

Chapter 8 Exercise 8A

Q. 1. (a) $R = 4\vec{j} + 5\vec{j} + \vec{j} = 10\vec{j}$ N
 $4(0) + 5(50) + 1(100) = 10(x)$
 $\Rightarrow x = 35$ cm

Answer: 35 cm from P

(b) $R = 2\vec{j} + \vec{j} + 2\vec{j} = 5\vec{j}$
 $2(0) + 1(2) + 2(5) = 5x$
 $\Rightarrow x = 2.4$ m

Answer: 2.4 m from P

(c) $R = \vec{j} - 7\vec{j} + \vec{j} = -5\vec{j}$
 $1(0) - 7(1) + 1(2) = -5(x)$
 $\Rightarrow x = 1$

Answer: 1 m from P

(d) $R = 3\vec{j} - 9\vec{j} + 3\vec{j} = -3\vec{j}$
 $3(1) - 9(3) + 3(5) = -3x$
 $\Rightarrow x = 3$

Answer: 3 m from P

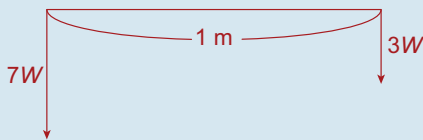
(e) $R = \vec{j} - 2\vec{j} - 3\vec{j} + \vec{j} = -3\vec{j}$
 $1(0) - 2(2) - 3(5) + 1(6) = -3(x)$
 $\Rightarrow x = 4\frac{1}{3}$

Answer: $4\frac{1}{3}$ m from P

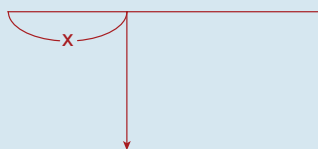
Q. 2. $R = 2W + W + 3W = 6W$
 $2W(0) + W(3) + 3W(6) = 6W(x)$
 $\Rightarrow x = 3\frac{1}{2}$

Answer: $3\frac{1}{2}$ M and $2\frac{1}{2}$ M from the ends.

Q. 3. Forces:



Resultant:



$R = 7W + 3W = 10W$
 $7W(0) + 3W(1) = 10W(x)$
 $\Rightarrow x = \frac{3}{10}$ m
 $= 30$ cm = Answer

Q. 4. Let the length of the plank be 1.
 Let d = the distance from the resultant's line of action from the left-hand end.

$$R = W + x$$

$$\therefore W(0) + x(1) = (W + x)d$$

$$\Rightarrow d = \frac{x}{W + x}$$

The remainder is $1 - \frac{x}{W + x} = \frac{W}{W + x}$

The ratio of these parts = $\frac{x}{W + x} : \frac{W}{W + x}$
 $= x : W$

Q. 5. (i) $R = 4W + W + KW = (5 + K)W$

Taking moments about p .

$$4W(0) + W(1) + KW(2) = (5 + K)W\left(\frac{7}{8}\right)$$

$$\Rightarrow 1 + 2K = \frac{7(5 + K)}{8}$$

$$\Rightarrow 8 + 16K = 35 + 7K$$

$$\Rightarrow K = 3$$

(ii) $4W(0) + W(1) + KW(2) = (5 + K)W\left(\frac{11}{10}\right)$

$$\Rightarrow 1 + 2K = \frac{11(5 + K)}{10}$$

$$\Rightarrow 10 + 20K = 55 + 11K$$

$$\Rightarrow K = 5$$

Exercise 8B

Q. 1. $\left. \begin{array}{l} 3N \text{ at } (2, 1) \\ 2N \text{ at } (4, 3) \\ 1N \text{ at } (10, 9) \end{array} \right\} = 6N \text{ at } (x, y)$

$$3(2) + 2(4) + 1(10) = 6(x)$$

$$\Rightarrow x = 4$$

$$3(1) + 2(3) + 1(9) = 6(y)$$

$$\Rightarrow y = 3$$

Answer: (4, 3)

Q. 2. $\left. \begin{array}{l} 1N \text{ at } (1, 1) \\ 2N \text{ at } (1, 7) \\ 3N \text{ at } (3, 1) \\ 4N \text{ at } (2, 3) \end{array} \right\} = 10N \text{ at } (x, y)$

$$1(1) + 2(1) + 3(3) + 4(2) = 10(x)$$

$$\Rightarrow x = 2$$

$$1(1) + 2(7) + 3(1) + 4(3) = 10(y)$$

$$\Rightarrow y = 3$$

Answer: (2, 3)

Q. 3. $3(4) + 2(9) = 5x$

$$\Rightarrow x = 6$$

$$3(1) + 2(-9) = 5y$$

$$\Rightarrow y = 3$$

Answer: (6, -3)

$$3(4) + 2(9) + 1(x) = 6(6)$$

$$\Rightarrow x = 6$$

$$3(1) + 2(-9) + 1(y) = 6(-2)$$

$$\Rightarrow y = 3$$

Answer: (6, 3)

Q. 4. $W(3) + 2W(12) = 3W(x)$

$$\Rightarrow x = 9$$

$$W(1) + 2W(19) = 3W(y)$$

$$\Rightarrow y = 13$$

Answer: (9, 13) = g

$$|pg| = \sqrt{(9 - 3)^2 + (13 - 1)^2}$$

$$= \sqrt{180}$$

$$= 6\sqrt{5}$$

$$|gq| = \sqrt{(12 - 9)^2 + (19 - 13)^2}$$

$$= \sqrt{45} = 3\sqrt{5}$$

$$|pg| : |gq| = 6\sqrt{5} : 3\sqrt{5}$$

$$= 2 : 1$$

Q. 5. $2N$ at (1, 2) }
 $3N$ at (1, 7) } = $10N$ at (x, y)
 $5N$ at (5, -1) }

$$2(1) + 3(1) + 5(5) = 10(x)$$

$$\Rightarrow x = 3$$

$$2(2) + 3(7) + 5(-1) = 10(y)$$

$$\Rightarrow y = 2$$

Answer: $3\vec{i} + 2\vec{j}$

$$\left. \begin{array}{l} 2N \text{ at } (1, 2) \\ 3N \text{ at } (1, 7) \\ 5N \text{ at } (5, -1) \\ 2N \text{ at } (x, y) \end{array} \right\} = 12N \text{ at } (4, 1)$$

$$2(1) + 3(1) + 5(5) + 2(x) = 12(4)$$

$$\Rightarrow x = 9$$

$$2(2) + 3(7) + 5(-1) + 2(y) = 12(1)$$

$$\Rightarrow y = -4$$

Answer: (9, -4)

Q. 6. $1(4) + 2(1) + 3(k) = 6\left(2\frac{1}{2}\right)$

$$k = 3$$

$$1(1) + 2h + 3(1) = 6(4)$$

$$h = 10$$

Q. 7. Centroid of triangle

$$pqr = \left(\frac{2 + 5 + 3}{3}, \frac{1 + 3 - 1}{3} \right)$$

$$= \left(\frac{10}{3}, 1 \right)$$

$$\left. \begin{array}{l} M \text{ at } (2, 1) \\ M \text{ at } (5, 3) \\ M \text{ at } (3, -1) \end{array} \right\} = 3M \text{ at } (x, y)$$

$$M(2) + M(5) + M(3) = 3M(x)$$

$$\Rightarrow x = \frac{10}{3}$$

$$M(1) + M(3) + M(-1) = 3M(y)$$

$$\Rightarrow y = 1$$

The centre of gravity is at $\left(\frac{10}{3}, 1\right)$ which is the centroid.

$$M(2) + M(5) + M(3) + 2M(x) = 5M(2)$$

$$\Rightarrow x = 0$$

$$M(1) + M(3) + M(-1) + 2M(y) = 5M(1)$$

$$\Rightarrow y = 1$$

Answer: (0, 1)

$$\text{Q. 8. (i) } \left. \begin{array}{l} W \text{ at } (6, 5) \\ 2W \text{ at } (7, -1) \\ 3W \text{ at } (2, 11) \\ 4W \text{ at } (6, 1) \end{array} \right\} = 10W \text{ at } (x, y)$$

$$W(6) + 2W(7) + 3W(2) + 4W(6) = 10W(x) \quad \dots \text{ divide by } W$$

$$\Rightarrow 10x = 50$$

$$\Rightarrow x = 5$$

$$W(5) + 2W(-1) + 3W(11) + 4W(1) = 10W(y) \quad \dots \text{ divide by } W$$

$$\Rightarrow 10y = 40$$

$$\Rightarrow y = 4$$

\Rightarrow The centre of gravity is at (5, 4)

$$\text{(ii) } \left. \begin{array}{l} W \text{ at } (6, 5) \\ 2W \text{ at } (7, -1) \\ 3W \text{ at } (2, 11) \\ kW \text{ at } (6, 1) \end{array} \right\} = (6 + k)W \text{ at } (11, y)$$

