Chapter 6 Exercise 6A

Q. 1. (i) Work $= Fs$

$= 80(30)$

$= 2{,}400$ J

(ii) Power $= \dfrac{\text{Work}}{\text{Time}}$

$= \dfrac{2{,}400}{10}$

$= 240$ W

Q. 2. (i) Work $= Fs$

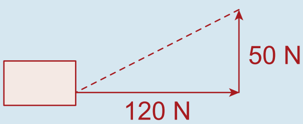
$= 200(12)$

$= 2{,}400$ J

(ii) Power $= \dfrac{2{,}400}{8}$

$= 300$ W

Q. 3.



(i) Work $= Fs$

$= 120(50)$

$= 6{,}000$ J

(ii) Power $= \dfrac{6{,}000}{60}$

$= 100$ W

Q. 4. Power $= Fv$

$= 600(20)$

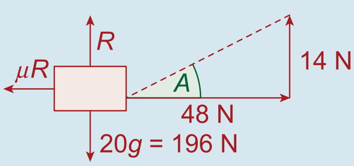
$= 12{,}000$ W

Q. 5. $P = Fv$

$\Rightarrow 100{,}000 = F(5)$

$\Rightarrow F = 20{,}000$ N

Q. 6. Since $\tan A = \dfrac{7}{24}$, $\cos A = \dfrac{24}{25}$, $\sin A = \dfrac{7}{25}$



(i) $R + 14 = 196$

$\Rightarrow R = 182$ N

(ii) $\mu R = \frac{1}{7}(182)$

$= 26$ N

(iii) Work $= Fs$

$= 48(30)$

$= 1{,}440$ J

(iv) $F = ma$

$\Rightarrow (48 - 26) = 20a$

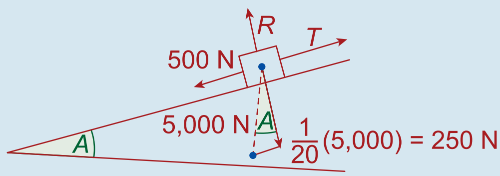
$\Rightarrow a = 1.1$ m/s^2

(v) Power $= \dfrac{\text{Work}}{\text{Time}}$

$= \dfrac{1{,}440}{20}$

$= 72$ W

Q. 7. $\sin A = \dfrac{1}{20}$



No acceleration $\Rightarrow$ $T = 500 + 250$

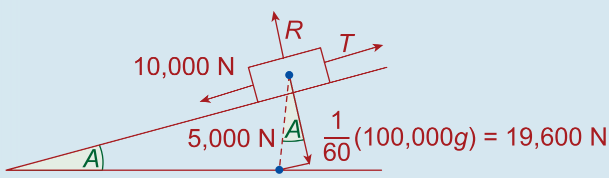
$= 750$ N

Power $= Fv$

$= 750(12)$

$= 9{,}000$ W

Q. 8.



No acceleration $\Rightarrow$ $T = 19{,}600 + 10{,}000$

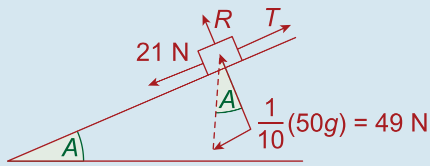
$= 29{,}600$ N

Fv $\Rightarrow$ $P = (29{,}600)(10)$

$= 296{,}000$ W

$= 296$ kW

Q. 9.



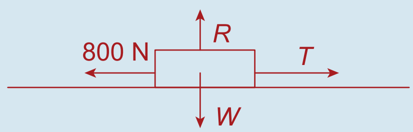
No acceleration $\Rightarrow$ $T = 21 + 49$

$= 70$ N

$P = Fv \Rightarrow 350 = (70)v$

$\Rightarrow v = 5$ m/s

Q. 10. (i)



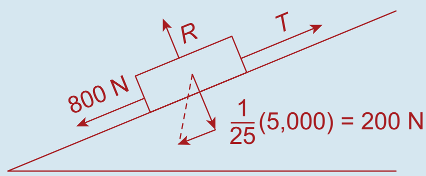
$T = 800$ N

$P = Fv \Rightarrow P = 800(50)$

$= 40{,}000$ W

$= 40$ kW

(ii)



