\sqrt{2g} \cos A}$$

$$= \sqrt{\frac{2}{g}} \left[ \frac{1}{\cos A} \right]$$

$$= \sqrt{\frac{2}{g}} \sqrt{26}$$

$$= \sqrt{\frac{52}{g}} = 2.3 \text{ s}$$

Secondly, look at the particle fired at an angle  $A$  where  $\tan A = 3 \Rightarrow \cos A = \frac{1}{\sqrt{10}}$

$$t = \sqrt{\frac{2}{g}} \left[ \frac{1}{\cos A} \right]$$

$$= \sqrt{\frac{2}{g}} \sqrt{10} = \sqrt{\frac{20}{g}} = 1.4 \text{ s}$$

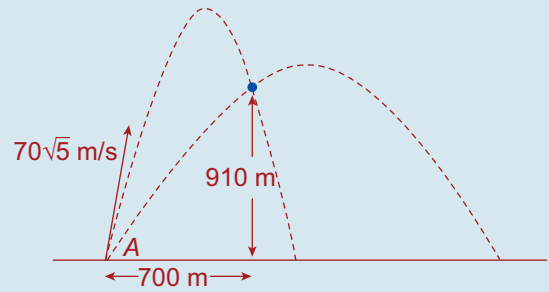
**Q. 4.**  $u_x = 70\sqrt{5} \cos A$        $u_y = 70\sqrt{5} \sin A$

$v_x = 70\sqrt{5} \cos A$

$v_y = 70\sqrt{5} \sin A - gt$

$s_x = 70t\sqrt{5} \cos A$

$s_y = 70t\sqrt{5} \sin A - \frac{1}{2}gt^2$



(i)  $s_x = 700$                       when               $s_y = 910$

$70t\sqrt{5} \cos A = 700$     when               $70t\sqrt{5} \sin A - \frac{1}{2}gt^2 = 910$

$\Rightarrow t = \frac{700}{70\sqrt{5} \cos A}$

$\Rightarrow t = \frac{10}{\sqrt{5} \cos A}$

$\frac{\sin A}{\cos A} = \tan A$

$\frac{1}{\cos^2 A} = 1 + \tan^2 A$

$\Rightarrow 70 \left[ \frac{10}{\sqrt{5} \cos A} \right] \sqrt{5} \sin A - \frac{1}{2}g \left[ \frac{100}{5 \cos^2 A} \right] = 910$

$\Rightarrow 700 \tan A - 10g(1 + \tan^2 A) = 910$

$\Rightarrow 700 \tan A - 98 - 98 \tan^2 A = 910$

$\Rightarrow 98 \tan^2 A - 700 \tan A + 1,008 = 0$  ... divide by 14

$\Rightarrow 7 \tan^2 A - 50 \tan A + 72 = 0$  ... let  $x = \tan A$

$\Rightarrow 7x^2 - 50x + 72 = 0$

$\Rightarrow (7x - 36)(x - 2)$

$\Rightarrow x = \frac{36}{7}$     **OR**     $x = 2$

$\Rightarrow \tan A = \frac{36}{7}$     **OR**     $\tan A = 2$

$\Rightarrow x = 79^\circ$     **OR**     $x = 63^\circ$

(ii) First particle is fired at an angle  $A$  where  $\tan A = \frac{36}{7} \Rightarrow \cos A = \frac{7}{\sqrt{1,345}}$

Time of Flight: Let  $s_x = 700$  (flight ends when it hits the target)

$\Rightarrow 70t\sqrt{5} \cos A = 700$

$\Rightarrow t\sqrt{5} \cos A = 10$

$\Rightarrow t = \frac{10}{\sqrt{5} \cos A}$

$= \frac{2\sqrt{5}}{\cos A}$

$= 2\sqrt{5} \left[ \frac{\sqrt{1,345}}{7} \right] = 23.43 \text{ s}$

Second particle is fired at an angle  $A$  where  $\tan A = 2 \Rightarrow \cos A = \frac{1}{\sqrt{5}}$

Time of Flight: Let  $s_x = 700$

$\Rightarrow 70t\sqrt{5} \cos A = 700$

$\Rightarrow t = \frac{2\sqrt{5}}{\cos A}$

$= 2\sqrt{5} \left[ \frac{\sqrt{5}}{1} \right] = 10 \text{ s}$

(iii)  $23.43 - 10 = 13.43$

$\Rightarrow 13.5$  seconds elapses between 1st and 2nd hit to the nearest half second.

