

## Chapter 7 Exercise 7A

Q. 1. (i)  $\frac{\text{NEW}}{\text{OLD}} = -e$   
 $\Rightarrow \frac{p}{-10} = \frac{-3}{5}$   
 $\Rightarrow p = 6 \text{ m/s}$   
 The new velocity =  $6\hat{j} \text{ m/s}$

(ii)  $\vec{I} = M\vec{V} - M\vec{u}$   
 $= 2(6\hat{j}) - 2(-10\hat{j})$   
 $= 32\hat{j} \text{ Ns}$

(iii)  $\text{K.E.}_{\text{before}} = \frac{1}{2}(2)(-10)^2$   
 $= 100 \text{ J}$

$\text{K.E.}_{\text{after}} = \frac{1}{2}(2)(6)^2$   
 $= 36 \text{ J}$

Loss =  $100 - 36$   
 $= 64 \text{ J}$

Q. 2. (i)  $\frac{\text{NEW}}{\text{OLD}} = -e$   
 $\Rightarrow \frac{v}{-20} = -0.5$   
 $\Rightarrow v = 10 \text{ m/s}$

(ii)  $\vec{I} = M\vec{V} - M\vec{u}$   
 $= (0.2)(10) - (0.2)(-20)$   
 $= 2 + 4 = 6 \text{ Ns}$

(iii)  $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.2)(-20)^2$   
 $= 40 \text{ J}$

$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.2)(10)^2$   
 $= 10 \text{ J}$

Loss =  $40 - 10$   
 $= 30 \text{ J}$

Q. 3. (i) To find speed at impact:  
 $v^2 = u^2 + 2as$   
 $\Rightarrow v^2 = 0^2 + 2(9.8)(2.5)$   
 $\Rightarrow v = 7 \text{ m/s}$

(ii)  $\frac{\text{NEW}}{\text{OLD}} = -e$   
 $\Rightarrow \frac{p}{-7} = \frac{-4}{7}$   
 $\Rightarrow p = 4$   
 $= 4 \text{ m/s}$

(iii)  $\vec{I} = M\vec{V} - M\vec{u}$   
 $= (1)(4\hat{j}) - (1)(-7\hat{j})$   
 $= 11\hat{j} \text{ Ns}$

(iv) Loss =  $\frac{1}{2}Mu^2 - \frac{1}{2}Mv^2$   
 $= \frac{1}{2}(1)(49) - \frac{1}{2}(1)(16)$   
 $= 16\frac{1}{2} \text{ J}$

Q. 4. (i)  $v^2 = u^2 + 2as$   
 $\Rightarrow v^2 = 0^2 + 2(9.8)(10)$   
 $\Rightarrow v = 14 \text{ m/s}$

(ii)  $\vec{I} = M\vec{V} - M\vec{u}$   
 $= (6)(0) - (6)(-14\hat{j})$   
 $= 84\hat{j} \text{ Ns}$

(iii) Loss =  $\frac{1}{2}Mu^2 - \frac{1}{2}Mv^2$   
 $= \frac{1}{2}(6)(14)^2 - \frac{1}{2}(6)(0)^2$   
 $= 588 \text{ J}$

Q. 5. (i)  $u = 0, a = 9.8, s = 22.5$   
 $v = \sqrt{u^2 + 2as}$   
 $= \sqrt{0 + 2(9.8)(22.5)}$   
 $= 21 \text{ m/s}$

(ii)  $\frac{\text{NEW}}{\text{OLD}} = -e$   
 $\Rightarrow \frac{v_1}{-21} = \frac{-5}{7}$   
 $\Rightarrow v_1 = 15 \text{ m/s}$

(iii)  $\vec{I} = M\vec{V}_1 - M\vec{v}$   
 $= (0.1)(15) - (0.1)(-21)$   
 $= 3.6 \text{ Ns}$

(iv)  $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.1)(21)^2$   
 $= 22.05 \text{ J}$

$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.1)(15)^2$   
 $= 11.25 \text{ J}$

Loss =  $22.05 - 11.25$   
 $= 10.8 \text{ J}$

**Q. 6.** (i) **Before** (Mass) **After**  
 $5\vec{i} - 8\vec{j}$  2kg  $5\vec{i} + p\vec{j}$

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{p}{-8} = -\frac{3}{4}$$

$$\Rightarrow p = 6$$

$$\text{Ans} = 5\vec{i} + 6\vec{j}$$

(ii)  $\frac{1}{2}Mu^2 - \frac{1}{2}Mv^2 = \frac{1}{2}(2)(25 + 64) - \frac{1}{2}(2)(25 + 36)$   
 $= 28 \text{ J}$

(iii)  $\vec{l} = M\vec{v} - M\vec{u}$   
 $= 2(5\vec{i} + 6\vec{j}) - 2(5\vec{i} - 8\vec{j})$   
 $= 28\vec{j} \text{ Ns}$

**Q. 7.** (i) **Before** (Mass) **After**  
 $8\vec{i} - 15\vec{j}$  0.2 kg  $8\vec{i} + p\vec{j}$

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{p}{-15} = -\frac{2}{5}$$

$$\Rightarrow 5p = 30 \Rightarrow p = 6$$

$$\Rightarrow \text{New velocity} = 8\vec{i} + 6\vec{j} \text{ m/s}$$

(ii) Speed before  $= \sqrt{8^2 + (-15)^2}$   
 $= 17 \text{ m/s}$

Speed after  $= \sqrt{8^2 + 6^2}$   
 $= 10 \text{ m/s}$

$$\Rightarrow \text{Fall in speed} = 17 - 10$$

$$= 7 \text{ m/s}$$

(iii)  $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.2)(17)^2$   
 $= 28.9 \text{ J}$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.2)(10)^2$$

$$= 10 \text{ J}$$

$$\text{Loss} = 28.9 - 10$$

$$= 18.9 \text{ J}$$

**Q. 8.** (i)  $\frac{\text{NEW}}{\text{OLD}} = -e$

$$\Rightarrow \frac{4}{-6} = -e$$

$$\Rightarrow e = \frac{2}{3}$$

(ii)  $\text{K.E.}_{\text{before}} = \frac{1}{2}(4)(64 + 36)$   
 $= 200 \text{ J}$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(4)(64 + 16)$$

$$= 160 \text{ J}$$

$$\% \text{ Loss} = \frac{40}{200} \times \frac{100}{1}$$

$$= 20\%$$

(iii)  $\tan A = \frac{6}{8} = \frac{3}{4}$ ,  $\tan B = \frac{4}{8} = \frac{1}{2}$

$$\frac{\tan B}{\tan A} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$$

**Q. 9.** (i) **Before** (Mass) **After**  
 $4\vec{i} + 3\vec{j}$  0.2  $p\vec{i} + 3\vec{j}$

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{p}{4} = -0.75$$

$$\Rightarrow p = -3$$

$$\Rightarrow \text{Velocity after impact} = -3\vec{i} + 3\vec{j} \text{ m/s}$$

(ii)  $\text{K.E.}_{\text{before}} = \frac{1}{2}(0.2)(4^2 + 3^2)$   
 $= 2.5 \text{ J}$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(0.2)((-3)^2 + 3^2)$$

$$= 1.8 \text{ J}$$

$$\text{Loss} = 2.5 - 1.8$$

$$= 0.7 \text{ J}$$

$$\% \text{ loss} = \frac{0.7}{2.5} \times 100 = 28\%$$

**Q. 10.** (i)  $u = 0, a = 9.8, s = 40$

$$v = \sqrt{u^2 + 2as}$$

$$= \sqrt{0 + 2(9.8)(40)}$$

$$= 28 \text{ m/s}$$

(ii) Let  $v_1$  = speed directly after impact with ground.

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{v_1}{-28} = -\frac{1}{2}$$

$$\Rightarrow v_1 = 14 \text{ m/s}$$

(iii)  $u = 14, a = -9.8, v = 0$

$$s = \frac{v^2 - u^2}{2a}$$

$$= \frac{0 - 14^2}{2(-9.8)}$$

$$= 10 \text{ m}$$

**Q. 11.** (i)  $u = 0, a = g, s = h$

$$v = \sqrt{u^2 + 2as}$$

$$= \sqrt{0 + 2gh}$$

$$= \sqrt{2gh}$$

(ii) Let  $v_1$  = the speed with which it first rises from the ground.

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{v_1}{-\sqrt{2gh}} = -e$$

$$\Rightarrow v_1 = e\sqrt{2gh}$$

(iii)  $u = e\sqrt{2gh}, a = -g, v = 0$

$$s = \frac{v^2 - u^2}{2a}$$

$$= \frac{0 - 2e^2gh}{-2g} = e^2h$$

**Q. 12.** Before (Mass) After

$$u \cos A \vec{i} + u \sin A \vec{j} \quad M \quad v \cos B \vec{i} + v \sin B \vec{j}$$

$u \cos A = u \cos B$  ... **Equation 1** ( $\vec{i}$ -velocity remains the same)

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{u \sin B}{-u \sin A} = -e$$

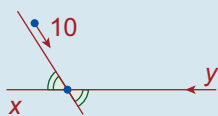
$$\Rightarrow eu \sin A = u \sin B \quad \dots \text{Equation 2}$$

Dividing 2 by 1 gives :  $e \tan A = \tan B$

$$\Rightarrow e = \frac{\tan B}{\tan A}$$

**Q. 13.** (i)  $\vec{u} = 20\left(\frac{3}{5}\right)\vec{i} - 20\left(\frac{4}{5}\right)\vec{j}$

$$= 12\vec{i} - 16\vec{j}$$



**Before (Mass) After**

$$12\vec{i} - 16\vec{j} \quad M \quad 12\vec{i} + p\vec{j}$$

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{p}{-16} = -\frac{3}{4} \Rightarrow p = 12$$