$T = 800 + 200 = 1{,}000$

$P = Fv \Rightarrow 40{,}000 = (1{,}000)v$

$\Rightarrow v = 40$ m/s

Q. 11. (i) Horizontal $= T\cos\theta$

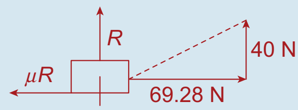
$= 80\ (0.8660)$

$= 69.28$ N

Vertical $= T\sin\theta$

$= 80(0.5)$

$= 40$ N



(ii) $R + 40 = 196 \Rightarrow R = 156$ N

(iii) $\mu R = \frac{1}{3}(156) = 52$ N

$F = ma \Rightarrow (69.28 - 52) = 20a$

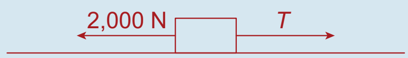
$\Rightarrow a = 0.864$ m/s

(iv) Work $= Fs$

$= 69.28(0.5)$

$= 34.64$ J

Q. 12. (i)



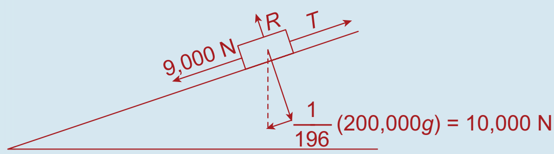
$P = Tv \Rightarrow 500{,}000 = T(10)$

$\Rightarrow T = 50{,}000$

$F = ma \Rightarrow (50{,}000 - 2{,}000)$

$= 200{,}000a$

$\Rightarrow a = 0.24$ m/s^2



(ii) $P = Tv \Rightarrow 500{,}000 = T(20)$

$\Rightarrow T = 25{,}000$ N

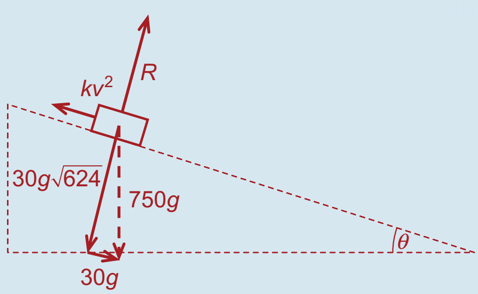
$F = ma$

$\Rightarrow (25{,}000 - 10{,}000 - 9{,}000)$

$= 200{,}000a$

$\Rightarrow a = 0.03$ m/s^2

Q. 13. (i)



$\sin\theta = \frac{1}{25}$

$\Rightarrow \cos\theta = \frac{\sqrt{624}}{25}$

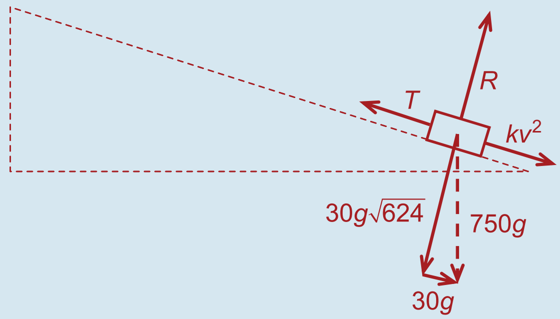
When train attains a speed of 30 m/s

$kv^2 = 30g$

$\Rightarrow k(30)^2 = 30g$

$\Rightarrow 30k = g \quad \Rightarrow k = \frac{g}{30}$

(ii)