**Q. 5.** (a)  $\sin^2 A + \cos^2 A = 1$  [ $\div \cos^2 A$ ]

$$\Rightarrow \tan^2 A + 1 = \sec^2 A$$

(b)  $\vec{u} = 35\sqrt{5} \cos\alpha \vec{i} + 35\sqrt{5} \sin\alpha \vec{j}$

$$s_x = 35\sqrt{5} \cos\alpha t = 350$$

$$\Rightarrow t = \frac{350}{35\sqrt{5} \cos\alpha}$$

$$= \frac{10}{\sqrt{5} \cos\alpha}$$

At  $t = \frac{10}{\sqrt{5} \cos\alpha}$

$$s_y = 35\sqrt{5} \sin\alpha t - 4.9t^2$$

$$= 210$$

$$\Rightarrow 35\sqrt{5} \sin\alpha \left( \frac{10}{\sqrt{5} \cos\alpha} \right) - 4.9 \left( \frac{100}{5 \cos^2 \alpha} \right) = 210$$

$$\Rightarrow 350 \tan\alpha - 98 \sec^2 \alpha = 210$$

$$\Rightarrow 25 \tan\alpha - 7 \sec^2 \alpha = 15$$

$$\Rightarrow 25 \tan\alpha - 7(\tan^2 \alpha + 1) = 15$$

$$\Rightarrow 25T - 7T^2 - 7 = 15 \quad (\text{where } T = \tan \alpha)$$

$$\Rightarrow 7T^2 - 25T + 22 = 0$$

$$\Rightarrow (7T - 11)(T - 2) = 0$$

$$\Rightarrow \tan\alpha = \frac{11}{7} \quad \text{OR} \quad 2$$

$$\left( \Rightarrow \cos\alpha = \frac{7}{\sqrt{170}} \quad \text{OR} \quad \frac{1}{\sqrt{5}} \right)$$

Now  $t = \frac{10}{\sqrt{5} \cos\alpha}$

$$= \frac{10}{7} \sqrt{34} \text{ s} \quad \text{OR} \quad 10 \text{ s}$$

**Q. 6. Bullet**

Angle of projection is  $\alpha$  where  $\tan\alpha = \frac{1}{2}$

$$\Rightarrow \cos\alpha = \frac{2}{\sqrt{5}} \text{ and } \sin\alpha = \frac{1}{\sqrt{5}}$$

$$u_x = 70\sqrt{5} \cos\alpha$$

$$= 70\sqrt{5} \left( \frac{2}{\sqrt{5}} \right)$$

$$= 140$$

$$u_y = 70\sqrt{5} \sin\alpha$$

$$= 70\sqrt{5} \left( \frac{1}{\sqrt{5}} \right)$$

$$= 70$$

$$v_x = 140$$

$$v_y = 70 - gt$$

$$s_x = 140t$$

$$s_y = 70t - \frac{1}{2}gt^2$$

Bullet will strike fighter when bullet has travelled 210 m vertically, i.e. when  $s_y = 210$ .

$$70t - \frac{1}{2}gt^2 = 210$$

$$\Rightarrow 4.9t^2 - 70t + 210 = 0 \quad \dots \text{ divide by } 0.7$$

$$\Rightarrow 7t^2 - 100t + 300 = 0$$

$$\Rightarrow (7t - 30)(t - 10) = 0$$

$$\Rightarrow t = \frac{30}{7}, t = 10 \quad \dots \text{ bullet will strike fighter on the way up}$$

$$\Rightarrow t = \frac{30}{7} \text{ s}$$

**Q. 7. Bullet**

(i) Let the angle of projection be  $\alpha$ .