New Velocity =  $12\vec{i} + 12\vec{j}$

$$\text{New Speed} = \sqrt{144 + 144}$$

$$= 12\sqrt{2} \text{ m/s}$$

(ii)  $\vec{l} = M\vec{v} - M\vec{u}$

$$= M(12\vec{i} + 12\vec{j}) - M(12\vec{i} - 16\vec{j})$$

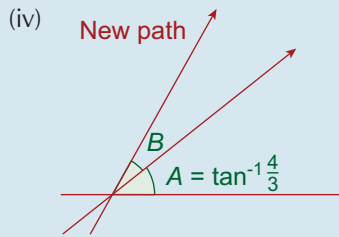
$$= 28M\vec{j} \text{ Ns}$$

Magnitude =  $28M \text{ Ns}$

(iii)  $\frac{1}{2}Mu^2 - \frac{1}{2}Mv^2$

$$= \frac{1}{2}M(144 + 256) - \frac{1}{2}M(144 + 144)$$

$$= 56M \text{ J}$$



$$\tan B = \frac{12}{12}$$

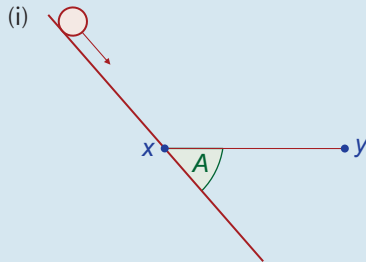
$$\tan B = 1$$

$$\Rightarrow B = 45^\circ$$

$$\begin{aligned} \tan(A + B) &= \pm \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \pm \frac{\frac{4}{3} + 1}{1 - (\frac{4}{3})(1)} = \pm 7 \end{aligned}$$

$$\tan(A + B) = -7 \text{ (Since } A + B > 90^\circ \text{)}$$

Q. 14.



$$\tan A = \frac{7}{24}$$

$$\Rightarrow \begin{cases} \cos A = \frac{24}{25} \\ \sin A = \frac{7}{25} \end{cases}$$

$$\begin{aligned} \vec{u} &= 25 \cos A \vec{i} - 25 \sin A \vec{j} \\ &= 24\vec{i} - 7\vec{j} \end{aligned}$$

Before	(Mass)	After
$24\vec{i} - 7\vec{j}$	0.1	$24\vec{i} + p\vec{j}$

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{p}{-7} = -\frac{3}{7} \Rightarrow p = 3$$

$$\Rightarrow \text{New Velocity} = 24\vec{i} + 3\vec{j}$$

$$\begin{aligned} \text{New Speed} &= \sqrt{24^2 + 3^2} \\ &= 24.2 \text{ m/s} \end{aligned}$$

(ii) 
$$\begin{aligned} \vec{I} &= M\vec{v} - M\vec{u} \\ &= 0.1(24\vec{i} + 3\vec{j}) - 0.1(24\vec{i} - 7\vec{j}) \\ &= 1\vec{j} \text{ Ns} \end{aligned}$$

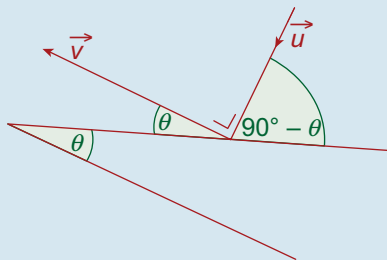
$$\text{Magnitude} = 1 \text{ Ns}$$

(iii) 
$$\begin{aligned} \text{K.E.}_{\text{before}} &= \frac{1}{2}(0.1)(24^2 + (-7)^2) \\ &= 31.25 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{K.E.}_{\text{after}} &= \frac{1}{2}(0.1)(24^2 + 3^2) \\ &= 29.25 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Loss} &= 31.25 - 29.25 \\ &= 2 \text{ J} \end{aligned}$$

Q. 15.



$$\begin{aligned} \vec{u} &= -u \cos(90^\circ - \theta)\vec{i} - u \sin(90^\circ - \theta)\vec{j} \\ &= -u \sin \theta \vec{i} - u \cos \theta \vec{j} \\ \vec{v} &= -v \cos \theta \vec{i} + v \sin \theta \vec{j} \end{aligned}$$

Before

$$-u \sin \theta \vec{i} - u \cos \theta \vec{j}$$

$\vec{i}$  - velocity is unchanged

$$\Rightarrow -u \sin \theta = -v \cos \theta$$

$$\Rightarrow u \sin \theta = v \cos \theta \quad \dots \text{Equation 1}$$

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{v \sin \theta}{-u \cos \theta} = \frac{-2}{3}$$

$$\Rightarrow 3v \sin \theta = 2u \cos \theta \quad \dots \text{Equation 2}$$

(Mass)

M

After

$$-v \cos \theta \vec{i} + u \sin \theta \vec{j}$$

Dividing 2 by 1 gives:

$$\frac{3v}{u} = \frac{2u}{v}$$

$$\Rightarrow v^2 = \frac{2}{3}u^2$$

$$\Rightarrow \frac{1}{2}Mv^2 = \frac{2}{3}\left(\frac{1}{2}Mu^2\right)$$

$$\Rightarrow \frac{2}{3} \text{ of the energy is preserved.}$$

$$\Rightarrow \frac{1}{3} \text{ of the energy has been lost.}$$

### Exercise 7B

**Q. 1.** (i) 

Before	(Mass)	After
$6\vec{i}$	2	$p\vec{i}$
$4\vec{i}$	1	$q\vec{i}$

$$\begin{array}{l} 2(6) + 1(4) = 2(p) + 1(q) \\ \Rightarrow 2p + q = 16 \dots \text{Equation 1} \end{array}$$

$$\Rightarrow 2p + q = 16 \dots \text{Equation 1}$$

$$\frac{p - q}{6 - 4} = -\frac{1}{2}$$

$$\Rightarrow 2p - 2q = -2$$

$$\Rightarrow p - q = -1 \dots \text{Equation 2}$$

Adding equations 1 and 2 we get:

$$3p = 15$$

$$\Rightarrow p = 5 \Rightarrow q = 6$$

$\Rightarrow$  Velocity of 2 kg sphere after impact is  $5\vec{i}$  m/s

Velocity of 1 kg sphere after impact is  $6\vec{i}$  m/s

$$\begin{aligned} \text{(ii) K.E.}_{\text{before}} &= \frac{1}{2}(2)(6)^2 + \frac{1}{2}(1)(4)^2 \\ &= 44 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{K.E.}_{\text{after}} &= \frac{1}{2}(2)(5)^2 + \frac{1}{2}(1)(6)^2 \\ &= 43 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{Loss} &= 44 - 43 \\ &= 1 \text{ J} \end{aligned}$$

**Q. 2.** (i) 

Before	(Mass)	After
$2\vec{i}$	1	$p\vec{i}$
$-6\vec{i}$	1	$q\vec{i}$

$$\begin{array}{l} 1(2) + 1(-6) = 1(p) + 1(q) \\ \Rightarrow p + q = -4 \dots \text{Equation 1} \end{array}$$

$$\frac{p - q}{2 + 6} = -\frac{3}{4}$$

$$\Rightarrow 4p - 4q = -24$$

$$\Rightarrow p - q = -6 \dots \text{Equation 2}$$

Adding equations 1 and 2 we get

$$2p = -10$$

$$\Rightarrow p = -5$$

$$\Rightarrow q = 1$$

$\Rightarrow$  velocities after impact are  $-5\vec{i}$  m/s and  $\vec{i}$  m/s

$\Rightarrow$  speeds after impact are 5 m/s and 1 m/s

$$\begin{aligned} \text{(ii) } \vec{I} &= M\vec{V} - M\vec{u} \\ &= 1(-5\vec{i}) - 1(2\vec{i}) \\ &= -7\vec{i} \text{ Ns} \dots \text{impulse imparted to first sphere} \end{aligned}$$

$\Rightarrow$  Impulse imparted to second sphere =  $7\vec{i}$  Ns

$\Rightarrow$  Magnitude of impulse imparted to each sphere = 7 Ns.

$$\begin{aligned} \text{(iii) K.E.}_{\text{before}} &= \frac{1}{2}(1)(2)^2 + \frac{1}{2}(1)(-6)^2 \\ &= 20 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{K.E.}_{\text{after}} &= \frac{1}{2}(1)(-5)^2 + \frac{1}{2}(1)(1)^2 \\ &= 13 \text{ J} \end{aligned}$$

$$\Rightarrow \text{Loss} = 20 - 13$$

$$\Rightarrow = 7 \text{ J}$$

$$\Rightarrow \% \text{ Loss} = \frac{7}{20} \times 100 = 35\%$$

**Q. 3.** (i) 

Before	(Mass)	After
$10\vec{i}$	3	$0\vec{i}$
$\vec{i}$	5	$q\vec{i}$

$$\begin{array}{l} 3(10) + 5(1) = 3(0) + 5(q) \\ \Rightarrow q = 7 \end{array}$$

$$\frac{0 - q}{10 - 1} = -e$$

$$\Rightarrow e = \frac{q}{9} = \frac{7}{9}$$

(ii) 7 m/s

$$\begin{aligned} \text{(iii) } \vec{I}_1 &= M\vec{V}_1 - M\vec{u}_1 \\ &= 3(0\vec{i}) - 3(10\vec{i}) \\ &= -30\vec{i} \text{ Ns} \end{aligned}$$

$$\begin{aligned} \vec{I}_2 &= M\vec{V}_2 - M\vec{u}_2 \\ &= 5(7\vec{i}) - 5(\vec{i}) = 30\vec{i} \text{ Ns} \end{aligned}$$

(iv)  $u = 7, v = 0, s = 2, a = ?$

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0 = 49 + 2a(2)$$

$$\Rightarrow a = \frac{-49}{4} = -12.25 \text{ m/s}^2$$

$\therefore$  Deceleration =  $-12.25 \text{ m/s}^2$

**Q. 4.** (i) 

Before	(Mass)	After
5	2	$v$
0	1	$3v$

$$2(5) + 1(0) = 2(v) + 1(3v)$$

$$\Rightarrow 5v = 10 \Rightarrow v = 2$$

(ii)  $\frac{v - 3v}{5 - 0} = -e$

$$\Rightarrow \frac{-4}{5} = -e \Rightarrow e = \frac{4}{5}$$

(iii)  $\text{K.E.}_{\text{before}} = \frac{1}{2}(2)(5)^2 + \frac{1}{2}(1)(0)^2$

$$= 25 \text{ J}$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(2)(2)^2 + \frac{1}{2}(1)(6)^2$$

$$= 22 \text{ J}$$

$$\text{Loss} = 25 - 22 = 3 \text{ J}$$

$$\% \text{ Loss} = \frac{3}{25} \times 100 = 12\%$$

**Q. 5.** (i) 

Before	(Mass)	After
$6\vec{i}$	5	$p\vec{i}$
$-4\vec{i}$	3	$q\vec{i}$

$$5(6) + 3(-4) = 5p + 3q$$

$$\Rightarrow 5p + 3q = 18$$

$$\frac{p - q}{6 + 4} = -\frac{1}{3}$$

$$\Rightarrow 3p - 3q = -10$$

$$\text{Solving these gives } p = 1, q = \frac{13}{3}.$$

(ii)  $\text{Loss} = \frac{1}{2}Mu^2 - \frac{1}{2}Mv^2$

$$= \frac{1}{2}(5)(6)^2 - \frac{1}{2}(5)(1)^2$$

$$= 87\frac{1}{2} \text{ J}$$

(iii)  $M\vec{v} - M\vec{u} = 3\left(\frac{13}{3}\vec{i}\right) - 3(-4\vec{i})$

$$= 25\vec{i} \text{ N s}$$

**Q. 6.** (i) 

Before	(Mass)	After
$10\vec{i}$	2	$p\vec{i}$
$0\vec{i}$	3	$q\vec{i}$

$$2(10) + 3(0) = 2p + 3q$$

$$\Rightarrow 2p + 3q = 20$$

$$\frac{p - q}{10 - 0} = -\frac{1}{2}$$

$$\Rightarrow p - q = -5$$

Solving these gives  $p = 1, q = 6$ .