$$W(6) + 2W(7) + 3W(2) + kW(6) = (6 + k)W(11) \quad \dots \text{ divide by } W$$

$$\Rightarrow 66 + 11k = 26 + 6k$$

$$\Rightarrow 5k = -40$$

$$\Rightarrow k = -8$$

$$W(5) + 2W(-1) + 3W(11) - 8W(1) = -2W(y) \quad \dots \text{ divide by } W$$

$$\Rightarrow 2y = -28$$

$$\Rightarrow y = -14$$

$$\text{Q. 9. } \left. \begin{array}{l} 2N \text{ at } (x, 5) \\ 3N \text{ at } (11, y) \\ 5N \text{ at } (5, -6) \end{array} \right\} = 10N \text{ at } (6, -5)$$

$$2(x) + 3(11) + 5(5) = 10(6)$$

$$\Rightarrow 2x + 58 = 60$$

$$\Rightarrow x = 1$$

$$2(5) + 3(y) + 5(-6) = 10(-5)$$

$$\Rightarrow 3y - 20 = -50$$

$$\Rightarrow y = -10$$

$$\text{Q. 10. } \left. \begin{array}{l} 1N \text{ at } (7, 8) \\ 4N \text{ at } (1, y) \\ xN \text{ at } (5, 2) \\ 2N \text{ at } (6, -3) \end{array} \right\} = (7 + x)N \text{ at } (4, 2)$$

$$1(7) + 4(1) + x(5) + 2(6) = (7 + x)(4)$$

$$\Rightarrow 5x + 23 = 28 + 4x$$

$$\Rightarrow x = 5$$

$$1(8) + 4(y) + 5(2) + 2(-3) = 12(2)$$

$$\Rightarrow 4y + 12 = 24$$

$$\Rightarrow y = 3$$

$$\text{Q. 11. } \left. \begin{array}{l} 3W \text{ at } (30, 20) \\ 2W \text{ at } (20, 10) \\ 5W \text{ at } (50, 20) \end{array} \right\} = 10W \text{ at } (x, y)$$

$$3W(30) + 2W(20) + 5W(50) = 10W(x)$$

\dots divide by $10W$

$$\Rightarrow x = 38$$

$$3W(20) + 2W(10) + 5W(20) = 10W(y)$$

\dots divide by $10W$

$$\Rightarrow y = 18$$

\Rightarrow Centre of gravity is at (38, 18)

Q. 12.
$$\left. \begin{array}{l} 3N \text{ at } \left(-\frac{7}{3}, 2\right) \\ 4N \text{ at } (1, 7) \\ 5N \text{ at } (3, 7) \\ 6N \text{ at } (4, -4) \end{array} \right\} = 18N \text{ at } (x, y)$$

$$3\left(-\frac{7}{3}\right) + 4(1) + 5(3) + 6(4) = 18(x)$$

$$\Rightarrow 18x = 36$$

$$\Rightarrow x = 2$$

$$3(2) + 4(7) + 5(7) + 6(-4) = 18(y)$$

$$\Rightarrow 18y = 45$$

$$\Rightarrow y = 2.5$$

\Rightarrow Centre of gravity is at (2, 2.5)

Q. 13.
$$\left. \begin{array}{l} 5 \text{ at } (3, -1) \\ 8 \text{ at } (4, 2) \\ 3 \text{ at } (-1, 5) \\ 2 \text{ at } (2, -6) \end{array} \right\} = 18 \text{ at } (x, y)$$

$$5(3) + 8(4) + 3(-1) + 2(2) = 18(x)$$

$$\Rightarrow 18x = 48$$

$$\Rightarrow x = \frac{8}{3}$$

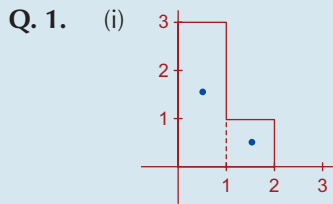
$$5(-1) + 8(2) + 3(5) + 2(-6) = 18(y)$$

$$\Rightarrow 18y = 14$$

$$\Rightarrow y = \frac{7}{9}$$

\Rightarrow Centre of gravity is at $\left(\frac{8}{3}, \frac{7}{9}\right)$

Exercise 8C



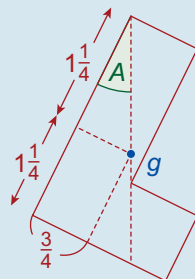
$$\left. \begin{array}{l} 3 \text{ at } \left(\frac{1}{2}, 1\frac{1}{2}\right) \\ 1 \text{ at } \left(1\frac{1}{2}, \frac{1}{2}\right) \end{array} \right\} = 4 \text{ at } (x, y)$$

$$3\left(\frac{1}{2}\right) + 1\left(1\frac{1}{2}\right) = 4x$$

$$\Rightarrow x = \frac{3}{4}$$

$$3\left(1\frac{1}{2}\right) + 1\left(\frac{1}{2}\right) = 4y$$

$$\Rightarrow y = 1\frac{3}{4}$$



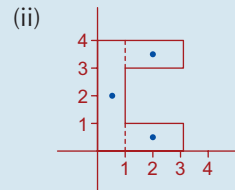
Answer: $\left(\frac{3}{4}, 1\frac{1}{4}\right)$

$$\tan A = \frac{\frac{3}{4}}{1\frac{3}{4}}$$

$$= \frac{3}{7}$$

$$= 0.4286$$

$\therefore A = 23^\circ 12'$



$$\left. \begin{array}{l} 4 \text{ at } \left(\frac{1}{2}, 2\right) \\ 2 \text{ at } \left(2, \frac{1}{2}\right) \\ 2 \text{ at } \left(2, 3\frac{1}{2}\right) \end{array} \right\} = 8 \text{ at } (x, y)$$

$$4\left(\frac{1}{2}\right) + 2(2) + 2(2) = 8(x)$$

$$\Rightarrow x = 1\frac{1}{4}$$

$$4(2) + 2\left(\frac{1}{2}\right) + 2\left(3\frac{1}{2}\right) = 8(y)$$

$$\Rightarrow y = 2$$

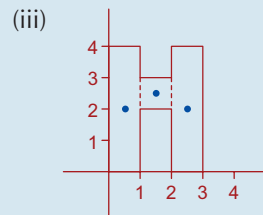
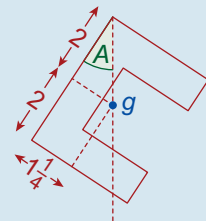
Answer: $\left(1\frac{1}{4}, 2\right)$

$$\tan A = \frac{1\frac{1}{4}}{2}$$

$$= \frac{5}{8}$$

$$= 0.625$$

$\therefore A = 32^\circ$



$$\left. \begin{array}{l} 4 \text{ at } \left(\frac{1}{2}, 2\right) \\ 1 \text{ at } \left(1\frac{1}{2}, 2\frac{1}{2}\right) \\ 4 \text{ at } \left(2\frac{1}{2}, 2\right) \end{array} \right\} = 9 \text{ at } (x, y)$$

$$4\left(\frac{1}{2}\right) + 1\left(1\frac{1}{2}\right) + 4\left(2\frac{1}{2}\right) = 9(x)$$

$$\Rightarrow x = \frac{3}{2}$$

$$4(2) + 1\left(2\frac{1}{2}\right) + 4(2) = 9(y)$$

$$\Rightarrow y = \frac{37}{18}$$

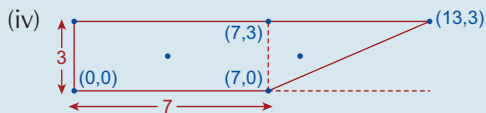
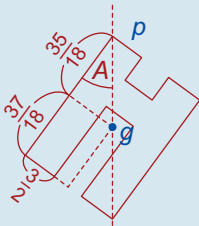
Answer: $\left(\frac{3}{2}, \frac{37}{18}\right)$

$$\tan A = \frac{\frac{3}{2}}{\frac{37}{18}}$$

$$= \frac{27}{35}$$

$$= 0.7714$$

$$\therefore A = 37^\circ 39'$$



Triangle piece:

Centroid is at $\left(\frac{7 + 7 + 13}{3}, \frac{3 + 3 + 0}{3}\right) = (9, 2)$

Area is $\frac{1}{2}(6)(3) = 9$ square units

$$21 \text{ at } \left(3\frac{1}{2}, 1\frac{1}{2}\right) \Bigg\} = 30 \text{ at } (x, y)$$

$$9 \text{ at } (9, 2)$$

$$21\left(3\frac{1}{2}\right) + 9(9) = 30(x)$$

$$\Rightarrow x = 5.15$$

$$21\left(1\frac{1}{2}\right) + 9(2) = 30(y)$$

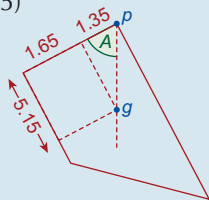
$$\Rightarrow y = 1.65$$

Answer: (5.15, 1.65)

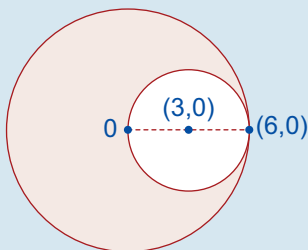
$$\tan A = \frac{5.15}{1.35}$$

$$= 3.8148$$

$$A = 75^\circ 19'$$



Q. 2.



Larger Circle: Area = 36π , Centre = (0, 0)