$u_x = v_x = 35 \cos \alpha = 28$  ... must match the speed of the bird

$$\Rightarrow \cos \alpha = \frac{4}{5}$$

$$\Rightarrow \tan \alpha = \frac{3}{4}$$

$$\Rightarrow \alpha = \tan^{-1} \frac{3}{4}$$

$$u_y = 35 \sin \alpha$$

$$= 35 \left( \frac{3}{5} \right)$$

$$= 21$$

$$\Rightarrow s_y = 21t - \frac{1}{2}gt^2$$

Bullet will reach bird when  $s_y = 5.6$

$$\Rightarrow 21t - 4.9t^2 = 5.6$$

$$\Rightarrow 4.9t^2 - 21t + 5.6 = 0 \quad \dots \text{divide by } 0.7$$

$$\Rightarrow 7t^2 - 30t + 8 = 0$$

$$\Rightarrow (7t - 2)(t - 4) = 0$$

$$\Rightarrow t = \frac{2}{7}, t = 4 \quad \dots \text{bullet will strike bird on the way up}$$

$$\Rightarrow t = \frac{2}{7} \text{ s}$$

**Q. 8. (i)  $u_x = 35 \cos A$**

$$u_y = 35 \sin A$$

$$x = s_x = 35t \cos A$$

$$y = s_y = 35t \sin A - \frac{1}{2}gt^2$$

$$250y = 250(\tan A)x - (1 + \tan^2 A)x^2$$

$$\Leftrightarrow 250 \left( 35t \sin A - \frac{1}{2}gt^2 \right) = 250(\tan A)(35t \cos A) - (1 + \tan^2 A)(35t \cos A)^2$$

$$\Leftrightarrow 8,750t \sin A - 1,225t^2 = 8,750 \tan A \cos A - (1 + \tan^2 A)(1,225t^2 \cos^2 A)$$

$$\Leftrightarrow 8,750 \sin A - 1,225t^2 = 8,750 \sin A - \left( \frac{1}{\cos^2 A} \right) (1,225t^2 \cos^2 A)$$

$$\Leftrightarrow 8,750 \sin A - 1,225t^2 = 8,750 \sin A - 1,225t^2 \quad \text{QED}$$

(ii)  $250y = 250(\tan A)x - (1 + \tan^2 A)x^2$  ... let  $x = 40$  and  $y = 20$

$$\Rightarrow 5,000 = 10,000 (\tan A) - (1 + \tan^2 A)(1,600)$$

$$\Rightarrow 1,600 \tan^2 A - 10,000 \tan A + 6,600 = 0 \quad \dots \text{divide by } 200$$

$$\Rightarrow 8 \tan^2 A - 50 \tan A + 33 = 0$$

$$\Rightarrow (4 \tan A - 3)(2 \tan A - 11) = 0$$

$$\Rightarrow \tan A = \frac{3}{4}, \tan A = \frac{11}{2}$$

Will reach target when  $x = 40$

$$\tan A = \frac{3}{4}$$

$$\Rightarrow \cos A = \frac{4}{5}$$

$$\Rightarrow x = 35t\left(\frac{4}{5}\right)$$

$$\Rightarrow = 40$$

$$\Rightarrow 28t = 40$$

$$\Rightarrow t = \frac{10}{7} \text{ s}$$

$$\tan A = \frac{11}{2}$$

$$\Rightarrow \cos A = \frac{2}{\sqrt{125}} = \frac{2}{5\sqrt{5}}$$

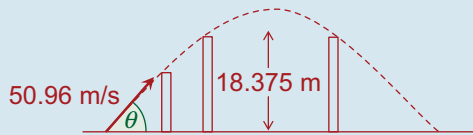
$$\Rightarrow x = 35t\frac{2}{5\sqrt{5}}$$

$$= 40$$

$$\Rightarrow \frac{14t}{\sqrt{5}} = 40$$

$$\Rightarrow t = \frac{20\sqrt{5}}{7} \text{ s}$$

Q. 9.



$$\tan \theta = \frac{5}{12} \Rightarrow \cos \theta = \frac{12}{13} \text{ and } \sin \theta = \frac{5}{13}$$

$$u_x = 50.96 \cos \theta = 50.96\left(\frac{12}{13}\right) = 47.04$$

$$u_y = 50.96 \sin \theta = 50.96\left(\frac{5}{13}\right) = 19.6$$

$$vc_x = 47.04$$

$$v_y = 19.6 - 9.8t$$

$$s_x = 47.04t$$

$$s_y = 19.6t - 4.9t^2$$

Passes over a wall 14.7 m high on its upward path.