**Answer:** 1 m/s and 6 m/s.

(ii)  $\text{K.E.}_{\text{Before}} = \frac{1}{2}(2)(10)^2 + \frac{1}{2}(3)(0)^2$

$$= 100 \text{ J}$$

$$\text{K.E.}_{\text{After}} = \frac{1}{2}(2)(1)^2 + \frac{1}{2}(3)(6)^2$$

$$= 55 \text{ J}$$

$$\text{Loss} = 100 - 55 = 45 \text{ J}$$

(iii) To find the acceleration:

$$v = u + at$$

$$\Rightarrow 0 = 6 + a(2)$$

$$\Rightarrow a = -3$$

To find the force:

$$F = Ma$$

$$\Rightarrow F = (3)(-3) = -9 \text{ N}$$

(i.e. it is a resistance force of 9 N)

To find the distance:

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0 = (6)^2 + 2(-3)s$$

$$\Rightarrow s = 6 \text{ m}$$

To find work done:

$$W = Fs = 9(6) = 54 \text{ J}$$

To find power:

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$

$$= \frac{54}{2} = 27 \text{ W}$$

**Q. 7.** (i) 

	Before	(Mass)	After
A:	$4\vec{i}$	$M$	$p\vec{i}$
B:	$0\vec{i}$	$M$	$q\vec{i}$

$$M(4) + M(0) = Mp + Mq$$

$$\Rightarrow p + q = 4$$

$$\frac{p - q}{4 - 0} = -\frac{1}{2}$$

$$\Rightarrow p - q = -2$$

Solving these gives  $p = 1, q = 3$ .

Speed of B is 3 m/s.

(ii)	<b>Before</b>	<b>(Mass)</b>	<b>After</b>
	B: $3\vec{i}$	M	$a\vec{i}$
	C: $0\vec{i}$	M	$b\vec{i}$

$$M(3) + M(0) = Ma + Mb$$

$$\Rightarrow a + b = 3$$

$$\frac{a - b}{3 - 0} = \frac{-1}{2}$$

$$\Rightarrow a - b = -\frac{3}{2}$$

Solving these gives  $a = \frac{3}{4}$ ,  $b = 2\frac{1}{4}$ .

Speed of B is  $\frac{3}{4}$  m/s.

Yes, because A will catch up with B, since  $v_A > v_B$

**Q. 8.**

(i)	<b>Before</b>	<b>(Mass)</b>	<b>After</b>
	A: $v$	4	0
	B: $-v$	2	$p$

$$4(v) + 2(-v) = 4(0) + 2(p)$$

$$\Rightarrow 2p = 2v \Rightarrow p = v$$

Speed of B after impact is the same as the speed before but in the opposite direction.

(ii) 
$$\frac{0 - p}{v - (-v)} = -e$$

$$\Rightarrow \frac{-v}{2v} = -e \Rightarrow e = \frac{1}{2}$$

(iii) 
$$\text{K.E.}_{\text{before}} = \frac{1}{2}(4)(v)^2 + \frac{1}{2}(2)(-v)^2$$

$$= 2v^2 + v^2 = 3v^2$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(4)(0)^2 + \frac{1}{2}(2)(v)^2$$

$$= v^2$$

$$\text{Loss} = 3v^2 - v^2 = 2v^2$$

$$\% \text{ Loss} = \frac{2v^2}{3v^2} \times 100 = 66\frac{2}{3}\%$$

**Q. 9.**

(i)	<b>Before</b>	<b>(Mass)</b>	<b>After</b>
	$2\vec{i}$	M	$11k\vec{i}$
	$\vec{i}$	M	$13k\vec{i}$

$$M(2) + M(1) = M(11k) + M(13k)$$

$$\Rightarrow k = \frac{1}{8}$$

$\therefore$  Their speeds will be  $\frac{11}{8}$  and  $\frac{13}{8}$ .

(ii) 
$$\frac{\frac{11}{8} - \frac{13}{8}}{2 - 1} = -e$$

$$\Rightarrow e = \frac{1}{4}$$

**Q. 10.**

(i)	<b>Before</b>	<b>(Mass)</b>	<b>After</b>
	A: $6\vec{i}$	M	$p\vec{i}$
	B: $0\vec{i}$	M	$q\vec{i}$

$$M(6) + M(0) = Mp + Mq$$

$$\Rightarrow p + q = 6$$

$$\frac{p - q}{6 - 0} = \frac{-2}{3}$$

$$\Rightarrow p - q = -4$$

Solving these gives  $p = 1$ ,  $q = 5$

Their speeds are (1, 5, 0) m/s.

(ii)

<b>Before</b>	<b>(Mass)</b>	<b>After</b>
B: $5\vec{i}$	M	$a\vec{i}$
C: $0\vec{i}$	M	$b\vec{i}$

$$M(5) + M(0) = Ma + Mb$$

$$\Rightarrow a + b = 5$$

$$\frac{a - b}{5 - 0} = \frac{-2}{3}$$

$$\Rightarrow a - b = \frac{-10}{3}$$

Solving these gives  $a = \frac{5}{6}$ ,  $b = \frac{25}{6}$ .

Their speeds now are  $(1, \frac{5}{6}, \frac{25}{6})$  m/s.

(iii)

<b>Before</b>	<b>(Mass)</b>	<b>After</b>
A: $1\vec{i}$	M	$c\vec{i}$
B: $\frac{5}{6}\vec{i}$	M	$d\vec{i}$

$$M(1) + M\left(\frac{5}{6}\right) = Mc + Md$$

$$\Rightarrow c + d = \frac{11}{6}$$

$$\frac{c - d}{1 - \frac{5}{6}} = \frac{-2}{3}$$

$$\Rightarrow c - d = -\frac{1}{9}$$

Solving these gives:  $c = \frac{31}{36}$ ,  $d = \frac{35}{36}$

Their speeds are  $(\frac{31}{36}, \frac{35}{36}, \frac{25}{6})$  m/s

Since  $v_A < v_B < v_C$ , there will be no further collisions.

**Q. 11.** Before (Mass) After

$$\begin{array}{ccc} 10\vec{i} & 10 & p\vec{i} \\ -5\vec{i} & 50 & q\vec{i} \end{array}$$

$$10(10) + 50(-5) = 10p + 50q$$

$$\Rightarrow p + 5q = -15$$

$$\frac{p - q}{10 + 5} = -\frac{1}{2}$$

$$\Rightarrow 2p - 2q = -15$$

$$\text{Solving these gives } p = -\frac{35}{4}, q = -\frac{5}{4}.$$

The speeds are  $8\frac{3}{4}$  m/s and  $1\frac{1}{4}$  m/s.

$$\vec{I}_1 = M\vec{v} - M\vec{u}$$

$$= 10\left(-\frac{35}{4}\right) - 10(10)$$

$$= -187.5 \text{ Ns}$$

The magnitude of the impulse = 187.5 Ns

**Q. 12.** (a) (i) Newton's law of restitution:  
For two bodies impinging directly, their relative velocity after impact is equal to a constant (e) times their relative velocity before impact and in the opposite direction.

(ii) In a closed system the total momentum will be conserved.

(b)  $v^2 = u^2 + 2as$

$$\Rightarrow v^2 = 0^2 + 2(9.8)(19.6)$$

$$\Rightarrow v = 19.6$$

$$\text{Rebound speed} = (0.8)(19.6)$$

$$= 15.68$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0^2 = (15.68)^2 + 2(-9.8)s$$

$$\Rightarrow s = 12.544 \text{ m}$$

(c) Gun    Bullet    Gun    Bullet

$$M_1u_1 + M_2u_2 = M_1v_1 + M_2v_2$$

$$\Rightarrow (2)(0) + (0.01)0 = (2)v_1 + (0.01)(300)$$

$$\Rightarrow v_1 = -1.5 \text{ m/s}$$

$\Rightarrow$  Initial speed of the gun = 1.5 m/s

To find acceleration:

$$v^2 = u^2 + 2as = (0)^2$$

$$= (-1.5)^2 + 2a(0.05)$$

$$\Rightarrow a = -22.5 \text{ m/s}^2$$

$$F = ma$$

$$\Rightarrow F = (2)(-22.5)$$

$$= -45 \text{ N}$$

A constant force of 45 N is required.

**Q. 13.** (i) Before (Mass) After

$$\begin{array}{ccc} u\vec{i} & M & 0\vec{i} \\ -v\vec{i} & 3M & q\vec{i} \end{array}$$

$$M(u) + 3M(-v) = M(0) + 3M(q)$$

$$\Rightarrow q = \frac{u - 3v}{3}$$

$$\frac{0 - q}{u + v} = -e$$

$$\Rightarrow e = \frac{q}{u + v}$$

$$= \frac{u - 3v}{3u + 3v} \quad \text{QED}$$

(ii)  $e \geq 0$

$$\therefore \frac{u - 3v}{3u + 3v} \geq 0$$

$$\therefore u - 3v \geq 0$$

$$\therefore u \geq 3v \quad \text{QED}$$

**Q. 14.** Before (Mass) After

$$\begin{array}{ccc} 5\vec{i} & 1 & p\vec{i} \\ \vec{i} & 2 & q\vec{i} \end{array}$$

$$1(5) + 2(1) = 1(p) + 2(q)$$

$$\Rightarrow p + 2q = 7$$

$$\Rightarrow p = 7 - 2q$$

$$\frac{p - q}{5 - 1} = -e$$

$$\Rightarrow -4e = p - q$$

$$= (7 - 2q) - q$$

$$= 7 - 3q$$

$$\therefore e = \frac{3q - 7}{4}$$

If there are to be no more collisions  $v_B \leq v_C$

$$\therefore q \leq 3$$

$$\text{If } q \leq 3 \text{ then } e = \frac{3q - 7}{4} \leq \frac{3(3) - 7}{4} = \frac{1}{2}$$