Smaller Circle: Area = 9π , Centre = (3, 0)

Remainder: Area = 27π ,
Centre of gravity is at (x, y)

$$9\pi \text{ at } (3, 0) \Bigg\} = 36\pi \text{ at } (0, 0)$$

$$27\pi \text{ at } (x, y)$$

$$\therefore 9\pi(3) + 27\pi(x) = 36\pi(0) \Rightarrow x = -1$$

$$9\pi(0) + 27\pi(y) = 36\pi(0) \Rightarrow y = 0$$

The distance (0, 0) to (-1, 0)

Answer: 1 cm

Q. 3. Full Square: Area = $4 \times 4 = 16 \text{ m}^2$

Small Square: Area = $1 \times 1 = 1 \text{ m}^2$

Remainder: Area = $16 - 1 = 15 \text{ m}^2$

$$1 \text{ at } (3.5, 3.5) \Bigg\} = 16 \text{ at } (2, 2)$$

$$15 \text{ at } (x, y)$$

$$1(3.5) + 15(x) = 16(2)$$

$$\Rightarrow 3.5 + 15x = 32$$

$$\Rightarrow 15x = 28.5$$

$$\Rightarrow x = 1.9$$

$$1(3.5) + 15(y) = 16(2)$$

$$\Rightarrow y = 1.9$$

\Rightarrow Centre of gravity of the remainder is at (1.9, 1.9)

Q. 4. (i) $\left(\frac{1 + 3 + 5}{3}, \frac{1 + 5 + 0}{3}\right) = (3, 2)$

(ii) $\left(\frac{1 + 9 + 11}{3}, \frac{2 + 6 + 1}{3}\right) = (7, 3)$

(iii) **Square:** Centre of gravity is at (1.5, 1.5)

Area of square = $3 \times 3 = 9$

Triangle: Centre of gravity is at $\left(\frac{3 + 3 + 9}{3}, \frac{0 + 3 + 0}{3}\right) = (5, 1)$

Area of triangle = $\frac{1}{2}(6)(3) = 9$

$$9 \text{ at } (1.5, 1.5) \Bigg\} = 18 \text{ at } (x, y)$$

$$9 \text{ at } (5, 1)$$

$$9(1.5) + 9(5) = 18(x)$$

$$\Rightarrow 18x = 58.5$$

$$\Rightarrow x = 3.25$$

$$9(1.5) + 9(1) = 18(y)$$

$$\Rightarrow 18y = 22.5$$

$$\Rightarrow y = 1.25$$

\Rightarrow Centre of gravity of lamina is at (3.25, 1.25)

(iv) **Rectangle:** Centre of gravity is at (8, 2)

$$\text{Area of rectangle} = 16 \times 4 = 64$$

Triangle: Centre of gravity is at (4, 5)

$$\text{Area of triangle} = \frac{1}{2}(16)(3) = 24$$

$$\left. \begin{array}{l} 64 \text{ at } (8, 2) \\ 24 \text{ at } (4, 5) \end{array} \right\} = 88 \text{ at } (x, y)$$

$$64(8) + 24(8) = 88(x)$$

$$\Rightarrow x = 8$$

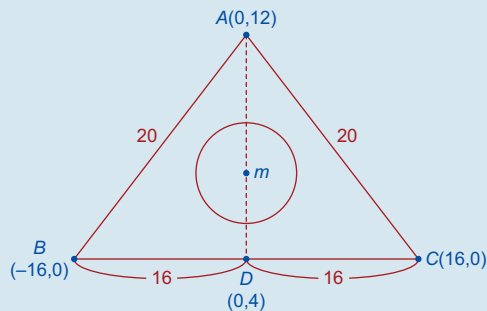
$$64(2) + 24(5) = 88(y)$$

$$\Rightarrow y = \frac{31}{11}$$

\Rightarrow Centre of gravity of lamina is at

$$\left(8, \frac{31}{11} \right)$$

Q. 5.



$$|AB|^2 = |BD|^2 + |AD|^2$$

$$\Rightarrow 20^2 = 16^2 + |AD|^2$$

$$\Rightarrow |AD| = 12$$

$$\text{Triangle: Area} = \frac{1}{2}(32)(12)$$

$$= 192$$

Taking D as the origin, the centre of gravity is at the centroid of $A(0, 12)$, $B(-16, 0)$ and $C(16, 0)$ which is at

$$\left(\frac{0 - 16 + 16}{3}, \frac{12 + 0 + 0}{3} \right) = (0, 4)$$

$$\text{Circle: Area} = \pi r^2$$

$$= \frac{22}{7} \times \frac{49}{4}$$

$$= \frac{77}{2}$$

$$= 38\frac{1}{2}$$

Centre of gravity is at $m(0, 6)$

$$\text{The remainder: Area} = 192 - 38\frac{1}{2}$$

$$= 153\frac{1}{2}$$

Centre of gravity is at (x, y)

$$\left. \begin{array}{l} 153\frac{1}{2} \text{ at } (x, y) \\ 38\frac{1}{2} \text{ at } (0, 6) \end{array} \right\} = 192 \text{ at } (0, 4)$$

$$153\frac{1}{2}(x) + 38\frac{1}{2}(0) = 192(0)$$

$$\Rightarrow x = 0$$

$$153\frac{1}{2}(y) + 38\frac{1}{2}(6) = 192(4)$$

$$\Rightarrow y = \frac{1,074}{307}$$

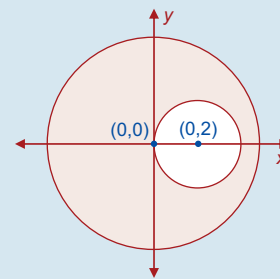
$$= 3.5 \text{ cm}$$

Q. 6. Area of full disc = $\pi(5^2) = 25\pi$

$$\text{Area of missing piece} = \pi(2^2) = 4\pi$$

$$\Rightarrow \text{Area of remainder} = 25\pi - 4\pi$$

$$= 21\pi$$



$$\left. \begin{array}{l} 21\pi \text{ at } (x, y) \\ 4\pi \text{ at } (2, 0) \end{array} \right\} = 25\pi \text{ at } (0, 0)$$

Taking moments around the y-axis:

$$21\pi(x) + 4\pi(2) = 25\pi(0)$$

$$\Rightarrow 21x + 8 = 0$$

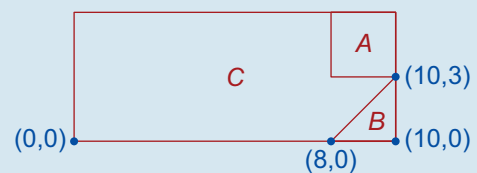
$$\Rightarrow 21x = -8$$

$$\Rightarrow x = -\frac{8}{21}$$

\Rightarrow Centre of gravity of the remainder is

$$\frac{8}{21} \text{ m} = 38 \text{ cm from } O.$$

Q. 7.



A: Area is $3 \times 2 = 6$

Centre of gravity is at $\left(9, 4\frac{1}{2} \right)$

B: Area = $\frac{1}{2}(2)(3) = 3$

Centre of gravity is at

$$\left(\frac{8 + 10 + 10}{3}, \frac{0 + 0 + 3}{3}\right) = \left(\frac{28}{3}, 1\right)$$

Rectangle: Area = $10 \times 6 = 60$

Centre of gravity is at (5, 3)

Remainder C: Area = $60 - 6 - 3 = 51$.

Centre of gravity is at (x, y)

$$\left. \begin{array}{l} 6 \text{ at } \left(9, 4\frac{1}{2}\right) \\ 3 \text{ at } \left(\frac{28}{3}, 1\right) \\ 51 \text{ at } (x, y) \end{array} \right\} = 60 \text{ at } (5, 3)$$

$$6(9) + 3\left(\frac{28}{3}\right) + 51(x) = 60(5)$$

$$\Rightarrow x = \frac{218}{51}$$

$$6\left(4\frac{1}{2}\right) + 3(1) + 51(y) = 60(3)$$

$$\Rightarrow y = \frac{150}{51}$$

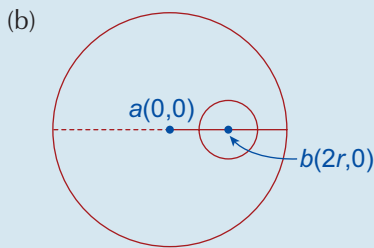
Answer: $\left(\frac{218}{51}, \frac{150}{51}\right) = (4.27, 2.94)$

Q. 8. (a) $4(2) + 5(x) + 1(5) + 3(1) = 13(2)$

$$\Rightarrow x = 2$$

$$4(3) + 5(4) + 1(y) + 3(7) = 13(4)$$

$$\Rightarrow y = -1$$



Larger circle: Area = πR^2

$$= \pi(4r)^2$$

$$= 16\pi r^2$$

Centre of gravity is at (0, 0)