Find  $s_x$  when  $s_y = 14.7$  to find the horizontal displacement of the first wall.

$$s_y = 14.7$$

$$\Rightarrow 19.6t - 4.9t^2 = 14.7 \quad \dots \text{ divide by } 4.9$$

$$\Rightarrow 4t - t^2 = 3$$

$$\Rightarrow t^2 - 4t + 3 = 0$$

$$\Rightarrow (t - 1)(t - 3) = 0$$

$$\Rightarrow t = 1 \quad \text{OR} \quad t = 3$$

$\Rightarrow$  Passes over wall after 1 second.

$$s_x = 47 - 04t \quad \dots \text{ let } t = 1$$

$$\Rightarrow s_x = 47.04 \text{ m}$$

Now find  $s_x$  when  $s_y = 18.375$  to find horizontal displacements of the two possible walls of height 18.375 m.

$$s_y = 18.375$$

$$\Rightarrow 19.6t - 4.9t^2 = 18.375 \quad \dots \text{ divide by } 4.9$$

$$\Rightarrow 4t - t^2 = 3.75$$

$$\Rightarrow t^2 - 4t + 3.75 = 0 \quad \dots \text{ multiply by } 4$$

$$\Rightarrow 4t^2 - 16t + 15 = 0$$

$$\Rightarrow (2t - 3)(2t - 5) = 0$$

$$\Rightarrow t = \frac{3}{2} \quad \text{OR} \quad t = \frac{5}{2}$$

$$s_x = 47.04t \quad \dots \text{ let } t = \frac{3}{2}$$

$$\Rightarrow s_x = 70.56$$

$$70.56 - 47.04 = 23.52$$

$\Rightarrow$  Second wall must not be less than 23.52 m from first wall.

$$s_x = 47.04t \quad \dots \text{ let } t = \frac{5}{2}$$

$$s_x = 117.6$$

$$117.6 - 47.04 = 70.56$$

$\Rightarrow$  Second wall must not be more than 70.56 m from first wall.

**Q. 10.** (i) Let initial velocity =  $u$  and let the angle of projection be  $\theta$

$$u_x = u \cos \theta \quad u_y = u \sin \theta$$

$$s_x = ut \cos \theta$$

$$s_y = ut \sin \theta - \frac{1}{2}gt^2$$

$$s_x = 27 \text{ and } s_y = 0 \text{ when } t = 3$$

$$\Rightarrow 3u \cos \theta = 27 \quad \text{and} \quad 3u \sin \theta - 44.1 = 0$$

$$\Rightarrow u \cos \theta = 9 \quad \text{Equation 1} \quad \Rightarrow u \sin \theta = 14.7 \quad \text{Equation 2}$$

$\Rightarrow$  Horizontal component of initial velocity = 9

Vertical component of initial velocity = 14.7

(ii) Need to find  $s_y$  when  $s_x = 5.4$

$$\Rightarrow 9t = 5.4$$

$$\Rightarrow t = 0.6$$

$$s_y = 14.7t - 4.9t^2 \quad \dots \text{ let } t = 0.6$$

$$\begin{aligned} \Rightarrow s_y &= 14.7(0.6) - 4.9(0.6)^2 \\ &= 7.056 \text{ m} \quad \dots \text{ height of wall} \end{aligned}$$

(iii) Need to find  $v_x$  and  $v_y$  when  $t = 0.6$

$$v_x = 9$$

$$\begin{aligned} v_y &= 14.7 - gt \\ &= 14.7 - 9.8(0.6) \\ &= 8.82 \end{aligned}$$

$$\vec{v} = 9\vec{i} + 8.82\vec{j}$$

$$\begin{aligned} \text{Speed} &= |\vec{v}| \\ &= \sqrt{9^2 + 8.82^2} \\ &= 12.6 \text{ m/s} \end{aligned}$$