**Answer:** MAX Value =  $\frac{1}{2}$



**Q. 15. Before (Mass) After**

$$\begin{array}{ccc} u_1 \vec{i} & 3 & v_1 \vec{i} \\ u_2 \vec{j} & 4 & q \vec{j} \end{array}$$

$$\begin{aligned} 1: 3u_1 + 4u_2 &= 3v_1 + 4q \\ \Rightarrow q &= \frac{1}{4}(3u_1 + 4u_2 - 3v_1) \end{aligned}$$

$$2: \frac{v_1 - q}{u_1 - u_2} = -e$$

$$\Rightarrow v_1 - q = -eu_1 + eu_2$$

$$\text{But } q = \frac{1}{4}(3u_1 + 4u_2 - 3v_1)$$

$$\begin{aligned} \therefore v_1 - \frac{1}{4}(3u_1 + 4u_2 - 3v_1) &= -eu_1 + eu_2 \\ \Rightarrow 4v_1 - 3u_1 - 4u_2 + 3v_1 &= -4eu_1 + 4eu_2 \\ \Rightarrow 7v_1 &= u_1(3 - 4e) + 4u_2(1 + e) \quad \mathbf{QED} \end{aligned}$$

$$I = 3v_1 - 3u_1. \text{ But } v_1 = \frac{1}{7}(u_1(3 - 4e) + 4u_2(1 + e))$$

$$\begin{aligned} \therefore I &= \frac{3}{7}(u_1(3 - 4e) + 4u_2(1 + e)) - 3u_1 \\ &= \frac{3}{7}(3u_1 - 4eu_1 + 4u_2 + 4eu_2 - 7u_1) \\ &= \frac{3}{7}(-4u_1 - 4eu_1 + 4u_2 + 4eu_2) \\ &= \frac{12}{7}(-u_1 - eu_1 + u_2 + eu_2) \\ &= \frac{12}{7}(-u_1(1 + e) + u_2(1 + e)) \\ &= \frac{12}{7}(1 + e)(u_2 - u_1) \quad \mathbf{QED} \end{aligned}$$

**Q. 16. (i) Before (Mass) After**

$$P: \quad 4u \quad 3m \quad 2u$$

$$Q: \quad 2u \quad 5m \quad q$$

$$3m(4u) + 5m(2u) = 3m(2u) + 5m(q)$$

... divide by  $m$

$$\Rightarrow 12u + 10u = 6u + 5q$$

$$\Rightarrow 5q = 16u$$

$$\Rightarrow q = \frac{16u}{5}$$

$$\frac{2u - q}{4u - 2u} = -e$$

$$\Rightarrow \frac{2u - q}{2u} = -e$$

$$\Rightarrow 2u - q = -2eu$$

$$\Rightarrow q = 2u(1 + e) \quad \dots \text{ but } q = \frac{16u}{5}$$

$$\Rightarrow 2u(1 + e) = \frac{16u}{5} \quad \dots \text{ multiply by } \frac{5}{2u}$$

$$\Rightarrow 5(1 + e) = 8$$

$$\Rightarrow 1 + e = 1.6$$

$$\Rightarrow e = 0.6$$

(ii) Let the velocity of  $P$  after impact be  $p$ .

$$3m(4u) + 5m(2u) = 3m(p) + 5m(q)$$

$$\Rightarrow 3p + 5q = 22u \quad \dots \mathbf{Equation 1}$$

$$\frac{p - q}{4u - 2u} = -e$$

$$\Rightarrow p - q = -2eu \quad \dots \text{ multiply by } -3$$

$$\Rightarrow -3p + 3q = 6eu \quad \dots \mathbf{Equation 2}$$

Adding equations **1** and **2** we get

$$8q = 22u + 6eu$$

$$\Rightarrow 8q = 2u(11 + 3e)$$

$$\Rightarrow q = \frac{u}{4}(11 + 3e) \quad \dots \text{ minimum value occurs when } e = 0$$

$$\Rightarrow q = \frac{u}{4}(11) = 2.75u$$

**Q. 17.** (i) **Before**      **(Mass)**      **After**

$$\begin{array}{ccc} u & 4m & p \\ 0 & 2m & q \end{array}$$

$$4m(u) + 2m(0) = 4m(p) + 2m(q) \quad \dots \text{divide by } 2m$$

$$\Rightarrow 2p + q = 2u \quad \dots \text{Equation 1}$$

$$\frac{p - q}{u - 0} = -e$$

$$\Rightarrow p - q = -eu \quad \dots \text{Equation 2}$$

Adding equations **1** and **2** we get

$$3p = u(2 - e)$$

$$\Rightarrow p = \frac{u}{3}(2 - e) \quad \dots \text{speed of } 4m \text{ sphere after impact.}$$

$$q = p + eu \quad \dots \text{from Equation 1}$$

$$\Rightarrow q = \frac{u}{3}(2 - e) + eu$$

$$\Rightarrow q = \frac{2u}{3} - \frac{eu}{3} + eu$$

$$\Rightarrow q = \frac{2u - eu + 3eu}{3}$$

$$\Rightarrow q = \frac{2u + 2eu}{3}$$

$$\Rightarrow q = \frac{2u}{3}(1 + e)$$

$$\begin{aligned} \text{(ii) K.E.}_{\text{before}} &= \frac{1}{2}(4m)(u)^2 \\ &= 2mu^2 \end{aligned}$$

$$\begin{aligned} \text{K.E.}_{\text{after}} &= \frac{1}{2}(4m)(p)^2 + \frac{1}{2}(2m)(q)^2 \\ &= 2m\left[\frac{u^2}{9}(2 - e)^2\right] + m\left[\frac{4u^2}{9}(1 + e)^2\right] \\ &= \frac{2mu^2}{9}[4 - 4e + e^2 + 2(1 + 2e + e^2)] \\ &= \frac{2mu^2}{9}[3e^2 + 6] \\ &= \frac{2mu^2}{3}(e^2 + 2) \end{aligned}$$

$$\begin{aligned} \text{Loss} &= 2mu^2 - \frac{2mu^2}{3}(e^2 + 2) \\ &= \frac{2mu^2}{3}(3 - e^2 - 2) \\ &= \frac{2mu^2}{3}(1 - e^2) \end{aligned}$$

**Q. 18.** (i) **Before**      **(Mass)**      **After**

$$\begin{array}{ccc} 2u & m & p \\ 0 & m & q \end{array}$$

$$m(2u) + m(0) = m(p) + m(q) \quad \dots \text{divide by } m$$

$$\Rightarrow p + q = 2u \quad \dots \text{Equation 1}$$

$$\frac{p - q}{2u - 0} = -e$$

$$\Rightarrow p - q = -2eu \quad \dots \text{Equation 2}$$

Adding equations **1** and **2** we get

$$2p = 2u(1 - e)$$

$$\Rightarrow p = u(1 - e) \quad \dots \text{speed of 1st sphere after impact}$$

$$q = p + 2eu \quad \dots \text{from Equation 1}$$

$$\Rightarrow q = u(1 - e) + 2eu$$

$$\Rightarrow q = u - eu + 2eu$$

$$\Rightarrow q = u + eu$$

$$\Rightarrow q = u(1 + e) \quad \dots \text{speed of second sphere after impact}$$

- (ii) Firstly, find how long it takes for sphere to hit wall.

$$\text{speed} = u(1 + e)$$

$$\text{distance} = 1 \text{ metre}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{1}{u(1 + e)} \text{ s}$$

Find the distance travelled by other sphere in this time.

$$\text{speed} = u(1 - e)$$

$$\text{time} = \frac{1}{u(1 + e)}$$

$$\text{distance} = \text{speed} \times \text{time}$$

$$= u(1 - e) \left[ \frac{1}{u(1 + e)} \right]$$

$$= \frac{1 - e}{1 + e}$$

$$\Rightarrow \text{distance apart} = 1 - \frac{1 - e}{1 + e}$$

$$= \frac{1 + e - 1 + e}{1 + e}$$

$$= \frac{2e}{1 + e}$$

- (iii) Find the speed of sphere after colliding with wall.

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{v}{u(1 + e)} = -e$$

$$\Rightarrow v = -eu(1 + e)$$

$$\vec{v}_A = u(1 - e)$$

$$\vec{v}_B = -eu(1 + e)$$

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$= u(1 - e) + eu(1 + e)$$

$$= u - eu + eu + e^2u$$

$$= u(1 + e^2)$$

$$\text{time} = \frac{\text{relative distance}}{\text{relative speed}}$$

$$= \frac{\frac{2e}{1 + e}}{u(1 + e^2)}$$

$$= \frac{2e}{u(1 + e)(1 + e^2)}$$

Look at how far the sphere has moved away from the wall in this time.

$$\text{distance} = \text{speed} \times \text{time}$$

$$\Rightarrow \text{distance} = eu(1 + e) \left[ \frac{2e}{u(1 + e)(1 + e^2)} \right]$$

$$= \frac{2e^2}{1 + e^2}$$

- Q. 19.** (i) Gain in K.E. = Loss in P.E.

$$\Rightarrow \frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \frac{1}{2}(0.6)v^2 = (0.6)g(0.5\cos 60^\circ)$$

$$\Rightarrow v^2 = \frac{g}{2}$$

$$\Rightarrow v = \sqrt{\frac{g}{2}} = 2.21 \text{ m/s}$$

- (ii) **Before (Mass) After**

$$\sqrt{\frac{g}{2}} \quad 0.6 \quad p$$

$$0 \quad 0.8 \quad q$$

$$0.6\left(\sqrt{\frac{g}{2}}\right) + 0.8(0) = 0.6(p) + 0.8(q)$$

... multiply by 5

$$\Rightarrow 3p + 4q = 3\sqrt{\frac{g}{2}} \quad \dots \text{Equation 1}$$

$$\frac{p - q}{\sqrt{\frac{g}{2}} - 0} = -\frac{1}{11}$$

$$\Rightarrow 11p - 11q = -\sqrt{\frac{g}{2}} \quad \dots \text{Equation 2}$$

$$\text{Eq. 1} (\times 11): \quad 33p + 44q = 33\sqrt{\frac{g}{2}}$$

$$\text{Eq. 2} (\times -3): \quad -33p + 33q = 3\sqrt{\frac{g}{2}} \quad \text{add}$$

$$77q = 36\sqrt{\frac{g}{2}}$$

$$\Rightarrow q = \frac{36}{77}\sqrt{\frac{g}{2}}$$

$$= 1.03 \text{ m/s}$$

$$p = \frac{3\sqrt{\frac{g}{2}} - 4q}{3} \quad \dots \text{from Equation 1}$$

$$\Rightarrow p = 0.84 \text{ m/s}$$

Q. 20. (i) Before (Mass) After

$$\begin{array}{ccc} u & m & p \\ 0 & 3m & q \end{array}$$

$$m(u) + 3m(0) = m(p) + 3m(q) \quad \dots \text{divide by } m$$

$$\Rightarrow p + 3q = u \quad \dots \text{Equation 1}$$

$$\frac{p - q}{u - 0} = -e$$

$$\Rightarrow -p + q = eu \quad \dots \text{Equation 2}$$

Adding equations 1 and 2 we get

$$4q = u(1 + e)$$

$$\Rightarrow q = \frac{u}{4}(1 + e) \quad \dots \text{speed of 2nd sphere after collision}$$

$$p = q - eu \quad \dots \text{from Equation 2}$$

$$\Rightarrow p = \frac{u}{4}(1 + e) - eu$$

$$\Rightarrow p = \frac{u + eu - 4eu}{4} = \frac{u - 3eu}{4}$$

$$\Rightarrow p = \frac{u}{4}(1 - 3e) \quad \dots \text{speed of 1st sphere after collision}$$