Smaller Circle: Area = πr^2

Centre of gravity is at $b(2r, 0)$

Remainder: Area = $16\pi r^2 - \pi r^2$

$$= 15\pi r^2$$

Centre of gravity is at (x, y)

$$\left. \begin{array}{l} 15\pi r^2 \text{ at } (x, y) \\ \pi r^2 \text{ at } (2r, 0) \end{array} \right\} = 16\pi r^2 \text{ at } (0, 0)$$

$$15\pi r^2(x) + \pi r^2(2r) = 16\pi r^2(0)$$

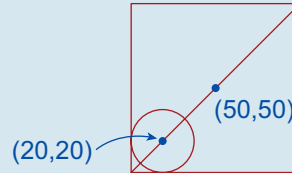
$$\Rightarrow x = -\frac{2r}{15}$$

$$15\pi r^2(y) + \pi r^2(0) = 16\pi r^2(0)$$

$$\Rightarrow y = 0$$

Answer: $\left(-\frac{2r}{15}, 0\right)$

Q. 9.



Circle: Area = πr^2

$$= \frac{22}{7} \times \frac{400}{1}$$

$$= 1,257$$

Centre of gravity is at (20, 20)

Square: Area = 100×100

$$= 10,000$$

Centre of gravity is at (50, 50)

Remainder: Area = $10,000 - 1,257$

$$= 8,743$$

Centre of gravity is at (x, y)

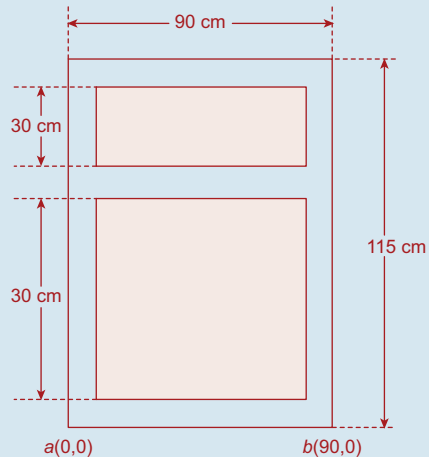
$$\left. \begin{array}{l} 1,257 \text{ at } (20, 20) \\ 8,743 \text{ at } (x, y) \end{array} \right\} = 10,000 \text{ at } (50, 50)$$

$$1,257(20) + 8,743(x) = 10,000(50)$$

$$\Rightarrow x = 54.3$$

Answer: 54 mm

Q. 10.



(i) Distance = $\frac{115}{2}$

$$= 57.5 \text{ cm}$$

(ii) **Whole rectangle:**

$$\begin{aligned} \text{Area} &= 90 \times 115 \\ &= 10,350 \end{aligned}$$

Centre of gravity is at (45, 75.5)

Larger Section:

$$\begin{aligned} \text{Area} &= 80 \times 70 \\ &= 5,600 \end{aligned}$$

Centre of gravity is at (45, 40)

Smaller Section:

$$\begin{aligned} \text{Area} &= 80 \times 30 \\ &= 2,400 \end{aligned}$$

Centre of gravity is at (45, 95)

Remainder:

$$\begin{aligned} \text{Area} &= 10,350 - 5,600 - 2,400 \\ &= 2,350 \end{aligned}$$

Centre of gravity is at (x, y)

$$\left. \begin{array}{l} 5,600 \text{ at } (45, 40) \\ 2,400 \text{ at } (45, 95) \\ 2,350 \text{ at } (x, y) \end{array} \right\} = 10,350 \text{ at } (45, 57.5)$$

Taking moments about the x-axis:

$$\begin{aligned} 5,600(40) + 2,400(95) + 2,350(y) \\ = 10,350(57.5) \\ \Rightarrow y = 60.9 \text{ cm} \end{aligned}$$

Q. 11. (i) Area $\triangle OPQ = \frac{1}{2}(\text{base})(\text{height})$
... take [OP] as the base

$$\begin{aligned} \Rightarrow \text{Area } \triangle OPQ &= \frac{1}{2}(12)(18) \\ &= 108 \text{ square units} \end{aligned}$$

Area $\triangle OQR = \frac{1}{2}(\text{base})(\text{height})$
... take [OR] as the base

$$\begin{aligned} \Rightarrow \text{Area } \triangle OQR &= \frac{1}{2}(30)(18) \\ &= 270 \text{ square units} \end{aligned}$$

(ii) Centre of Gravity of $\triangle OPQ$

$$\begin{aligned} &= \left(\frac{0 + 0 + 18}{3}, \frac{0 + 12 + 18}{3} \right) \\ &= (6, 10) \end{aligned}$$

Centre of Gravity of $\triangle OQR$

$$\begin{aligned} &= \left(\frac{0 + 18 + 30}{3}, \frac{0 + 18 + 0}{3} \right) \\ &= (16, 6) \end{aligned}$$

$$\left. \begin{array}{l} \text{(iii) } 108 \text{ at } (6, 10) \\ 270 \text{ at } (16, 6) \end{array} \right\} = 378 \text{ at } (x, y)$$

Taking moments around the y-axis:

$$\begin{aligned} (108)(6) + (270)(16) &= (378)(x) \\ \Rightarrow 378x &= 4,968 \\ \Rightarrow x &= 13.14 \end{aligned}$$

Taking moments around the x-axis:

$$\begin{aligned} (108)(10) + (270)(6) &= (378)(y) \\ \Rightarrow 378y &= 2,700 \\ \Rightarrow y &= 7.14 \end{aligned}$$

\Rightarrow Centre of Gravity of lamina is at (13.14, 7.41)

Q. 12. (i) Area $\triangle OPQ = \frac{1}{2} |x_1y_2 - x_2y_1|$
 $= \frac{1}{2} |(18)(6) - (-6)(18)|$
 $= 108$

Area $\triangle OQR = \frac{1}{2}(\text{base})(\text{height})$
... take [OR] as the base

$$\begin{aligned} \Rightarrow \text{Area } \triangle OQR &= \frac{1}{2}(36)(18) \\ &= 324 \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{Area } \triangle OPQ : \text{Area } \triangle OQR &= 108 : 324 \\ &= 1 : 3 \end{aligned}$$

(ii) Centre of Gravity of $\triangle OPQ$

$$\begin{aligned} &= \left(\frac{0 - 6 + 18}{3}, \frac{0 + 6 + 18}{3} \right) \\ &= (4, 8) \end{aligned}$$

Centre of Gravity of $\triangle OQR$

$$\begin{aligned} &= \left(\frac{0 + 18 + 36}{3}, \frac{0 + 18 + 0}{3} \right) \\ &= (18, 6) \end{aligned}$$

$$\left. \begin{array}{l} \text{(iii) } 1 \text{ at } (4, 8) \\ 3 \text{ at } (18, 6) \end{array} \right\} = 4 \text{ at } (x, y)$$

Taking moments around the y-axis:

$$\begin{aligned} (1)(4) + (3)(18) &= (4)(x) \\ \Rightarrow 4x &= 58 \\ \Rightarrow x &= 14.5 \end{aligned}$$

Taking moments around the x-axis:

$$\begin{aligned} (1)(8) + (3)(6) &= (4)(y) \\ 4y &= 26 \\ \Rightarrow y &= 6.5 \end{aligned}$$

\Rightarrow Centre of Gravity of lamina is at (14.5, 6.5)

Exercise 8D

- Q. 1.** (i) The centre of gravity of the cylinder is at a height $\frac{1}{2}h = \frac{1}{2}(24) = 12$ cm above the table.
- (ii) The centre of gravity of the cone is at a height $\frac{1}{4}h = \frac{1}{4}(24) = 6$ cm above the table.
- (iii) The centre of gravity of the solid hemisphere is at a height $\frac{3}{8}r = \frac{3}{8}(24) = 9$ cm above the table.
- (iv) The centre of gravity of the hemispherical shell is at a height $\frac{1}{2}r = \frac{1}{2}(24) = 12$ cm above the table.

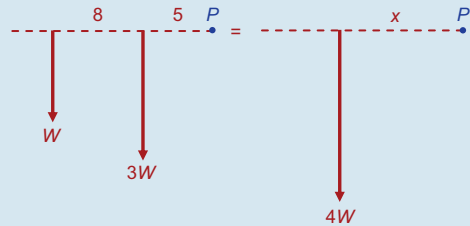
- Q. 2.** The centre of gravity of the solid hemisphere is at a height $\frac{3}{8}r = \frac{3}{8}(16) = 6$ cm above the table. The centre of gravity of the hemispherical shell is at a height $\frac{1}{2}r = \frac{1}{2}(16) = 8$ cm above the table. The difference in the heights of their centres of gravity above the table is $8 - 6 = 2$ cm.