$T = kv^2 + 30g$ ... maximum speed attained is 20 m/s

$\Rightarrow T = \left(\frac{g}{30}\right)(20)^2 + 30g$

$\Rightarrow T = \frac{40}{3}g + 30g$

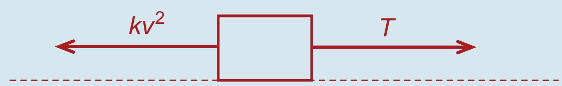
$\Rightarrow T = \frac{130g}{3}$ N

$P = Tv$

$= \left(\frac{130g}{3}\right)(20)$

$= \frac{2{,}600g}{3}$ watts

(iii) $P = \frac{2{,}600g}{3}$



At maximum speed

$T = kv^2$ ... but $T = \frac{P}{v} = \frac{2{,}600g}{3v}$

$\Rightarrow \frac{2{,}600g}{3v} = \left(\frac{g}{30}\right)v^2$

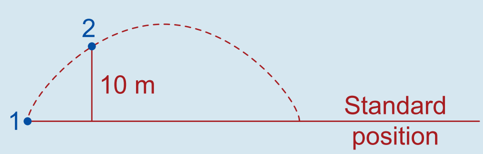
$\Rightarrow 3v^3 = 78{,}000$

$\Rightarrow v^3 = 26{,}000$

$\Rightarrow v = 29.625$ m/s

Exercise 6B

Q. 1.



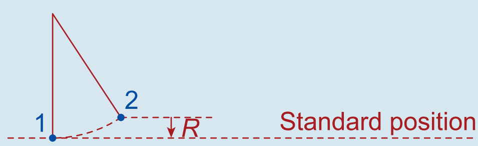
$\frac{1}{2}M(20)^2 + Mg(0) = \frac{1}{2}Mv^2 + Mg(10)$

$\Rightarrow 200 = \frac{1}{2}v^2 + 98$

$\Rightarrow v = \sqrt{204}$

$= 14.28$ m/s

Q. 2.



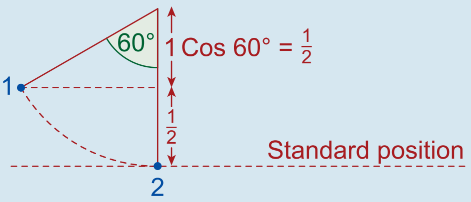
$\frac{1}{2}M(2)^2 + Mg(0) = \frac{1}{2}M(0)^2 + Mgh$

$\Rightarrow 2 = 9.8h$

$\Rightarrow h = \frac{2}{9.8}$

$= \frac{10}{49}$ m

Q. 3.



$$\frac{1}{2}M(0) + Mg\left(\frac{1}{2}\right) = \frac{1}{2}Mv^2 + Mg(0)$$

$$\Rightarrow v^2 = g$$

$$\Rightarrow v = \sqrt{g} = 3.13 \text{ m/s}$$

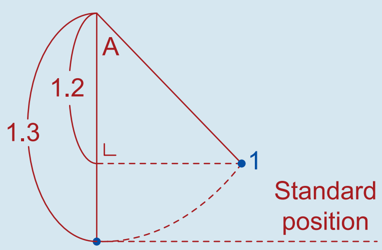
Q. 4. $\frac{1}{2}mu^2 + mgh_1 = \frac{1}{2}mv^2 + mgh_2$