(ii)  $K.E._{\text{before}} = \frac{1}{2}mu^2$

$$K.E._{\text{after}} = \frac{1}{2}mp^2 + \frac{1}{2}(3m)q^2$$

$$= \frac{m}{2} \left[ \frac{u^2}{16}(1 - 6e + 9e^2) \right] + \frac{3m}{2} \left[ \frac{u^2}{16}(1 + 2e + e^2) \right]$$

$$= \frac{mu^2}{32} [1 - 6e + 9e^2 + 3 + 6e + 3e^2]$$

$$= \frac{mu^2}{32} [12e^2 + 4]$$

$$= \frac{mu^2}{8} (3e^2 + 1)$$

$$\text{Loss} = \frac{mu^2}{2} - \frac{mu^2}{8} (3e^2 + 1)$$

$$= \frac{mu^2}{2} \left[ 1 - \frac{3e^2}{4} - \frac{1}{4} \right]$$

$$= \frac{mu^2}{2} \left[ \frac{4 - 3e^2 - 1}{4} \right]$$

$$= \frac{mu^2}{8} (3 - 3e^2)$$

$$= \frac{3mu^2}{8} (1 - e^2)$$

(iii) Let  $e = \frac{1}{4}$

$$\Rightarrow \text{Loss} = \frac{3mu^2}{8} \left( \frac{15}{16} \right) = \frac{45mu^2}{128}$$

$$\% \text{ Loss} = \frac{\frac{45mu^2}{128}}{\frac{mu^2}{2}} \times 100$$

$$= \frac{45}{64} \times 100 = 70\%$$

(iv) Loss in K.E. is maximised when  $e = 0$

$$\Rightarrow \text{Loss} = \frac{3mu^2}{8}$$

$$\% \text{ Loss} = \frac{\frac{3mu^2}{8}}{\frac{mu^2}{2}} \times 100$$

$$= \frac{3}{4} \times 100$$

$$= 75\%$$

### Exercise 7C

<b>Q. 1.</b>	(i)	<b>Before</b>	<b>(Mass)</b>	<b>After</b>
		$4\vec{i} + 3\vec{j}$	$M$	$p\vec{i} + 3\vec{j}$
		$\vec{i} + 2\vec{j}$	$M$	$q\vec{i} + 2\vec{j}$

$$M(4) + M(1) = M(p) + M(q)$$

$$\Rightarrow p + q = 5$$

$$\frac{p - q}{4 - 1} = -\frac{1}{3}$$

$$\Rightarrow p - q = -1$$

Solving these gives  $p = 2, q = 3$ .

The new velocities are  $2\vec{i} + 3\vec{j}$  and  $3\vec{i} + 2\vec{j}$

$$(ii) \text{ K.E.}_{\text{before}} = \frac{1}{2}(M)(4^2 + 3^2) + \frac{1}{2}(M)(1^2 + 2^2) \\ = 15M \text{ J}$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}M(2^2 + 3^2) + \frac{1}{2}M(3^2 + 2^2) \\ = 13M \text{ J}$$

$$\text{Loss} = 15M - 13M = 2M \text{ J}$$

<b>Q. 2.</b>	<b>Before</b>	<b>(Mass)</b>	<b>After</b>
	$3\vec{i} + 4\vec{j}$	$2$	$p\vec{i} + 4\vec{j}$
	$-4\vec{i} + 3\vec{j}$	$3$	$q\vec{i} + 3\vec{j}$

$$2(3) + 3(-4) = 2(p) + 3(q)$$

$$\Rightarrow 2p + 3q = -6$$

$$\frac{p - q}{3 + 4} = -\frac{3}{7}$$

$$\Rightarrow p - q = -3$$

Solving these gives  $p = -3, q = 0$

(i) Their velocities are  $-3\vec{i} + 4\vec{j}, 0\vec{i} + 3\vec{j}$

$$(ii) \text{ K.E.}_{\text{before}} = \frac{1}{2}(2)(3^2 + 4^2) + \frac{1}{2}(3)((-4)^2 + 3^2) \\ = 62\frac{1}{2} \text{ J}$$

$$\text{K.E.}_{\text{after}} =$$

$$\frac{1}{2}(2)((-3)^2 + 4^2) + \frac{1}{2}(3)(0^2 + 3^2)$$

$$= 38\frac{1}{2} \text{ J}$$

$$\text{Loss} = 62\frac{1}{2} - 38\frac{1}{2}$$

$$= 24 \text{ J}$$

$$(iii) \vec{l}_1 = M\vec{v} - M\vec{u} \\ = 2(-3\vec{i} + 4\vec{j}) - 2(3\vec{i} + 4\vec{j}) \\ = -12\vec{i} \text{ Ns}$$

The magnitude of the impulse is 12 Ns.

<b>Q. 3.</b>	<b>Before</b>	<b>(Mass)</b>	<b>After</b>
	$6\vec{i} + \vec{j}$	$M$	$0\vec{i} + \vec{j}$
	$-2\vec{i} - 5\vec{j}$	$2M$	$q\vec{i} - 5\vec{j}$

$$M(6) + 2M(-2) = M(0) + 2Mq$$

$$\Rightarrow q = 1$$

(i) Its velocity is  $\vec{i} - 5\vec{j}$ .

$$(ii) \frac{0 - 1}{6 + 2} = -e \Rightarrow e = \frac{1}{8}$$

<b>Q. 4.</b>	<b>Before</b>	<b>(Mass)</b>	<b>After</b>
	$5\vec{i} + 5\vec{j}$	$2M$	$p\vec{i} + 5\vec{j}$
	$0\vec{i} + 0\vec{j}$	$M$	$q\vec{i} + 0\vec{j}$

$$2M(5) + M(0) = 2Mp + Mq$$

$$\Rightarrow 2p + q = 10$$

$$\frac{p - q}{5 - 0} = -\frac{1}{2}$$

$$\Rightarrow 2p - 2q = -5$$

Solving these gives  $p = \frac{21}{2}, q = 5$

(i) Their velocities are  $2\frac{1}{2}\vec{i} + 5\vec{j}; 5\vec{i} + 0\vec{j}$

$$(ii) \vec{l}_1 = M\vec{v} - M\vec{u} \\ = 2M\left(2\frac{1}{2}\vec{i} + 5\vec{j}\right) - 2M(5\vec{i} + 5\vec{j}) \\ = -5M\vec{i} \text{ Ns}$$

$$\vec{l}_2 = M(5\vec{i} + 0\vec{j}) - M(0\vec{i} + 0\vec{j}) \\ = 5M\vec{i} \text{ Ns}$$

$$(iii) \text{ K.E.}_{\text{before}} = \frac{1}{2}(2M)(5^2 + 5^2) + \frac{1}{2}(M)(0^2 + 0^2) \\ = 50M \text{ J}$$

$$\text{K.E.}_{\text{after}} =$$

$$\frac{1}{2}(2M)\left(\left(\frac{5}{2}\right)^2 + 5^2\right) + \frac{1}{2}(M)(5^2 + 0^2)$$

$$= 43\frac{3}{4}M \text{ J}$$

$$\text{Percentage Loss} = \frac{6\frac{1}{4}M}{50M} \times \frac{100}{1}$$

$$= 12\frac{1}{2}\%$$

$$(iv) m_1 = \frac{5}{5} = 1, m_2 = \frac{5}{2\frac{1}{2}} = 2$$

$$\begin{aligned} \therefore \tan \theta &= \pm \frac{1-2}{1+(1)(2)} \\ &= \pm \frac{1}{3} = 0.3333 \\ \therefore \theta &= 18^\circ 26' \text{ (Since } \theta \text{ is acute)} \end{aligned}$$

**Q. 5.** (i)  $u_1 = 5 \cos \theta \vec{i} + 5 \sin \theta \vec{j}$

$$= 5\left(\frac{4}{5}\right)\vec{i} + 5\left(\frac{3}{5}\right)\vec{j}$$

$$= 4\vec{i} + 3\vec{j}$$

$$u_2 = -4\sqrt{2} \cos 45^\circ \vec{i} + 4\sqrt{2} \sin 45^\circ \vec{j}$$

$$= -4\vec{i} + 4\vec{j}$$

Before	(Mass)	After
$4\vec{i} + 3\vec{j}$	2	$p\vec{i} + 3\vec{j}$
$-4\vec{i} + 4\vec{j}$	3	$q\vec{i} + 4\vec{j}$

$$2(4) + 3(-4) = 2p + 3q$$

$$\Rightarrow 2p + 3q = -4$$

$$\frac{p-q}{4+4} = \frac{-7}{8}$$

$$\Rightarrow p - q = -7$$

Solving these gives  $p = -5, q = 2$ .

(ii)  $-5\vec{i} + 3\vec{j}, 2\vec{i} + 4\vec{j}$  m/s

(iii)  $K.E._{\text{before}} = \frac{1}{2}(2)(4^2 + 3^2)$

$$+ \frac{1}{2}(3)((-4)^2 + 4^2)$$

$$= 73 \text{ J}$$

$$K.E._{\text{after}} = \frac{1}{2}(2)((-5)^2 + 3^2)$$

$$+ \frac{1}{2}(3)(2^2 + 4^2)$$

$$= 64 \text{ J}$$

$$\text{Loss} = 73 - 64 = 9 \text{ J}$$

**Q. 6.**

Before	(Mass)	(After)
$5\vec{i} + 4\vec{j}$	5	$p\vec{i} + 4\vec{j}$
$-2\vec{i} - 3\vec{j}$	10	$q\vec{i} - 3\vec{j}$

$$5(5) + 10(-2) = 5p + 10q$$

$$\Rightarrow p + 2q = 1$$

$$\frac{p-q}{5+2} = \frac{1}{7}$$

$$\Rightarrow p - q = -1$$

Solving these gives:  $p = -\frac{1}{3}, q = \frac{2}{3}$ .