- Q. 3.** (i) $\frac{1}{4}h = 5$
 $\Rightarrow h = 20$ cm
- (ii) $V = \frac{1}{3}\pi r^2 h$
 $= \frac{1}{3}\pi(3^2)(20)$
 $= 60\pi$ cm³

- Q. 4.** (i) $\frac{3}{8}r = 3$
 $\Rightarrow r = 8$ cm
- (ii) $V = \frac{2}{3}\pi r^3$
 $= \frac{2}{3}\pi(8^3)$
 $= \frac{1,024\pi}{3}$ cm³

- Q. 5.** (i) $\frac{1}{2}r = 3$
 $\Rightarrow r = 6$ cm
- (ii) Curved Surface Area = $2\pi r^2$
 $= 2\pi(6^2)$
 $= 72\pi$ cm²

- Q. 6.** W acts through a point $\frac{3}{8}r = 3$ cm from the base of the cylinder.
 $3W$ acts through a point $\frac{1}{2}h = 5$ cm from the base of the hemisphere.
 Let P be the point at the right extreme end of the central axis.
 Here, then, is the diagram of the forces.
 The total weight of the compound body is $4W$ which acts through a point which is x cm from P .



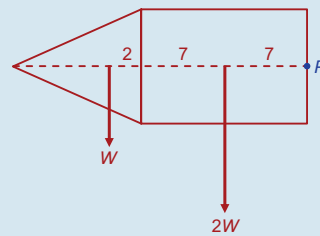
By the Principle of Moments:

$$W(13) + 3W(5) = 4W(x) \quad \dots \text{divide by } W$$

$$\Rightarrow 28 = 4x$$

$$\Rightarrow x = 7 \text{ cm from } P$$

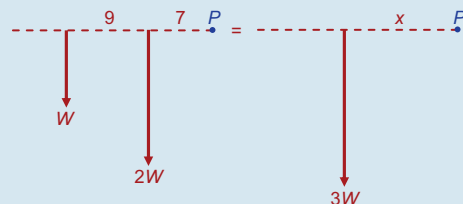
- Q. 7.**



$2W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(14) = 7$ cm from P .

W acts through a point $\frac{1}{4}(8) = 2$ cm from the base of the cone, i.e. 16 cm from P .

Here, then, is the diagram of the forces.
 The total weight of the compound body is $3W$, which acts through a point which is x cm from P .



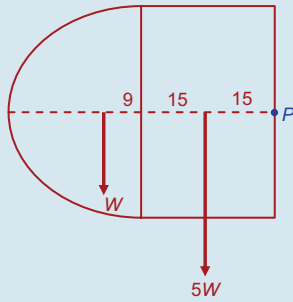
By the Principle of Moments,

$$W(16) + 2W(7) = 3W(x) \quad \dots \text{divide by } W$$

$$\Rightarrow 30 = 3x$$

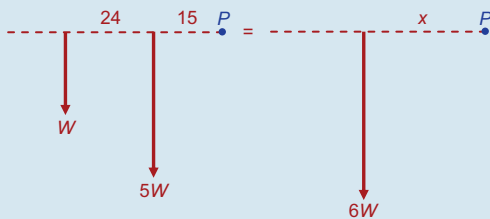
$$\Rightarrow x = 10 \text{ cm}$$

Q. 8.



$5W$ acts through a point
 $\frac{1}{2}h = \frac{1}{2}(30) = 15$ mm from P .
 W acts through a point $\frac{3}{8}(24) = 9$ mm
 from the base of the hemisphere,
 i.e. 39 mm from P .

Here, then, is the diagram of the forces.
 The total weight of the compound body is $6W$, which acts through a point which is x mm from P .



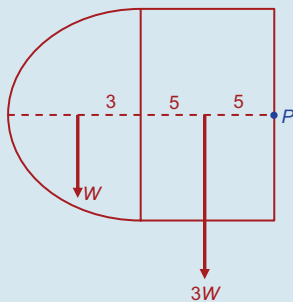
By the Principle of Moments:

$$W(39) + 5W(15) = 6W(x) \quad \dots \text{divide by } W$$

$$\Rightarrow 114 = 6x$$

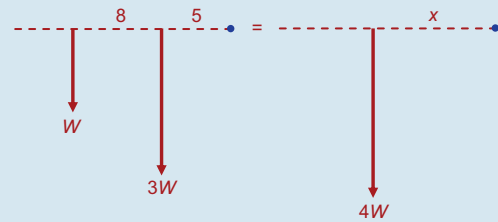
$$\Rightarrow x = 19 \text{ mm}$$

Q. 9.



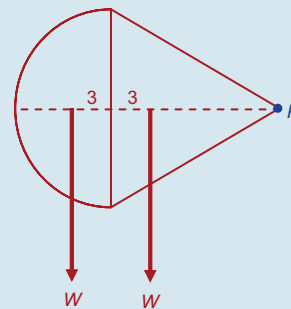
$3W$ acts through a point
 $\frac{1}{2}h = \frac{1}{2}(10) = 5$ mm from P .
 W acts through a point $\frac{1}{2}(6) = 3$ mm from
 the base of the hemisphere, i.e. 13 mm
 from P .

Here, then, is the diagram of the forces.
 The total weight of the compound body is $4W$, which acts through a point which is x mm from P .

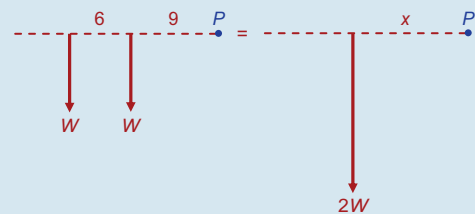


By the Principle of Moments,
 $W(13) + 3W(5) = 4W(x) \quad \dots \text{divide by } W$
 $\Rightarrow 28 = 4x$
 $\Rightarrow x = 7$ mm

Q. 10.



W acts through a point $\frac{1}{4}(12) = 3$ cm
 from the base of the hemisphere.
 W acts through a point $\frac{3}{8}(8) = 3$ cm from
 the base of the cone, i.e. 15 cm from P .
 Here, then, is the diagram of the forces.
 The total weight of the compound body
 is $2W$ which acts through a point which is
 x cm from P .

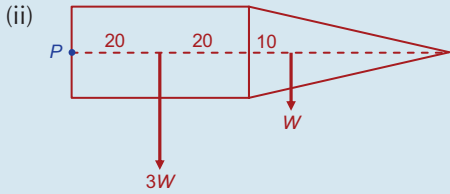


By the Principle of Moments,
 $W(15) + W(9) = 2W(x) \quad \dots \text{divide by } W$
 $\Rightarrow 24 = 2x$
 $\Rightarrow x = 12$ cm

i.e. the centre of the compound body is
 12 cm from P . This is at the plane where
 the bases of the two solids meet.

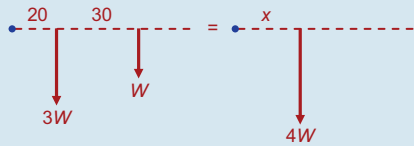
Q. 11. (i) $V_{\text{cylinder}} = \pi r^2 h$
 $= \pi(12^2)(40)$
 $= 5,760\pi \text{ cm}^3$

$$\begin{aligned}
 v_{\text{cone}} &= \frac{1}{3}\pi r^2 h \\
 &= \frac{1}{3}\pi(12^2)(40) \\
 &= \frac{1}{3}(5,760\pi) \text{ cm}^3 \\
 \Rightarrow v_{\text{cylinder}} &= 3(v_{\text{cone}}) \\
 \Rightarrow \text{weight of cylinder} &= 3(\text{weight of cone})
 \end{aligned}$$



Let W be the weight of the cone. Therefore $3W$ is the weight of the cylinder. W acts through a point $\frac{1}{4}h = \frac{1}{4}(40) = 10$ cm from the base of the cylinder. $3W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(40) = 20$ cm from the base of the cone.

Here, then, is the diagram of the forces. The total weight of the compound body is $4W$, which acts through a point which is x cm from P .

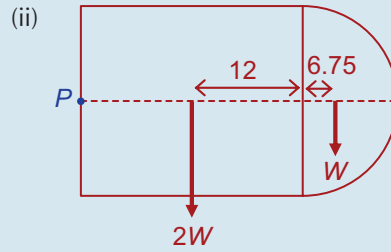


By the Principle of Moments:

$$\begin{aligned}
 3W(20) + W(50) &= 4W(x) \\
 &\dots \text{ divide by } W \\
 \Rightarrow 110 &= 4x \\
 \Rightarrow x &= 27.5 \text{ cm}
 \end{aligned}$$

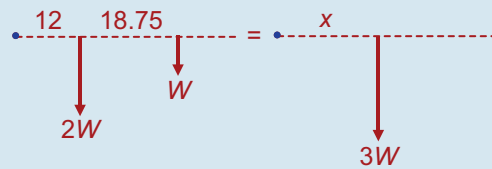
Q. 12. (i) $v_{\text{cylinder}} = \pi r^2 h = \pi(18^2)(24) = 7,776\pi \text{ cm}^3$

$$\begin{aligned}
 v_{\text{hemisphere}} &= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(18^3) \\
 &= 3,888\pi \text{ cm}^3 \\
 \Rightarrow v_{\text{cylinder}} &= 2(v_{\text{hemisphere}}) \\
 \Rightarrow \text{weight of cylinder} : \text{weight of cone} &= 2 : 1
 \end{aligned}$$



Let W be the weight of the hemisphere. Therefore $2W$ is the weight of the cylinder. W acts through a point $\frac{3}{8}r = \frac{3}{8}(18) = 6.75$ cm from the top of the cylinder. $2W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(24) = 12$ cm from the base of the hemisphere.