$$\frac{1}{2}(0.4)(7)^2 + (0.4)g(1) = \frac{1}{2}(0.4)(0)^2 + (0.4)g(h)$$

$$\Rightarrow 13.72 = 3.92h$$

$$\Rightarrow h = 3.5 \text{ m}$$

Q. 5.



$$\frac{1}{2}M(0) + Mg(0.1) = \frac{1}{2}Mv^2 + Mg(0)$$

$$\Rightarrow v^2 = 0.2g$$

$$\Rightarrow v = \sqrt{0.2g}$$

$$= \sqrt{\frac{g}{5}}$$

Q. 6. (i) $\frac{1}{2}mu^2 + mgh_1 = \frac{1}{2}mv^2 + mgh_2$

$$\Rightarrow \frac{1}{2}m(14)^2 + mg(0.5) = \frac{1}{2}m(0)^2 + mgh$$

… divide by m

$$\Rightarrow 102.9 = 9.8h$$

$$\Rightarrow h = 10.5 \text{ m}$$

(ii) $\frac{1}{2}mu^2 + mgh_1 = \frac{1}{2}mv^2 + mgh_2$

$$\Rightarrow \frac{1}{2}m(0)^2 + mg(10.5) = \frac{1}{2}mv^2 + mg(0)$$

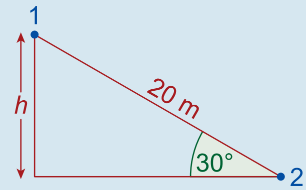
…divide by m

$$\Rightarrow 102.9 = \frac{v^2}{2}$$

$$\Rightarrow v^2 = 205.8$$

$$\Rightarrow v = 14.35 \text{ m/s}$$

Q. 7. Since no force, apart from the gravitational force, does work on the sleigh.



$$h = 20 \sin 30°$$

$$= 20\left(\frac{1}{2}\right)$$

$$= 10 \text{ m}$$

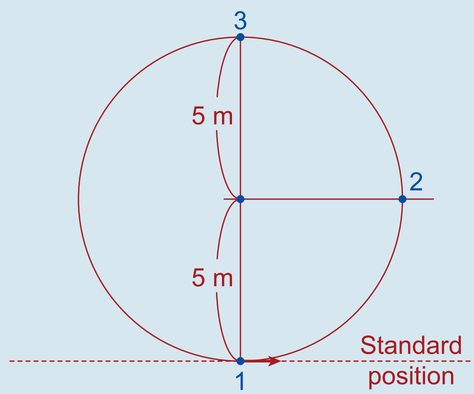
$$\frac{1}{2}M(0)^2 + Mg(10) = \frac{1}{2}Mv^2 + Mg(0)$$

$$\Rightarrow v^2 = 20g$$

$$= 196$$

$$\Rightarrow v = 14 \text{ m/s}$$

Q. 8.



(i) From position 1 to position 2

$$\frac{1}{2}M(14)^2 + Mg(0) = \frac{1}{2}Mv^2 + Mg(5)$$

$$\Rightarrow 98 = \frac{1}{2}v^2 + 49$$

$$\Rightarrow v^2 = 98$$

$$\Rightarrow v = \sqrt{98}$$

$$= 7\sqrt{2} \text{ m/s}$$

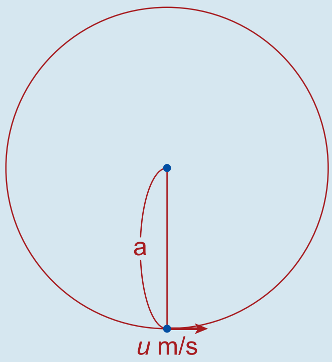
(ii) From position 1 to position 3

$$\frac{1}{2}M(14)^2 + Mg(0) = \frac{1}{2}Mv^2 + Mg(10)$$

$$\Rightarrow 98 = \frac{1}{2}v^2 + 98$$

$\Rightarrow v = 0$ It will just reach (3)

Q. 9.



(i) K.E. + P.E. $= \frac{1}{2}M(2ga) + Mg(0)$

$= Mga$

(ii) $Mga = \frac{1}{2}M(0)^2 + Mgh$

$\Rightarrow h = a$

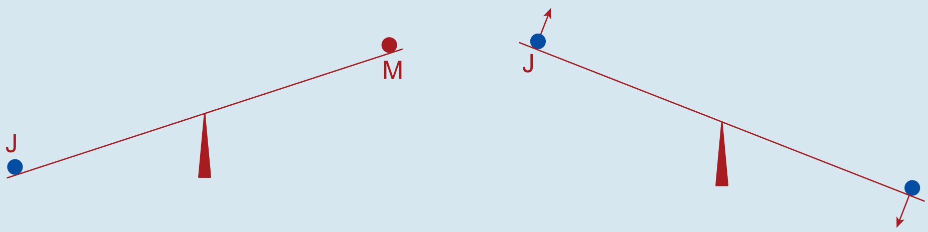
$= a$

(iii) $Mga = \frac{1}{2}Mv^2 + Mg\left(\frac{1}{2}a\right)$

$\Rightarrow v^2 = ga$

$\Rightarrow v = \sqrt{ga}$ m/s

Q. 10.



Energy Before = **Energy After**

$$\underbrace{\frac{1}{2}M(0)^2 + Mg(0)}_{\text{Johnny}} + \underbrace{\frac{1}{2}\left(\frac{3}{2}M\right)(0)^2 + \frac{3}{2}Mg\left(\frac{1}{2}\right)}_{\text{Margery}} = \underbrace{\frac{1}{2}Mv^2 + Mg\left(\frac{1}{2}\right)}_{\text{Johnny}} + \underbrace{\frac{1}{2}\left(\frac{3}{2}M\right)v^2 + \frac{3}{2}Mg(0)}_{\text{Margery}}$$