(i)  $-\frac{1}{3}\vec{i} + 4\vec{j}$

(ii)  $\frac{2}{3}\vec{j} - 3\vec{j}$

$$K.E._{\text{before}} =$$

$$\frac{1}{2}(5)(25 + 16) + \frac{1}{2}(10)(4 + q)$$

$$= 102.5 + 65$$

$$= 167.5 \text{ J}$$

$$K.E._{\text{after}} =$$

$$\frac{1}{2}(5)\left(\frac{1}{9} + 16\right) + \frac{1}{2}(10)\left(\frac{4}{9} + 9\right)$$

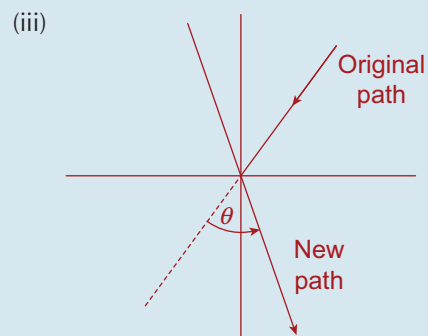
$$= \frac{725}{18}$$

$$= \frac{850}{18}$$

$$= \frac{1,575}{18}$$

$$= 87.5 \text{ J}$$

$$\text{Loss} = 167.5 - 87.5 = 80 \text{ J} \quad \text{QED}$$



$$M_1 = \frac{-3}{-2} = \frac{3}{2}, \quad M_2 = \frac{-3}{\frac{2}{5}} = \frac{-9}{2}$$

$$\tan \theta = \pm \frac{\frac{3}{2} + \frac{9}{2}}{1 - \frac{27}{4}} = \pm \frac{24}{23}$$

$$\tan \theta = \frac{24}{23}, \text{ since } \theta \text{ is acute.}$$

**Q. 7.**

Before	(Mass)	(After)
$p\vec{i} + q\vec{j}$	4	$0\vec{i} + q\vec{j}$
$0\vec{i} + 0\vec{j}$	$m$	$r\vec{i} + 0\vec{j}$

Momentum in the  $\vec{i}$ -direction is conserved

$$\Rightarrow 4p = mr$$

**N.E.L.**

$$\frac{0-r}{p-0} = \frac{-4}{7}$$

$$\Rightarrow 4p = 7r$$

$$\Rightarrow m = 7$$

**Q. 8.** Before Mass After

$8\vec{i} + 4\vec{j}$	$m$	$x\vec{i} + 4\vec{j}$
$0\vec{i} + 0\vec{j}$	$2m$	$y\vec{i} + 0\vec{j}$

(i)  $8\vec{i} + 4\vec{j} \perp x\vec{i} + 4\vec{j}$

$$\therefore \frac{4}{8} \cdot \frac{4}{x} = -1$$

$$\Rightarrow x = -2$$

$\therefore$  New velocity =  $-2\vec{i} + 4\vec{j}$  m/s

(ii)  $m(8) + 2m(0) = m(x) + 2my$

$$8m = -2m + 2my$$

$$\therefore y = 5$$

$\therefore$  Its velocity =  $5\vec{i}$  m/s

(iii)  $\frac{x-y}{8-0} = -e$

$$\therefore \frac{-2-5}{8} = -e$$

$$\therefore e = \frac{7}{8}$$

**Q. 9.** (i) Before (Mass) After

$p\vec{i} + q\vec{j}$	4	$r\vec{i} + q\vec{j}$
$x\vec{i} + y\vec{j}$	2	$0\vec{i} + 0\vec{j}$

Momentum in the  $\vec{i}$ -direction is conserved

$$\Rightarrow 4p + 2x = 4r$$

$$\Rightarrow 2p + x = 2r \dots \text{Equation 1}$$

**N.E.L.**

$$\frac{r-0}{p-x} = -\frac{1}{2}$$

$$\Rightarrow p - x = -2r \dots \text{Equation 2}$$

Adding equations 1 and 2 we get

$$3p = 0$$

$$\Rightarrow p = 0$$

$\vec{j}$ -velocity of 2nd sphere is unchanged

$$\Rightarrow y = 0$$

$\Rightarrow$  Velocities before impact were  $q\vec{j}$  and  $x\vec{i}$  ... these are  $\perp$  to each other.

(ii) **4 kg mass:**

$$\text{K.E.}_{\text{before}} = \frac{1}{2}(4)q^2 = 2q^2$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(4)(r^2 + q^2)$$

$$= 2(r^2 + q^2)$$

$$\text{Gain} = 2(r^2 + q^2) - 2q^2$$

$$= 2r^2$$

**2 kg mass:**

$$\text{K.E.}_{\text{before}} = \frac{1}{2}(2)x^2 = x^2$$

... but  $x = 2r$  (from **Equation 1**)

$$\Rightarrow \text{K.E.}_{\text{before}} = (2r)^2 = 4r^2$$

$$\text{K.E.}_{\text{after}} = 0$$

$$\Rightarrow \text{Loss} = 4r^2$$

$$\Rightarrow \text{Gain in K.E. of 4 kg mass} = \frac{1}{2}$$

(Loss in K.E. of 2 kg mass)

**Q. 10.** (i) First collision between P and Q

	Before	(Mass)	After
P:	$12u$	4	$p$
Q:	0	8	$q$

$$4(12u) + 8(0) = 4(p) + 8(q)$$

$$\Rightarrow p + 2q = 12u \dots \text{Equation 1}$$

$$\frac{p-q}{12u-0} = -\frac{1}{4}$$

$$\Rightarrow -p + q = 3u \dots \text{Equation 2}$$

Adding equations 1 and 2 we get

$$3q = 15u$$

$$\Rightarrow q = 5u \dots \text{speed of Q after 1st collision}$$

$$p = q - 3u \dots \text{from Equation 2}$$

$$\Rightarrow p = 5u - 3u$$

$$\Rightarrow p = 2u \dots \text{speed of P after 1st collision}$$

Collision of Q with the wall

$$\frac{\text{NEW}}{\text{OLD}} = -e$$

$$\Rightarrow \frac{v}{5u} = -e$$

$$\Rightarrow v = -5eu$$

Second collision between  $P$  and  $Q$

	Before	(Mass)	After
$P$ :	$2u$	$4$	$r$
$Q$ :	$-5eu$	$8$	$0$

$$4(2u) + 8[-5eu] = 4r$$

$$\Rightarrow r = 2u - 10eu$$

$$\frac{r - 0}{2u + 5eu} = -\frac{1}{4}$$

$$\Rightarrow 4r = -2u - 5eu$$

$$\Rightarrow 4(2u - 10eu) = -2u - 5eu$$

$$\Rightarrow 8 - 40e = -2 - 5e$$

$$\Rightarrow 35e = 10$$

$$\Rightarrow e = \frac{2}{7}$$

(ii)  $r = 2u - 10eu$

$$\Rightarrow r = 2u - 10\left(\frac{2}{7}\right)u$$

$$\Rightarrow r = 2u - \frac{20}{7}u$$

$$\Rightarrow r = -\frac{6}{7}u \quad \dots \text{ final speed of } P$$

(iii)  $\text{K.E.}_{\text{before}} = \frac{1}{2}(4)(12u)^2$

$$= 288u^2 \text{ J}$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(4)\left(\frac{6}{7}u\right)^2$$

$$= \frac{72}{49}u^2 \text{ J}$$

$$\text{Loss} = 288u^2 - \frac{72}{49}u^2$$

$$= 286.53u^2 \text{ J}$$

## Exercise 7D

Q. 1.	Before	(Mass)	After
	$u \cos A \vec{i} + u \sin A \vec{j}$	$M$	$v \cos B \vec{i} + v \sin B \vec{j}$
	$0\vec{i} + 0\vec{j}$	$M$	$q\vec{i} + 0\vec{j}$

$$u \sin A = v \sin B \quad \dots \text{ Equation 1}$$

$$M(u \cos A) + M(0) = M(v \cos B) + Mq$$

$$\Rightarrow q = u \cos A - v \cos B \quad \dots \text{ Equation 2}$$

$$\frac{v \cos B - q}{u \cos A - 0} = -\frac{1}{4}$$

$$\Rightarrow 4v \cos B - 4q = -u \cos A \quad \dots \text{ Equation 3}$$

$$\text{But } q = u \cos A - v \cos B$$

$\therefore$  3 reads:

$$4v \cos B - 4u \cos A + 4v \cos B = -u \cos A$$

$$\Rightarrow 3u \cos A = 8v \cos B \quad \dots \text{ Equation 4}$$

Dividing equation 1 by equation 4, we get:

$$\frac{u \sin A}{3u \cos A} = \frac{v \sin B}{8v \cos B}$$

$$\Rightarrow 8 \tan A = 3 \tan B \quad \text{QED}$$



<b>Q. 2. Before</b>	<b>(Mass)</b>	<b>After</b>
$2u \cos A \vec{i} + 2u \sin A \vec{j}$	$2M$	$p\vec{i} + 2u \sin A \vec{j}$
$-u\vec{i} + 0\vec{j}$	$M$	$0\vec{i} + 0\vec{j}$

$$(i) \quad 2M(2u \cos A) + M(-u) = 2M(p) + M(0)$$

$$\Rightarrow p = 2u \cos A - \frac{1}{2}u$$

$$\frac{p - 0}{2u \cos A + u} = \frac{-5}{118}$$

$$\Rightarrow 118p = -10u \cos A - 5u$$

$$\text{But } p = 2u \cos A - \frac{1}{2}u$$

$$\therefore 118\left(2u \cos A - \frac{1}{2}u\right)$$

$$= -10u \cos A - 5u$$

$$\Rightarrow 236 \cos A - 59 = -10 \cos A - 5$$

$$\Rightarrow \cos A = \frac{9}{41}$$

$$\Rightarrow \sin A = \frac{40}{41}$$

$$(ii) \quad p = 2u \cos A - \frac{1}{2}u$$

$$= 2u\left(\frac{9}{41}\right) - \frac{1}{2}u = \frac{-5u}{82}$$

$$2u \sin A = 2u\left(\frac{40}{41}\right) = \frac{80u}{41} = \frac{160u}{82}$$

$\therefore$  Velocity after impact

$$= \frac{u}{82}(-5\vec{i} + 160\vec{j})$$

$$= \frac{5u}{82}(-\vec{i} + 32\vec{j}) \text{ m/s}$$

**Q. 3. (i) Before (Mass) After**

$$4u \quad m \quad p$$

$$2u \quad m \quad q$$

$$m(4u) + m(2u) = m(p) + m(q)$$

... divide by  $m$

$$\Rightarrow p + q = 6u \quad \text{Equation 1}$$

$$\frac{p - q}{4u - 2u} = -e$$

$$\Rightarrow p - q = -2eu \quad \text{Equation 2}$$

Adding equations 1 and 2 we get

$$2p = 2u(3 - e)$$

$$\Rightarrow p = u(3 - e)$$

$$q = 6u - p \quad \text{from Equation 1}$$

$$\Rightarrow q = 6u - u(3 - e)$$

$$\Rightarrow q = 3u + eu$$

$$\Rightarrow q = u(3 + e)$$

$$\begin{aligned} \text{K.E.}_{\text{before}} &= \frac{1}{2}(m)(4u)^2 + \frac{1}{2}(m)(2u)^2 \\ &= 10mu^2 \end{aligned}$$

$$\begin{aligned} \text{K.E.}_{\text{after}} &= \frac{1}{2}(m)[u(3 - e)^2] \\ &\quad + \frac{1}{2}(m)[u(3 + e)^2] \\ &= \frac{1}{2}(m)[u^2(9 - 6e + e^2 + 9 \\ &\quad + 6e + e^2)] \\ &= \frac{1}{2}(m)[u^2(18 + 2e^2)] \\ &= mu^2(9 + e^2) \end{aligned}$$

$$\begin{aligned} \text{Loss} &= 10mu^2 - mu^2(9 + e^2) \\ &= mu^2 - mu^2e^2 \\ &= mu^2(1 - e^2) \end{aligned}$$