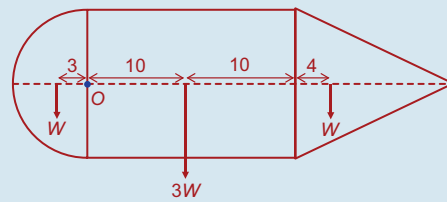
Here, then, is the diagram of the forces. The total weight of the compound body is $3W$, which acts through a point which is x cm from P .



By the Principle of Moments:

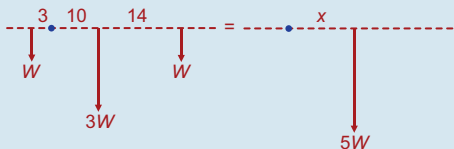
$$\begin{aligned}
 2W(12) + W(30.75) &= 3W(x) \\
 &\dots \text{ divide by } W \\
 \Rightarrow 3x &= 54.75 \\
 \Rightarrow x &= 18.25 \text{ cm}
 \end{aligned}$$

Q. 13.



Let W be the weight of the hemisphere and of the cone. Therefore $3W$ is the weight of the cylinder. The weight of the hemisphere, W , acts through a point $\frac{3}{8}r = \frac{3}{8}(8) = 3$ cm from the base of the cylinder. The weight of the cylinder, $3W$, acts through a point $\frac{1}{2}h = \frac{1}{2}(20) = 10$ cm from the base of the cylinder. The weight of the cone, W , acts through a point $\frac{1}{4}h = \frac{1}{4}(16) = 4$ cm from the top of the cylinder.

Here, then, is the diagram of the forces. The total weight of the compound body is $5W$ which acts through a point which is x cm from O .



By the Principle of Moments:

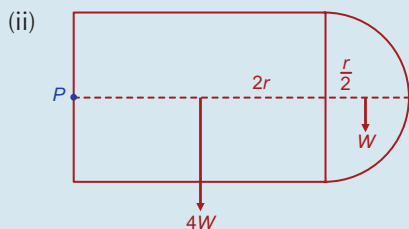
$$W(-3) + 3W(10) + W(24) = 5W(x)$$

... divide by W

$$\Rightarrow 5x = 51$$

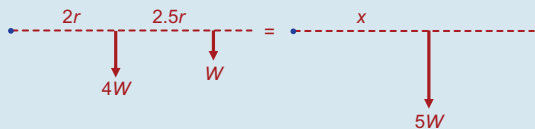
$$\Rightarrow x = 10.2 \text{ cm}$$

Q. 14. (i) $CSA_{\text{cylinder}} = 2\pi rh = 2\pi(r)(4r) = 8\pi r^2$
 $CSA_{\text{hemisphere}} = 2\pi r^2$
 $\Rightarrow CSA_{\text{cylinder}} = 4(CSA_{\text{hemisphere}})$
 $\Rightarrow W_{\text{cylinder}} = 4(W_{\text{hemisphere}})$



Let W be the weight of the hemisphere. Therefore $4W$ is the weight of the cylinder. W acts through a point $\frac{r}{2}$ from the top of the cylinder. $4W$ acts through a point $\frac{1}{2}h = \frac{1}{2}(4r) = 2r$ from the base of the hemisphere.

Here, then, is the diagram of the forces. The total weight of the compound body is $5W$ which acts through a point which is a distance of x from P .



By the Principle of Moments:

$$4W(2r) + W(4.5r) = 5W(x)$$

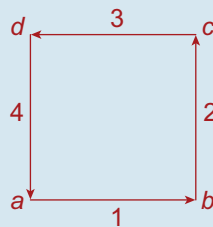
... divide by W

$$\Rightarrow 5x = 12.5r$$

$$\Rightarrow x = 2.5r.$$

Exercise 8E

Q. 1.



$$\vec{R} = \vec{i} + 2\vec{j} - 3\vec{i} - 4\vec{j}$$

$$= -2\vec{i} - 2\vec{j}$$

$$|\vec{R}| = \sqrt{(-2)^2 + (-2)^2}$$

$$= \sqrt{8} \text{ N}$$

$$\vec{db} = 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\vec{i}\right) - 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\vec{j}\right)$$

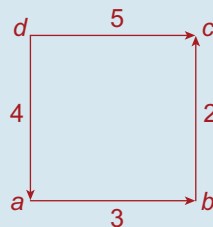
$$= 3\vec{i} - 3\vec{j}$$

The new resultant $= (-2\vec{i} - 2\vec{j}) + (3\vec{i} - 3\vec{j})$

$$= \vec{i} - 5\vec{j}$$

$$\therefore |\vec{R}| = \sqrt{1^2 + (-5)^2} = \sqrt{26} \text{ N}$$

Q. 2.



$$\vec{R} = 3\vec{i} - 2\vec{i} + 5\vec{i} - 4\vec{j}$$

$$= 8\vec{i} - 6\vec{j}$$

$$|\vec{R}| = \sqrt{8^2 + (-6)^2}$$

$$= \sqrt{100}$$

$$= 10 \text{ N}$$

Let x = the distance of its line of action from a .

The moment of the sum = the sum of the moments. (Taking moments about a)

$$-10(x) = 3(0) - 2(1) - 5(1) + 4(0)$$

$$\Rightarrow x = \frac{7}{10} \text{ m}$$

$$= 70 \text{ cm}$$

Let it intersect at a distance k from a , therefore a distance $(1 - k)$ from b .

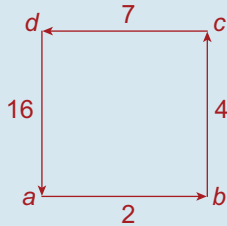
Taking moments about the point of intersection.

$$10(0) = 3(0) - 2(1-k) - 5(1) + 4(k)$$

$$\Rightarrow k = \frac{7}{6} \text{ m}$$

Answer: $1\frac{1}{6}$ m from a.

Q. 3.



$$(i) \vec{R} = 2\vec{i} + 4\vec{j} - 7\vec{i} - 16\vec{j} \\ = -5\vec{i} - 12\vec{j}$$

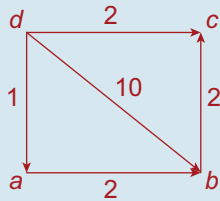
$$(ii) \therefore |\vec{R}| = \sqrt{(-5)^2 + (-12)^2} \\ = \sqrt{169} \\ = 13 \text{ N}$$

(iii) **Taking moments about a:**

$$13(x) = 2(0) + 4(1) + 7(1) + 16(0)$$

$$\Rightarrow x = \frac{11}{13} \text{ m}$$

Q. 4.



$$(i) \vec{R} = 2\vec{i} + 2\vec{j} + 2\vec{i} - \vec{j} + (8\vec{i} - 6\vec{j}) \\ = 12\vec{i} - 5\vec{j}$$

$$\therefore |\vec{R}| = \sqrt{12^2 + (-5)^2} \\ = \sqrt{169} \\ = 13 \text{ N}$$

(ii) **Taking moments about d:**

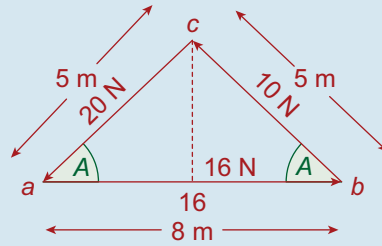
$$13(x) = 2(3) + 2(4) + 2(0) + 1(0) + 10(0)$$

$$\Rightarrow x = \frac{14}{13} \text{ m}$$

$$(iii) 13x = 2(3) + 2(4) + 2(0) + 1(0) \\ + 10(0) + 12$$

$$\Rightarrow x = 2 \text{ m}$$

Q. 5.



$$|ad|^2 + |dc|^2 = |ac|^2$$

$$\Rightarrow 4^2 + |dc|^2 = 5^2$$

$$\Rightarrow |dc| = 3 \text{ m}$$

$$\therefore \tan A = \frac{3}{4}$$

$$\therefore \sin A = \frac{3}{5}$$

$$\therefore \cos A = \frac{4}{5}$$

$$\vec{ab} = 16\vec{i}$$

$$\vec{bc} = -10 \cos A \vec{i} + 10 \sin A \vec{j} \\ = -8\vec{i} + 6\vec{j}$$

$$\vec{ca} = -20 \cos A \vec{i} - 20 \sin A \vec{j} \\ = -16\vec{i} - 12\vec{j}$$

$$\Rightarrow \vec{R} = -8\vec{i} - 6\vec{j}$$

$$\therefore |\vec{R}| = \sqrt{(-8)^2 + (-6)^2} \\ = \sqrt{100} \\ = 10 \text{ N}$$