$$\Rightarrow \frac{3}{4}Mg = \frac{1}{2}Mv^2 + \frac{1}{2}Mg + \frac{3}{4}Mv^2$$

$$\Rightarrow Mg = 5Mv^2$$

$$\Rightarrow v = \sqrt{\frac{g}{5}}$$

$$= \sqrt{\frac{9.8}{5}}$$

$$= \sqrt{1.96}$$

$$= 1.4 \text{ m/s}$$

Exercise 6C

Q. 1. $I = Mv - Mu$

$= (0.125)(40) - (0.125)(0)$

$= 5$ Ns

Q. 2. $I = Mv - Mu$

$= \left(\frac{1}{4}\right)(20) - \frac{1}{4}(-40)$

$= 15$ Ns

Q. 3. $M_1U_1 + M_2U_2 = M_1V_1 + M_2V_2$

(Hammer) (Stake) (Hammer) (Stake)

$\Rightarrow 4(2) + 1(0) = 4(0) + 1(V_2)$

$\Rightarrow V_2 = 8$ m/s

(i) $\vec{I} = M\vec{v} - M\vec{u}$

$= (1)(-8\vec{j}) - (1)(0)$

$= -8\vec{j}$ Ns

(ii) $\vec{I} = M\vec{v} - M\vec{u}$

$= 4(\vec{0}) - 4(-2\vec{j})$

$= 8\vec{j}$ Ns

Q. 4. $I = |M\vec{v} - M\vec{u}|$

$= |(0.1)(0) - (0.1)(8)|$

$= 0.8$ Ns

Q. 5. $I = |M\vec{v} - M\vec{u}|$

$= |2(0) - 2(5)|$

$= 10$ Ns

Q. 6. (i) $4m(12) + m(0) = 5m(v)$ … divide by m

$\Rightarrow 5v = 48$

$\Rightarrow v = 9.6$ m/s

(ii) $5m(12) + nm(0) = (5 + n)m(7.5)$

… divide by m

$\Rightarrow 60 = 37.5 + 7.5n$

$\Rightarrow 7.5n = 22.5$

$\Rightarrow n = 3$

Q. 7. $M_1U_1 + M_2U_2 = M_1V_1 + M_2V_2$

$\Rightarrow (0.15)(200) + (3)(0) = (0.15)(100) + (3)V_2$

$\Rightarrow V_2 = 5$ m/s

Q. 8. $M_1\overrightarrow{U_1} + M_2\overrightarrow{U_2} = (M_1 + M_2)\overrightarrow{v^2}$

$(0.1)400\vec{i} + (3)(10\vec{j}) = (0.1 + 3)\overrightarrow{v^2}$

$\Rightarrow 3.1\overrightarrow{v^2} = 40\vec{i} + 30\vec{j}$

$\Rightarrow \vec{v} = \frac{10}{31}(40\vec{j} + 30\vec{j})$

$\Rightarrow |\overrightarrow{v^2}| = \frac{10}{31}\sqrt{40^2 + 30^2}$

$= \frac{500}{31} = 12.9$ m/s

Q. 9. $M_1U_1 + M_2U_2 = (M_1 + M_2)\vec{V}$

$(0.1)(200) + M_2(0) = (0.1 + M_2)10$

$\Rightarrow 20 = 1 + 10\,M_2$

$\Rightarrow M_2 = 1.9$ kg

Q. 10. $M_1U_1 + M_2U_2 = (M_1 + M_2)V$

$(0.05)U_1 + (1.45)(0) = (0.05 + 1.45)(4)$

$\Rightarrow 0.05U_1 = 6$

$\Rightarrow U_1 = 120$ m/s

Q. 11. **Step 1:** Find the speed of the joint mass after impact.

$\frac{1}{2}MV_1{}^2 + Mgh_1 = \frac{1}{2}MV_2{}^2 + Mgh_2$

$\frac{1}{2}(3)V_1{}^2 + (3)g(0) = \frac{1}{2}(3)(0)^2 + (3)(9.8)(10)$, since it rises 10 m.