(ii) Firstly, calculate velocity of first sphere before impact:

$$4u \cos 30^\circ \vec{i} + 4u \sin 30^\circ \vec{j}$$

$$= 4u\left(\frac{\sqrt{3}}{2}\right)\vec{i} + 4u\left(\frac{1}{2}\right)\vec{j}$$

$$= 2u\sqrt{3}\vec{i} + 2u\vec{j}$$

<b>Before</b>	<b>(Mass)</b>	<b>After</b>
$2u\sqrt{3}\vec{i} + 2u\vec{j}$	$m$	$p\vec{i} + 2u\vec{j}$
$0\vec{i} + 0\vec{j}$	$m$	$q\vec{i} + 0\vec{j}$

Momentum in the  $\vec{i}$ -direction is conserved

$$\Rightarrow m(2u\sqrt{3}) + m(0) = m(p) + m(q)$$

... divide by  $m$

$$\Rightarrow p + q = 2u\sqrt{3} \quad \text{Equation 3}$$

**N.E.L.**

$$\frac{p - q}{2u\sqrt{3} - 0} = -e$$

$$\Rightarrow p - q = -2eu\sqrt{3} \quad \text{Equation 4}$$

Adding equations 3 and 4 we get

$$2p = 2u\sqrt{3}(1 - e)$$

$$\Rightarrow p = u\sqrt{3}(1 - e)$$

$\Rightarrow$  velocity of 1st sphere after impact

$$= u\sqrt{3}(1 - e)\vec{i} + 2u\vec{j}$$

$$q = 2u\sqrt{3} - p \quad \text{from Equation 3}$$

$$\Rightarrow q = 2u\sqrt{3} - u\sqrt{3}(1 - e)$$

$$= u\sqrt{3} + eu\sqrt{3}$$

$$\Rightarrow q = u\sqrt{3}(1 + e)$$

$\Rightarrow$  velocity of 2nd sphere after impact  $= u\sqrt{3}(1 + e)\hat{i}$

$$\text{K.E.}_{\text{before}} = \frac{1}{2}(m)(4u)^2$$

$$= 8mu^2$$

$$\text{K.E.}_{\text{after}} = \frac{1}{2}(m)[\{u\sqrt{3}(1 - e)\}^2 + \{2u\}^2]$$

$$+ \frac{1}{2}(m)[u\sqrt{3}(1 + e)]^2$$

$$= \frac{m}{2}[3u^2(1 - 2e + e^2) + 4u^2 + 3u^2(1 + 2e + e^2)]$$

$$= \frac{m}{2}[10u^2 + 6e^2u^2]$$

$$= mu^2(5 + 3e^2)$$

$$\text{Loss} = 8mu^2 - mu^2(5 + 3e^2)$$

$$= 3mu^2 - 3mu^2e^2$$

$$= 3mu^2(1 - e^2)$$

**Q. 4. (i) Before (Mass) After**

A:  $\frac{u}{2}\hat{i} + \frac{u\sqrt{3}}{2}\hat{j}$        $m$        $p\hat{i} + \frac{u\sqrt{3}}{2}\hat{j}$

B:  $0\hat{i} + 0\hat{j}$        $m$        $q\hat{i} + 0\hat{j}$

Momentum in the  $\hat{i}$ -direction is conserved

$$\Rightarrow m\left(\frac{u}{2}\right) + m(0) = m(p) + m(q) \quad \dots \text{divide by } m$$

$$\Rightarrow p + q = \frac{u}{2}$$

$$\Rightarrow 2p + 2q = u \quad \text{Equation 1}$$

$$\frac{p - q}{\frac{u}{2} - 0} = -e$$

$$\Rightarrow p - q = -\frac{eu}{2}$$

$$2p - 2q = -eu \quad \text{Equation 2}$$

Adding equations 1 and 2 we get

$$4p = u(1 - e)$$

$$\Rightarrow p = \frac{u}{4}(1 - e)$$

$$\tan \theta = \frac{u\sqrt{3}}{2p}$$

$$\Rightarrow p = \frac{u\sqrt{3}}{2 \tan \theta}$$

$$\Rightarrow \frac{u\sqrt{3}}{2 \tan \theta} = \frac{u}{4}(1 - e) \quad \dots \text{multiply by } \frac{2}{u}$$

$$\Rightarrow \frac{\sqrt{3}}{\tan \theta} = \frac{1}{2}(1 - e)$$

$$\Rightarrow (1 - e)\tan \theta = 2\sqrt{3}$$

$$\Rightarrow \tan \theta = \frac{2\sqrt{3}}{1 - e}$$

$$\tan \theta = \frac{\frac{u\sqrt{3}}{2}}{p} = \frac{u\sqrt{3}}{2p}$$

$$q = \frac{u}{2} - p \quad \dots \text{from Equation 1}$$

$$\Rightarrow q = \frac{u}{2} - \frac{u}{4}(1 - e) = \frac{u}{4} + \frac{ue}{4} = \frac{u}{4}(1 + e)$$

(ii)  $K.E._{\text{before}} = \frac{1}{2}mu^2 \text{ J}$

$$K.E._{\text{after}} = \frac{1}{2}m\left[p^2 + \left(\frac{u\sqrt{3}}{2}\right)^2\right] + \frac{1}{2}mq^2$$

$$= \frac{1}{2}m\left[\frac{u^2}{16}(1 - e)^2 + \frac{3u^2}{4}\right]$$

$$+ \frac{1}{2}m\left[\frac{u^2}{16}(1 + e)^2\right] \dots \text{let } e = 0$$

$$= \left[\frac{1}{2}m\frac{u^2}{16} + \frac{3u^2}{4} + \frac{u^2}{16}\right]$$

$$= \frac{1}{2}m\left(\frac{7u^2}{8}\right)$$

$$= \frac{7mu^2}{16} \text{ J}$$

$$\text{Loss} = \frac{mu^2}{2} - \frac{7mu^2}{16} = \frac{mu^2}{16} \text{ J}$$

$$\Rightarrow \% \text{ loss} = \frac{\frac{1}{16}mu^2}{\frac{1}{2}mu^2} \times 100 = 12\frac{1}{2}\%$$

(iii)  $K.E._{A(\text{after})} = \frac{1}{2}m\left[p^2 + \left(\frac{u\sqrt{3}}{2}\right)^2\right]$

$$= \frac{1}{2}m\left[\frac{u^2}{16}(1 - e)^2 + \frac{3u^2}{4}\right]$$

$$= \frac{1}{2}m\left[\frac{u^2}{16}(1 - 2e + e^2) + \frac{3u^2}{4}\right]$$

$$= \frac{1}{2}m\left[\frac{13u^2}{16} - \frac{2eu^2}{16} + \frac{e^2u^2}{16}\right]$$

$$= \frac{mu^2}{32}[13 - 2e + e^2]$$

$$K.E._{B(\text{after})} = \frac{1}{2}mq^2 = \frac{1}{2}m\left[\frac{u^2}{16}(1 + e)^2\right]$$

$$= \frac{mu^2}{32}(1 + 2e + e^2)$$

$$\frac{K.E._{A(\text{after})}}{K.E._{B(\text{after})}} = \frac{7}{1}$$

$$\Rightarrow \frac{13 - 2e + e^2}{1 + 2e + e^2} = 7$$

$$\Rightarrow 13 - 2e + e^2 = 7 + 14e + 7e^2$$

$$\Rightarrow 6e^2 + 16e - 6 = 0$$

$$\Rightarrow 3e^2 + 8e - 3 = 0$$

$$\Rightarrow (3e - 1)(e + 3) = 0$$

$$\Rightarrow e = \frac{1}{3} \quad 0 \leq e \leq 1$$

**Q. 5.** (i) 

<b>Before</b>	<b>(Mass)</b>	<b>After</b>
A: $p\vec{i} + q\vec{j}$	$m$	$r\vec{i} + q\vec{j}$
B: $0\vec{i} + 0\vec{j}$	$2m$	$t\vec{i} + 0\vec{j}$

$$\tan \theta = \frac{q}{p}$$

$$\frac{q}{p} \times \frac{q}{r} = -1 \quad \dots \text{ new path at right angles to old path}$$

$$\Rightarrow \frac{q^2}{pr} = -1$$

$$\Rightarrow r = -\frac{q^2}{p}$$

Also,  $\sqrt{r^2 + q^2} = 0.2$

$$\Rightarrow r^2 + q^2 = 0.04$$

$$\Rightarrow \frac{q^4}{p^2} + q^2 = 0.04$$

$$\Rightarrow \frac{q^4 + p^2q^2}{p^2} = 0.04$$

$$\Rightarrow \frac{q^2(p^2 + q^2)}{p^2} = 0.04 \dots \sqrt{p^2 + q^2} = 0.06$$

so,  $p^2 + q^2 = 0.36$

$$\Rightarrow \frac{0.36q^2}{p^2} = 0.04 \quad \dots \text{ divide by } 0.04$$

$$\Rightarrow \frac{9q^2}{p^2} = 1$$

$$\Rightarrow \frac{q^2}{p^2} = \frac{1}{9}$$

$$\Rightarrow \frac{q}{p} = \frac{1}{3} = \tan \theta$$

(ii) Momentum in the  $\vec{i}$ -direction is conserved

$$m(p) + 2m(0) = m(r) + 2m(t)$$

...divide by  $m$

$$\Rightarrow r + 2t = p$$

$$\Rightarrow t = \frac{p - r}{2} \quad \text{Equation 1}$$

**N.E.L.**

$$\frac{r - t}{p - 0} = -e$$

$$\Rightarrow r - t = -pe \quad \text{Equation 2}$$

$$\Rightarrow r - \frac{p - r}{2} = -pe$$

$$\Rightarrow 2r - p + r = -2pe$$

$$\Rightarrow 3r - p = -2pe$$

$$\begin{aligned} \Rightarrow e &= \frac{p - 3r}{2p} \\ &= \frac{p + \frac{3q^2}{p}}{2p} \\ &= \frac{p^2 + 3q^2}{2p^2} \\ &= \frac{1}{2} + \frac{3(q^2)}{2(p^2)} \\ &= \frac{1}{2} + \frac{3(1)}{2(9)} \\ &= \frac{1}{2} + \frac{1}{6} = \frac{2}{3} \end{aligned}$$