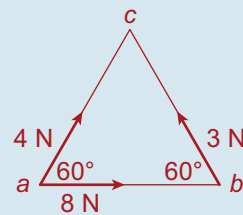
Taking moments about c:

Moment of the sum = the sum of the moments

$$10(x) = 16(3) + 10(0) + 20(0)$$

$$\Rightarrow x = 4.8 \text{ m}$$

Q. 6.



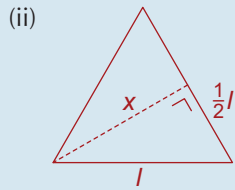
$$(i) \vec{ab} = 8\vec{i}$$

$$\vec{bc} = -8 \cos 60^\circ \vec{i} + 8 \sin 60^\circ \vec{j}$$

$$= -8\left(\frac{1}{2}\right)\vec{i} + 8\left(\frac{\sqrt{3}}{2}\right)\vec{j}$$

$$= -4\vec{i} + 4\sqrt{3}\vec{j}$$

$$\begin{aligned}\vec{ca} &= 4 \cos 60^\circ \vec{i} + 4 \sin 60^\circ \vec{j} \\ &= 2\vec{i} + 2\sqrt{3}\vec{j} \\ \Rightarrow \vec{R} &= 6\vec{i} + 6\sqrt{3}\vec{j} \\ \therefore |\vec{R}| &= \sqrt{6^2 + (6\sqrt{3})^2} \\ &= \sqrt{36 + 108} \\ &= \sqrt{144} \\ &= 12 \text{ N}\end{aligned}$$



$$\begin{aligned}x^2 + \left(\frac{1}{2}l\right)^2 &= l^2 \\ \Rightarrow x &= \frac{\sqrt{3}}{2}l\end{aligned}$$

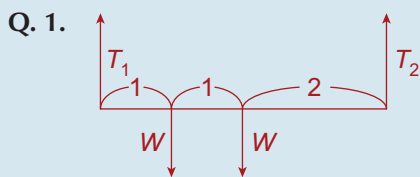
Taking moments about a:

Moment of the sum = the sum of the moments

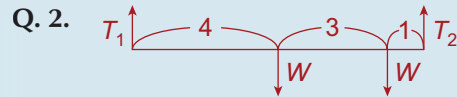
$$\begin{aligned}12(x) &= 8(0) + 8\left(\frac{\sqrt{3}}{2}l\right) + 4(0) \\ \Rightarrow 12x &= 4\sqrt{3}l \\ \Rightarrow x &= \frac{\sqrt{3}}{3}l \\ &= \frac{1}{\sqrt{3}}l \text{ m}\end{aligned}$$

The moment of the forces about a was $4\sqrt{3}l$ N m, so the moment $M = 4\sqrt{3}l$ N m.

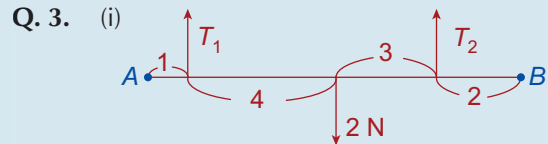
Exercise 8F



$$\begin{aligned}\textcircled{1} \quad T_1 + T_2 &= 2W \\ \textcircled{2} \quad W(1) + W(2) &= T_2(4) \\ \Rightarrow T_2 &= \frac{3}{4}W \\ \Rightarrow T_1 &= 1\frac{1}{4}W\end{aligned}$$

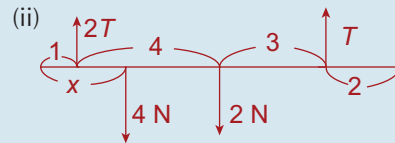


$$\begin{aligned}\textcircled{1} \quad T_1 + T_2 &= 2W \\ \textcircled{2} \quad W(4) + W(7) &= T_2(8) \\ \Rightarrow T_2 &= \frac{11}{8}W \\ \Rightarrow T_1 &= \frac{5}{8}W\end{aligned}$$

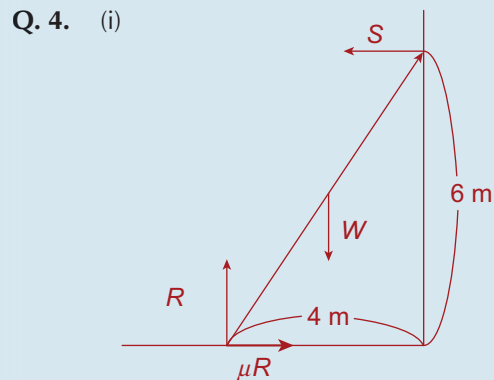


$$\begin{aligned}\textcircled{1} \quad T_1 + T_2 &= 2 \\ \textcircled{2} \quad 2(5) &= T_1(1) + T_2(8) \\ \Rightarrow T_1 + T_2 &= 10\end{aligned}$$

Solving these gives $T_1 = \frac{8}{7}$ N,
 $T_2 = \frac{6}{7}$ N



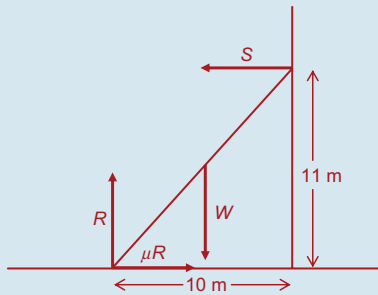
$$\begin{aligned}\textcircled{1} \quad T + 2T &= 6 \\ \Rightarrow T &= 2 \text{ N} \\ \textcircled{2} \quad 4(x) + 2(5) &= 4(1) + 2(8) \\ \Rightarrow x &= 2\frac{1}{2} \text{ cm}\end{aligned}$$



$$\begin{aligned}\textcircled{1} \quad R &= W \\ \textcircled{2} \quad \mu R &= S\end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad W(2) &= S(6) \\ \Rightarrow S &= \frac{1}{3}W \\ \text{(ii) } \textcircled{2} \quad \mu W &= \frac{1}{3}W \\ \Rightarrow \mu &= \frac{1}{3} \end{aligned}$$

Q. 5.



$$\begin{aligned} \textcircled{1} \quad R &= W \\ \textcircled{2} \quad \mu R &= S \\ \textcircled{3} \quad W(5) &= S(11) \Rightarrow S = \frac{5}{11}W \\ \textcircled{2} \quad \mu W &= \frac{5}{11}W \Rightarrow \mu = \frac{5}{11} \end{aligned}$$

Q. 6.

- (a) (i) Friction
(ii) Moment
- (b) $\textcircled{1} \quad R = 245$
 $\textcircled{2} \quad 0.8R = S$
 $\textcircled{3} \quad 245(a \cos \alpha) = S(2a \sin \alpha)$

Equation $\textcircled{2}$

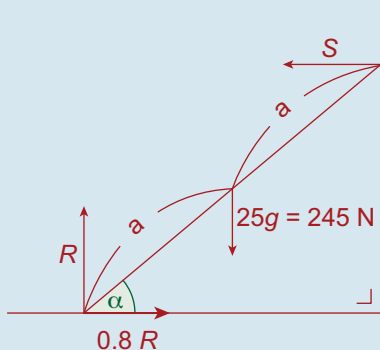
$$\Rightarrow S = 0.8(245) = 196$$

Equation $\textcircled{3}$

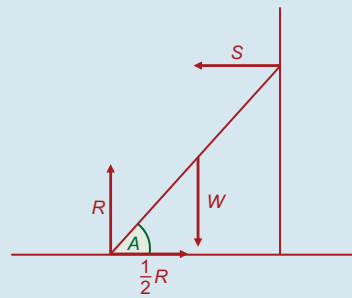
$$\Rightarrow 245 \cos \alpha = (196)2 \sin \alpha$$

$$\Rightarrow 245 = 392 \tan \alpha$$

$$\begin{aligned} \Rightarrow \tan \alpha &= \frac{245}{392} \\ &= \frac{5}{8} \end{aligned}$$



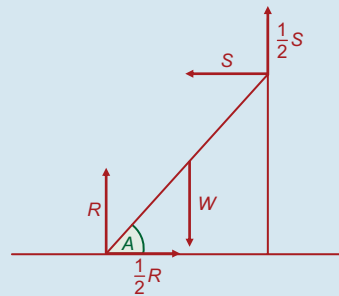
Q. 7.



Let the ladder have a length of $2l$.

$$\begin{aligned} \textcircled{1} \quad R &= W \\ \textcircled{2} \quad \frac{1}{2}R &= S \\ \textcircled{3} \quad W(l \cos A) &= S(2l \sin A) \\ \Rightarrow S &= \frac{W \cos A}{2 \sin A} = \frac{W}{2 \tan A} \\ \textcircled{2} \quad \frac{1}{2}W &= \frac{W}{2 \tan A} \Rightarrow \tan A = 1 \\ \Rightarrow A &= 45^\circ \end{aligned}$$

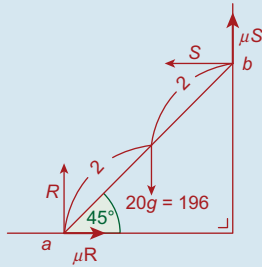
Q. 8.