$\Rightarrow V_1{}^2 = 196$

$\Rightarrow V_1 = 14$ m/s

Step 2: To find speed of the bullet before impact:

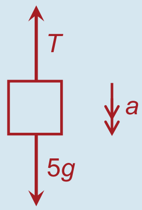
$M_1U_1 + M_2U_2 = (M_1 + M_2)V$

$(0.1)U_1 + (2.9)(0) = (0.1 + 2.9)(14)$

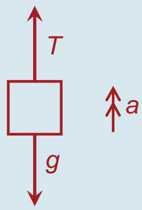
$\Rightarrow U_1 = 420$ m/s

Exercise 6D

Q. 1. (i) Let a = the common acceleration of the particles during the first second.



$5g - T = 5a$ **Equation 1**



$T - g = a$ **Equation 2**

Adding equations **1** and **2** we get:

$4g = 6a$

$\Rightarrow a = \frac{2}{3}g$ m/s^2

After 1 second, the distance travelled by the 5 kg particle will be given by

$s = ut + \frac{1}{2}at^2$

$\Rightarrow s = (0)(1) + \frac{1}{2}\left(\frac{2}{3}g\right)(1)^2$

$\Rightarrow s = \frac{1}{3}g$ m

(ii) Firstly, we must calculate the speed just before the 2 kg mass is picked up:

$v = u + at = 0 + \left(\frac{2}{3}g\right)(1)$

$= \frac{2}{3}g$ m/s

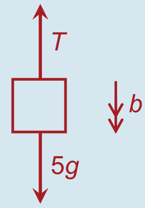
$m_1u = m_2v$

$\Rightarrow 6\left(\frac{2}{3}g\right) = 8v$

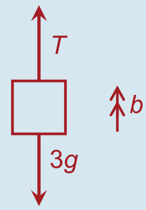
$\Rightarrow 8v = 4g$

$\Rightarrow v = \frac{1}{2}g$ m/s ... speed directly after 2 kg mass is picked up.

Let b = the new common acceleration of the particles.



$5g - T = 5b$ **Equation 3**



$T - 3g = 3b$ **Equation 4**

Adding equations **3** and **4** we get:

$2g = 8b$

$\Rightarrow b = \frac{1}{4}g$ m/s^2

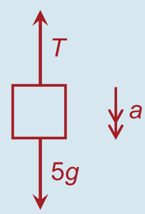
$\Rightarrow$ The distance travelled in the 2nd second is given by

$s = ut + \frac{1}{2}at^2$

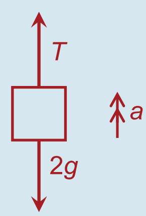
$\Rightarrow s = \left(\frac{1}{2}g\right)(1) + \frac{1}{2}\left(\frac{1}{4}g\right)(1)^2$

$\Rightarrow s = \frac{5}{8}g$ m

Q. 2. Let a = the common acceleration of the particles during the first 3 seconds.



$5g - T = 5a$ **Equation 1**



$T - 2g = 2a$ **Equation 2**

Adding equations **1** and **2** we get:

$3g = 7a$

$\Rightarrow a = \frac{3}{7}g$ m/s^2

After 3 seconds, the speed of each particle will be given by

$v = u + at$

$\Rightarrow v = 0 + \left(\frac{3}{7}g\right)(3)$

$\Rightarrow v = \frac{9}{7}g$ m/s

At this point, the 2 kg mass picks up a particle of mass 4 kg.

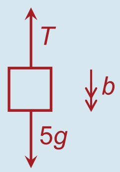
$m_1 u = m_2 v$

$\Rightarrow 7\left(\frac{9}{7}g\right) = 11v$

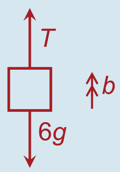
$\Rightarrow 9g = 11v$

$\Rightarrow v = \frac{9}{11}g$ m/s … speed directly after 4 kg mass is picked up.