**Q. 11.** (i)  $s_x = ut$

$$s_y = -\frac{1}{2}gt^2$$

$$s_y = -0.1 \text{ when } s_x = 2$$

$$ut = 2$$

$$\Rightarrow t = \frac{2}{u}$$

$$-\frac{1}{2}gt^2 = -0.1 \quad \dots \text{ let } t = \frac{2}{u}$$

$$\Rightarrow \frac{1}{2}g \left( \frac{4}{u^2} \right) = 0.1$$

$$\Rightarrow \frac{2g}{u^2} = 0.1$$

$$\Rightarrow u^2 = 20g$$

$$\Rightarrow u = \sqrt{20g} \text{ m/s}$$

$$t = \frac{2}{u}$$

$$= \frac{2}{\sqrt{20g}}$$

$$= \frac{1}{7} \text{ s}$$

(ii)  $u = \sqrt{20g}$

$$= 14 \text{ m/s}$$

(iii)  $v_x = 14$

$$\begin{aligned} v_y &= -gt \\ &= -9.8 \left( \frac{1}{7} \right) \\ &= -1.4 \end{aligned}$$

$$\vec{v} = 14\vec{i} - 1.4\vec{j}$$

$$\begin{aligned} \text{Speed} &= |\vec{v}| = \sqrt{14^2 + (-1.4)^2} \\ &= 14.07 \text{ m/s} \end{aligned}$$



**Q. 12.** (i)  $v_x = u \cos \alpha$

$$v_y = u \sin \alpha - gt$$

$$s_x = ut \cos \alpha$$

$$s_y = ut \sin \alpha - \frac{1}{2}gt^2$$

Range:  $s_x$  when  $s_y = 0$

$$ut \sin \alpha - \frac{1}{2}gt^2 = 0$$

$$\Rightarrow 2ut \sin \alpha - gt^2 = 0$$

$$\Rightarrow t(2u \sin \alpha - gt) = 0$$

$$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \quad \quad \quad \underbrace{t = \frac{2u \sin \alpha}{g}}_{\text{Time of Flight}}$$

Point of  
Projection

Time of  
Flight

$$\Rightarrow \text{Range} = u \left[ \frac{2u \sin \alpha}{g} \right] \cos \alpha = \frac{2u^2 \sin \alpha \cos \alpha}{g}$$

(ii) Passes through  $3\vec{i} + \vec{j}$

$$\Rightarrow s_x = 3 \text{ when } s_y = 1$$

$$\Rightarrow ut \cos \alpha = 3 \text{ when } ut \sin \alpha - \frac{1}{2}gt^2 = 1$$

$$\Rightarrow t = \frac{3}{u \cos \alpha} \Rightarrow u \left[ \frac{3}{u \cos \alpha} \right] \sin \alpha - \frac{1}{2}g \left[ \frac{9}{u^2 \cos^2 \alpha} \right] = 1$$

$$\Rightarrow 3 \tan \alpha - \frac{9g}{2u^2} (1 + \tan^2 \alpha) = 1$$

$$\Rightarrow 6u^2 \tan \alpha - 9g(1 + \tan^2 \alpha) = 2u^2$$

$$\Rightarrow 2u^2(3 \tan \alpha - 1) = 9g(1 + \tan^2 \alpha)$$

$$\Rightarrow 2u^2 = \frac{9g(1 + \tan^2 \alpha)}{3 \tan \alpha - 1}$$

Passes through  $\vec{i} + 3\vec{j} \Rightarrow s_x = 1 \text{ when } s_y = 3$

$$\Rightarrow ut \cos \alpha = 1 \quad \text{when} \quad ut \sin \alpha - \frac{1}{2}gt^2 = 3$$

$$\Rightarrow t = \frac{1}{u \cos \alpha} \Rightarrow u \left[ \frac{1}{u \cos \alpha} \right] \sin \alpha - \frac{1}{2}g \left[ \frac{1}{u^2 \cos^2 \alpha} \right] = 3$$

$$\Rightarrow \tan \alpha - \frac{g}{2u^2} (1 + \tan^2 \alpha) = 3 \quad \dots \quad 2u^2 = \frac{9g(1 + \tan^2 \alpha)}{3 \tan \alpha - 1}$$

$$\Rightarrow \tan \alpha - g \left[ \frac{3 \tan \alpha - 1}{9g(1 + \tan^2 \alpha)} \right] (1 + \tan^2 \alpha) = 3$$