(iii)  $\text{K.E.}_{\text{before}} = \frac{1}{2}m(0.6)^2 = 0.18m$

$$\begin{aligned} \text{K.E.}_{\text{after}} &= \frac{1}{2}m(0.2)^2 + \frac{1}{2}(2m)t^2 \\ &= 0.02m + mt^2 \end{aligned}$$

$$p = 0.6 \cos \theta = 0.6 \left( \frac{3}{\sqrt{10}} \right) = \frac{1.8}{\sqrt{10}}$$

$$r + 2t = \frac{1.8}{\sqrt{10}} \quad \dots \text{ from Equation 1}$$

$$r - t = -\frac{2p}{3} \quad \dots \text{ from Equation 2}$$

$$\Rightarrow -r + t = \frac{3.6}{3\sqrt{10}}$$

$$= \frac{1.2}{\sqrt{10}}$$

$$\text{Add } 3t = \frac{3}{\sqrt{10}}$$

$$\Rightarrow t = \frac{1}{\sqrt{10}}$$

$$\Rightarrow \text{K.E.}_{\text{after}} = 0.02m + 0.1m = 0.12m$$

$$\text{Loss} = 0.18m - 0.12m = 0.06m$$

**Q. 6.** (a)  $\frac{b}{a} \times \frac{d}{c} = -1$

$$\Rightarrow \frac{b}{a} = -\frac{c}{d}$$

$$\Rightarrow ac = -bd$$

(b)  $\cos \theta = \frac{3}{7}$

$$\Rightarrow \sin \theta = \frac{\sqrt{40}}{7}$$

Before	(Mass)	After
$\frac{3v^2}{7}\vec{i} + \frac{v\sqrt{40}}{7}\vec{j}$	2m	$p\vec{i} + \frac{v\sqrt{40}}{7}\vec{j}$
$-u\vec{i} + 0\vec{j}$	m	$q\vec{i} + 0\vec{j}$
$\frac{3v}{7}(p) = -\frac{v\sqrt{40}}{7}\left(\frac{v\sqrt{40}}{7}\right)$		... from part (a)
$\Rightarrow 3vp = -\frac{40v^2}{7}$		
$\Rightarrow p = -\frac{40v}{21}$		

Momentum in the  $\vec{i}$ -direction is conserved

$$\begin{aligned} \Rightarrow 2m\left(\frac{3v}{7}\right) + m(-u) &= 2m\left(\frac{-40v}{21}\right) + m(q) \\ &\dots \text{ multiply by } \frac{21}{m} \end{aligned}$$

$$\Rightarrow 18v - 21u = -80v + 21q$$

$$\Rightarrow 98v - 21u = 21q$$

$$\Rightarrow 14v - 3q = 3u \quad \dots \text{ Equation 1}$$

**N.E.L.**

$$\frac{-\frac{40v}{21} - q}{\frac{3v}{7} + u} = -\frac{3}{4}$$

$$\Rightarrow \frac{-40v - 21q}{9v + 21u} = -\frac{3}{4}$$

$$\Rightarrow 160v + 84q = 27v + 63u$$

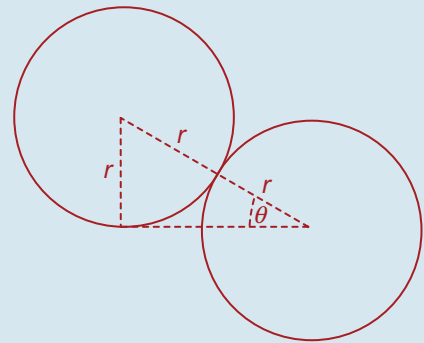
$$\Rightarrow 133v + 84q = 63u \quad \dots \text{ Equation 2}$$

$$392v - 84q = 84u \quad \dots \text{ from Equation 1}$$

$$\hline 525v = 147u$$

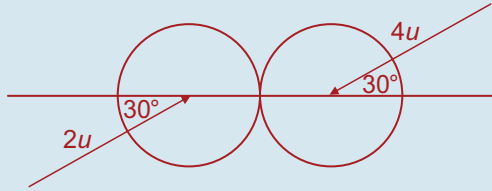
$$v = \frac{147u}{525} = \frac{7u}{25}$$

**Q. 7.** (i)



$$\sin \theta = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

(ii) Rotate diagram as shown:



Before	(Mass)	After
$u\sqrt{3}\vec{i} + u\vec{j}$	$2m$	$p\vec{i} + u\vec{j}$
$-2u\sqrt{3}\vec{i} - 2u\vec{j}$	$m$	$q\vec{i} - 2u\vec{j}$

Momentum in the  $\vec{i}$ -direction is conserved

$$\Rightarrow 2m(u\sqrt{3}) + m(-2u\sqrt{3}) = 2m(p) + m(q)$$

... divide by  $m$

$$\Rightarrow 2p + q = 0 \quad \text{Equation 1}$$

N.E.L.

$$\frac{p - q}{u\sqrt{3} + 2u\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow p - q = -3u \quad \text{Equation 2}$$

Adding equations 1 and 2 we get

$$3p = -3u$$

$$\Rightarrow p = -u$$

Speed of 1st sphere after impact

$$= \sqrt{p^2 + u^2}$$

$$= \sqrt{(-u)^2 + u^2}$$

$$= \sqrt{2u^2}$$

$$= u\sqrt{2}$$

$$q = -2p \quad \dots \text{ from Equation 1}$$

$$\Rightarrow q = 2u$$

Speed of 2nd sphere after impact

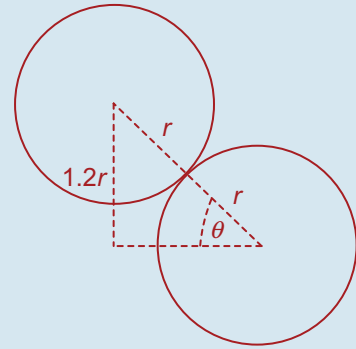
$$= \sqrt{q^2 + (-2u)^2}$$

$$= \sqrt{(2u)^2 + (-2u)^2}$$

$$= \sqrt{8u^2}$$

$$= 2u\sqrt{2}$$

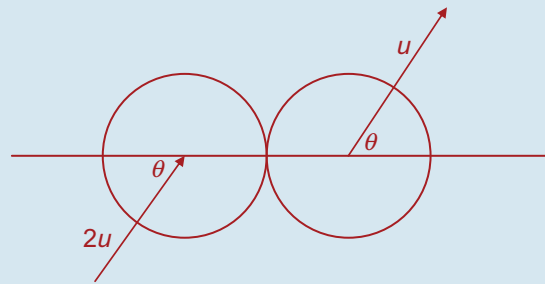
Q. 8. (i) Point of collision:



$$\sin \theta = \frac{1.2r}{2r} = \frac{3}{5}$$

$$\Rightarrow \cos \theta = \frac{4}{5}$$

Rotate diagram as shown in the diagram below:



Before	(Mass)	After
$\frac{8u}{5}\vec{i} + \frac{6u}{5}\vec{j}$	$m$	$p\vec{i} + \frac{6u}{5}\vec{j}$
$\frac{4u}{5}\vec{i} + \frac{3u}{5}\vec{j}$	$m$	$q\vec{i} + \frac{3u}{5}\vec{j}$

Momentum in the  $\vec{i}$ -direction is conserved

$$\Rightarrow m\left(\frac{8u}{5}\right) + m\left(\frac{4u}{5}\right) = m(p) + m(q)$$

... multiply by  $\frac{5}{m}$

$$\Rightarrow 5p + 5q = 12u \quad \text{Equation 1}$$

N.E.L.

$$\frac{p - q}{\frac{8u}{5} - \frac{4u}{5}} = -\frac{1}{2}$$

$$\Rightarrow \frac{5p - 5q}{4u} = -\frac{1}{2}$$

$$\Rightarrow 5p - 5q = -2u \quad \text{Equation 2}$$

Adding equations 1 and 2 we get

$$10p = 10u$$

$$\Rightarrow p = u$$

⇒ Velocity of 1st sphere after impact

$$= u\vec{i} + \frac{6u}{5}\vec{j}$$

⇒ Speed of 1st sphere after impact

$$= \sqrt{u^2 + \left(\frac{6u}{5}\right)^2}$$

$$= \sqrt{u^2 + \frac{36u^2}{25}}$$

$$= \sqrt{\frac{25u^2 + 36u^2}{25}}$$

$$= \sqrt{\frac{61u^2}{25}}$$

$$= \frac{u}{5}\sqrt{61}$$

$$= \frac{\sqrt{61}}{5}u$$

$$q = \frac{1}{5}(5p + 2u) \quad \dots \text{ from Equation 2}$$

$$\Rightarrow q = \frac{1}{5}(5u + 2u)$$

$$\Rightarrow q = \frac{7u}{5}$$

⇒ Velocity of 2nd sphere after impact

$$\frac{7u}{5}\vec{i} + \frac{3u}{5}\vec{j}$$

⇒ Speed of 2nd sphere after impact

$$\sqrt{\left(\frac{7u}{5}\right)^2 + \left(\frac{3u}{5}\right)^2} = \sqrt{\frac{49u^2 + 9u^2}{25}}$$

$$= \sqrt{\frac{58u^2}{25}} = \frac{u}{5}\sqrt{58} = \frac{\sqrt{58}}{5}u$$

(ii) Velocities after impact

$$\text{are } \vec{v}_1 = u\vec{i} + \frac{6u}{5}\vec{j} \text{ and } \vec{v}_2 = \frac{7u}{5}\vec{i} + \frac{3u}{5}\vec{j}$$

$$\text{The slope of } \vec{v}_1 \text{ is given by } m_1 = \frac{\frac{6u}{5}}{u}$$

$$= \frac{6}{5}$$

$$\text{The slope of } \vec{v}_2 \text{ is given by } m_2 = \frac{\frac{3u}{5}}{\frac{7u}{5}} = \frac{3}{7}$$

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\Rightarrow \tan \theta = \pm \frac{\frac{6}{5} - \frac{3}{7}}{1 + \frac{18}{35}}$$

$$= \pm \frac{42 - 15}{35 + 18}$$

$$= \pm \frac{27}{53}$$

Take the plus case to find the acute angle

$$\Rightarrow \theta = \tan^{-1} \frac{27}{53} = 27^\circ$$