Let b = the new common acceleration of the particles.



$5g - T = 5b$ **Equation 3**



$T - 6g = 6b$ **Equation 4**

Adding equations **3** and **4** we get:

$-g = 11b$

$\Rightarrow b = -\frac{1}{11}g$ m/s^2

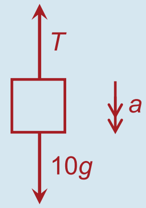
$\Rightarrow$ The distance travelled by the 5 kg mass before stopping is given by

$s = \frac{v^2 - u^2}{2a}$

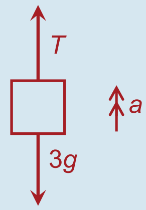
$$\Rightarrow s = \frac{0^2 - \left(\frac{9}{11}g\right)^2}{2\left(-\frac{1}{11}g\right)} = \frac{\frac{81g^2}{121}}{\frac{2g}{11}}$$

$$= \frac{81g^2}{121} \times \frac{11}{2g} = \frac{81}{22}g \text{ m}$$

Q. 3. (i) Let a = the common acceleration of the particles during the first 2 seconds.



$10g - T = 10a$ **Equation 1**



$T - 3g = 3a$ **Equation 2**

Adding equations **1** and **2** we get:

$7g = 13a$

$\Rightarrow a = \frac{7}{13}g$ m/s^2

After 2 seconds, the distance travelled by the 10 kg particle will be given by

$s = ut + \frac{1}{2}at^2$

$\Rightarrow s = (0)(2) + \frac{1}{2}\left(\frac{7}{13}g\right)(2)^2$

$\Rightarrow s = \frac{14}{13}g$ m

(ii) Firstly, we must calculate the speed just before the 2 kg mass is picked up:

$v = u + at = 0 + \left(\frac{7}{13}g\right)(2)$

$= \frac{14}{13}g$ m/s

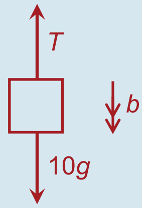
$m_1 u = m_2 v$

$\Rightarrow 13\left(\frac{14}{13}g\right) = 15v$

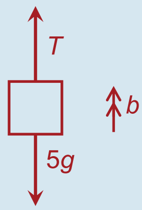
$\Rightarrow 15v = 14g$

$\Rightarrow v = \frac{14}{15}g$ m/s … speed directly after 2 kg mass is picked up.

Let b = the new common acceleration of the particles.



$10g - T = 10b$ **Equation 3**



$T - 5g = 5b$ **Equation 4**

Adding equations **3** and **4** we get:

$5g = 15b$

$\Rightarrow b = \frac{1}{3}g \text{ m/s}^2$

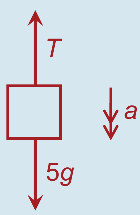
$\Rightarrow$ The distance travelled in the next 2 second period is given by

$s = ut + \frac{1}{2}at^2$

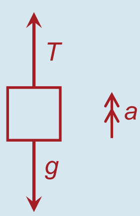
$\Rightarrow s = \left(\frac{14}{15}g\right)(2) + \frac{1}{2}\left(\frac{1}{3}g\right)(2)^2$

$\Rightarrow s = \frac{38}{15}g \text{ m}$

Q. 4. (i) Let a = the common acceleration of the particles during the first 2 seconds.



$5g - T = 5a$ **Equation 1**



$T - g = a$ **Equation 2**

Adding equations **1** and **2** we get:

$4g = 6a$

$\Rightarrow a = \frac{2}{3}g \text{ m/s}^2$

After 2 seconds, the 5 kg mass hits the table. The 1 kg mass now behaves like a projectile.

Firstly, we must find the speed of the two particles just as the 5 kg mass hits the table.