$$\Rightarrow \tan \alpha - \frac{3 \tan \alpha - 1}{9} = 3 \quad \dots \text{ multiply by 9}$$

$$\Rightarrow 9 \tan \alpha - 3 \tan \alpha + 1 = 27$$

$$\Rightarrow 6 \tan \alpha = 26$$

$$\Rightarrow \tan \alpha = \frac{13}{3} \quad \dots \text{ as required}$$

$$\Rightarrow \cos \alpha = \frac{3}{\sqrt{178}} \quad \text{and} \quad \sin \alpha = \frac{13}{\sqrt{178}}$$

$$2u^2 = \frac{9g(1 + \tan^2 \alpha)}{3 \tan \alpha - 1} = \frac{9g\left(1 + \frac{169}{9}\right)}{13 - 1} = \frac{9g + 169g}{12} = \frac{178g}{12}$$

$$\text{Range} = \frac{2u^2 \sin \alpha \cos \alpha}{g} = \frac{\left(\frac{178g}{12}\right)\left(\frac{13}{\sqrt{178}}\right)\left(\frac{3}{\sqrt{178}}\right)}{g} = \frac{13}{4} \text{ m}$$

<p><b>Q. 13.</b> <math>\sin \theta = \frac{3}{5} \Rightarrow \cos \theta = \frac{4}{5}</math></p> <p><math>u_x = u \cos \theta = \frac{4u}{5} \quad u_y = u \sin \theta = \frac{3u}{5}</math></p> <p><math>v_x = \frac{4u}{5}</math></p> <p><math>v_y = \frac{3u}{5} - gt</math></p> <p><math>s_x = \frac{4u}{5}t</math></p> <p><math>s_y = \frac{3u}{5}t - \frac{1}{2}gt^2</math></p> <p><math>s_x = 240 \quad \text{when} \quad s_y = 90</math></p>	<p><math>\frac{4u}{5}t = 240</math></p> <p><math>\Rightarrow t = \frac{300}{u}</math></p> <p><math>\Rightarrow \frac{3u}{5}\left[\frac{300}{u}\right] - \frac{1}{2}g\left[\frac{9,0000}{u^2}\right] = 90</math></p> <p><math>\Rightarrow 180 - \frac{441,000}{u^2} = 90</math></p> <p><math>\Rightarrow \frac{441,000}{u^2} = 90</math></p> <p><math>\Rightarrow u^2 = 4,900</math></p> <p><math>\Rightarrow u = 70 \text{ m/s}</math></p>
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<p><b>Q. 14.</b> (i) <b>Bullet</b></p> <p><math>v_x = 70 \cos \theta</math></p> <p><math>v_y = 70 \sin \theta - gt</math></p> <p><math>s_x = 70t \cos \theta</math></p> <p><math>s_y = 70t \sin \theta - \frac{1}{2}gt^2</math></p>	<p><b>Target</b></p> <p><math>v_x = 42\sqrt{2} \cos 45^\circ = 42</math></p> <p><math>v_y = 42\sqrt{2} \sin 45^\circ = 42</math></p> <p><math>s_x = 42t</math></p> <p><math>s_y = 42t + 10</math></p>
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If bullet is to hit target, then x-velocities must match

$$\Rightarrow 70 \cos \theta = 42 \Rightarrow \cos \theta = \frac{42}{70} = \frac{3}{5} \Rightarrow \tan \theta = \frac{4}{3}$$

(ii) Hits target when y-displacements match

$$\Rightarrow 70t \sin \theta - \frac{1}{2}gt^2 = 42t + 10 \quad \dots \sin \theta = \frac{4}{5}$$

$$\Rightarrow 70t\left(\frac{4}{5}\right) - \frac{1}{2}gt^2 = 42t + 10$$

$$\Rightarrow 56t - 4.9t^2 = 42t + 10$$

$$\Rightarrow 4.9t^2 - 14t + 10 = 0$$

$$\Rightarrow 49t^2 - 140t + 100 = 0$$

$$\Rightarrow (7t - 10)(7t - 10) = 0$$

$$\Rightarrow t = \frac{10}{7} \text{ seconds}$$

Horizontal distance =  $s_x = 42t = 42\left(\frac{10}{7}\right) = 60 \text{ m}$