$u = 0, \quad a = \frac{2}{3}g, \quad t = 2$

$v = u + at$

$\Rightarrow v = 0 + \left(\frac{2}{3}g\right)(2)$

$\Rightarrow v = \frac{4}{3}g \text{ m/s}$

Now, examine the motion of the 1 kg mass after the 5 kg mass hits the table:

$u = \frac{4}{3}g, \quad a = -g, \quad v = 0$

$s = \frac{v^2 - u^2}{2a}$

$\Rightarrow s = \frac{0 - \frac{16g^2}{9}}{-2g}$

$\Rightarrow s = \frac{8}{9}g \text{ m}$

(ii) The 1 kg mass falls back down. When the string becomes taut, the speed will once again be $\frac{4}{3}g$ m/s.

Using Conservation of Momentum for the system, we have:

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

$\Rightarrow (1)\left(\frac{4}{3}g\right) + (5)(0) = 6v$

$\Rightarrow 6v = \frac{4}{3}g$

$\Rightarrow v = \frac{2}{9}g \text{ m/s}$ ... speed at which 5 kg mass begins to rise.

Q. 5. **Part 1:** Motion of 2 kg falling mass:

$u = 0, \quad a = g, \quad s = 1$

$v = \sqrt{u^2 + 2as}$

$\Rightarrow v = \sqrt{0^2 + 2g(1)}$

$\Rightarrow v = \sqrt{2g} \text{ m/s}$

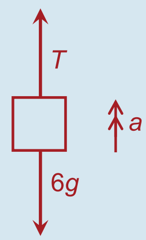
Part 2: Coalescence of 2 kg masses:

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

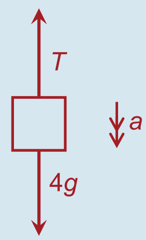
$\Rightarrow 2\sqrt{2g} + 8(0) = 10v$

$\Rightarrow v = \dfrac{\sqrt{2g}}{5}$ m/s … speed at which connected particles start to move.

Part 3: Calculate acceleration of system (which should be negative):



$T - 6g = 6a$ **Equation 1**



$4g - T = 4a$ **Equation 2**

Adding equations **1** and **2** gives

$-2g = 10a$

$\Rightarrow a = -\frac{1}{5}g$ m/s^2

Part 4: Upward motion of 6 kg mass:

$u = \dfrac{\sqrt{2g}}{5}, \quad a = -\frac{1}{5}g, \quad v = 0$

$s = \dfrac{v^2 - u^2}{2a}$

$\Rightarrow s = \dfrac{0 - \frac{2g}{25}}{-\frac{2g}{5}}$

$= \dfrac{2g}{25} \times \dfrac{5}{2g}$

$= \dfrac{1}{5}$ m

$= 20$ cm

Q. 6. (i) Motion of 1 kg mass while falling:

$u = 0, \quad s = 2.5, \quad a = g$

$v = \sqrt{u^2 + 2as}$

$\Rightarrow v = \sqrt{0 + 2g(2.5)}$

$\Rightarrow v = \sqrt{5g}$

$= 7$ m/s

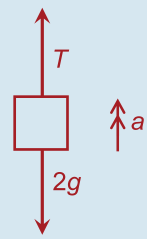
Conservation of momentum once string becomes taut:

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$

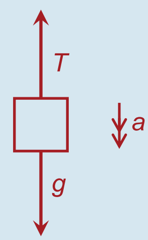
$\Rightarrow (1)(7) + (2)(0) = 3v$

$\Rightarrow v = \dfrac{7}{3}$ m/s

(ii) Motion of system:



$T - 2g = 2a$ **Equation 1**



$g - T = a$ **Equation 2**

Adding equations **1** and **2** we get:

$-g = 3a$

$\Rightarrow a = -\frac{1}{3}g$ m/s^2

Consider subsequent motion of 1 kg mass:

$u = \dfrac{7}{3}, \quad a = -\frac{1}{3}g, \quad v = 0$

$s = \dfrac{v^2 - u^2}{2a}$

$\Rightarrow s = \dfrac{0 - \frac{49}{9}}{-\frac{2g}{3}} = \dfrac{49}{9} \times \dfrac{3}{2g} = \dfrac{5}{6}$ m

Since the motion of the system began with the 1 kg mass being 1 m from the table, it follows that the 1 kg mass will not reach the table.