Let the ladder have a length of $2l$.

$$\begin{aligned} \textcircled{1} \quad R + \frac{1}{2}S &= W \Rightarrow R = W - \frac{1}{2}S \\ \textcircled{2} \quad \frac{1}{2}R &= S \\ \textcircled{3} \quad W(l \cos A) &= S(2l \sin A) + \frac{1}{2}S(2l \cos A) \\ &\dots \text{ divide by } l \cos A \\ \Rightarrow W &= 2S \tan A + S \\ \Rightarrow S(2 \tan A + 1) &= W \\ \Rightarrow S &= \frac{W}{2 \tan A + 1} \\ \textcircled{2} \quad \frac{1}{2}\left(W - \frac{1}{2}S\right) &= S \\ \Rightarrow \frac{1}{2}W - \frac{1}{4}S &= S \Rightarrow 2W - S = 4S \\ \Rightarrow 2W &= 5S \\ \Rightarrow 2W &= \frac{5W}{2 \tan A + 1} \\ \Rightarrow 2 \tan A + 1 &= \frac{5}{2} \\ \Rightarrow \tan A &= \frac{3}{4} \Rightarrow A = 37^\circ \end{aligned}$$

Q. 9.



① $R + \mu S = 196$

② $\mu R = S$

③ **Taking moments about a:**

$$196(2 \cos 45^\circ) = S(4 \sin 45^\circ) + \mu S(4 \cos 45^\circ)$$

But $\cos 45^\circ = \sin 45^\circ = \frac{1}{\sqrt{2}}$

$$\therefore 196(2) = S(4) + \mu S(4)$$

$$\Rightarrow S + \mu S = 98$$

$$\Rightarrow S(1 + \mu) = 98$$

$$\Rightarrow S = \frac{98}{1 + \mu}$$

Now $\mu R = S$

$$\Rightarrow R = \frac{1}{\mu} S = \frac{1}{\mu} \left(\frac{98}{1 + \mu} \right) = \frac{98}{\mu(1 + \mu)}$$

Putting these into equation ①, we get

$$\frac{98}{\mu(1 + \mu)} + \mu \left(\frac{98}{1 + \mu} \right) = 196.$$

... Multiply by $\mu(1 + \mu)$

$$\Rightarrow 98 + 98\mu^2 = 196\mu(1 + \mu).$$

... Divide by 98.

$$\Rightarrow 1 + \mu^2 = 2\mu(1 + \mu)$$

$$\Rightarrow 1 + \mu^2 = 2\mu + 2\mu^2$$

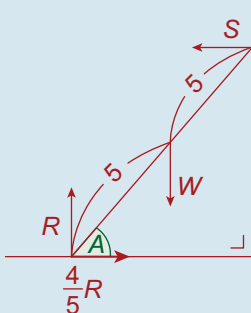
$$\Rightarrow \mu^2 + 2\mu - 1 = 0$$

$$\Rightarrow \mu = \frac{-2 \pm \sqrt{4 + 4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$= -1 \pm \sqrt{2}$$

Since $\mu > 0$, $\mu = \sqrt{2} - 1$ **Q.E.D.**

Q. 10. (i)



① $R = W$

② $\frac{4}{5}R = S$

③ **Taking moments about the foot of the ladder:**

$$W(5 \cos A) = S(10 \sin A)$$

$$\Rightarrow W \cos A = 2S \sin A$$

But $S = \frac{4}{5}R = \frac{4}{5}W$.

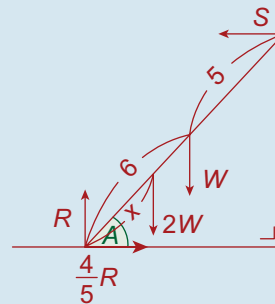
$$\therefore W \cos A = 2 \left(\frac{4}{5}W \right) \sin A$$

$$\Rightarrow \cos A = \frac{8}{5} \sin A$$

$$\Rightarrow 1 = \frac{8}{5} \tan A$$

$$\Rightarrow \tan A = \frac{5}{8}$$

(ii)



Since $\tan A = \frac{3}{4}$, $\cos A = \frac{4}{5}$, $\sin A = \frac{3}{5}$

① $R = 2W + W = 3W$

② $\frac{4}{5}R = S$

$$\Rightarrow S = \frac{4}{5}(3W) = \frac{12}{5}W$$

③ $2W(x \cos A) + W(5 \cos A)$

$$= S(10 \sin A)$$

$$\Rightarrow 2W \left(\frac{4}{5}x \right) + W(4) = S(6)$$

$$\Rightarrow \frac{8}{5}xW + 4W = 6S$$

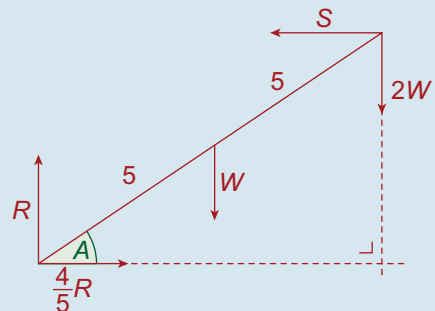
But $S = \frac{12}{5}W$

$$\therefore \frac{8}{5}xW + 4W = 6 \left(\frac{12}{5}W \right)$$

$$\Rightarrow 8x + 20 = 72$$

$$\Rightarrow x = 6.5 \text{ m}$$

(iii)

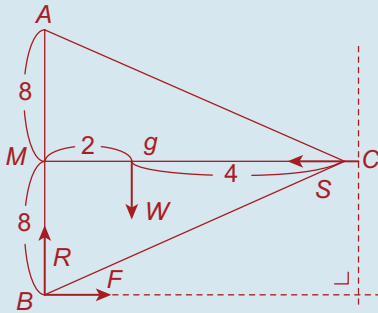


Assume it is just on the point of slipping when the man reaches the top.

- ① $R = W + 2W = 3W$
- ② $\frac{4}{5}R = S$
 $\Rightarrow S = \frac{4}{5}(3W) = \frac{12}{5}W$
- ③ $W(5 \cos A) + 2W(10 \cos A)$
 $= S(10 \sin A)$
 $\Rightarrow 5W = 2S \tan A$
 But $S = \frac{12}{5}W$,
 $\therefore 5W = \frac{24}{5}W \tan A$
 $\Rightarrow \tan A = \frac{25}{24}$

Exercise 8G

Q. 1. (i)



$|MC| = 6$ (from Pythagoras' Theorem)

The centroid, g , is 2 cm from m , 4 cm from C .

Assume it is on the point of slipping. Therefore, $F = \mu R$.

- (ii) ① $R = W$
- ② $\mu R = S$
- ③ **Taking moments about b :**

$$W(2) = S(8)$$

$$\Rightarrow S = \frac{1}{4}W$$

Equation ②

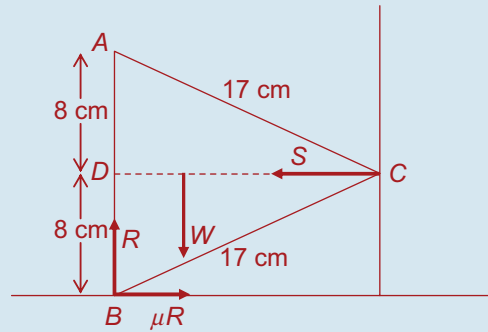
$$\Rightarrow \mu R = S$$

$$\Rightarrow \mu(W) = \frac{1}{4}W$$

$$\Rightarrow \mu = \frac{1}{4}$$

The least value of is $\frac{1}{4}$.

Q. 2.



(i) Using Pythagoras' Theorem

$$|CD|^2 + 8^2 = 17^2$$

$$\Rightarrow |CD| = \sqrt{17^2 - 8^2}$$

$$= 15 \text{ cm}$$

$[CD]$ is a median of the triangle. The centroid therefore lies $\frac{2}{3}$ of the way along $[CD]$.

$$\frac{2}{3}(15) = 10$$

\Rightarrow The centre of gravity is 10 cm from C .

(ii) ① $R = W$

$$\textcircled{2} \mu R = S$$

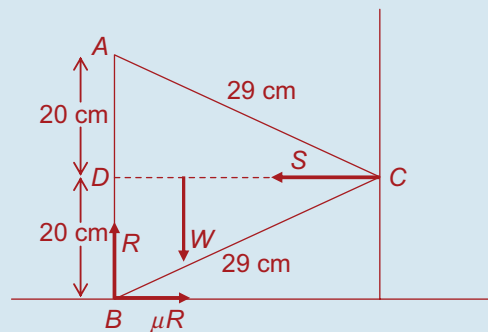
$$\textcircled{3} W(5) = S(8)$$

$$\Rightarrow S = \frac{5}{8}W$$

$$\textcircled{2} \mu W = \frac{5}{8}W$$

$$\Rightarrow \mu = \frac{5}{8}$$

Q. 3.



Using Pythagoras' Theorem

$$|CD|^2 + 20^2 = 29^2$$

$$\Rightarrow |CD| = \sqrt{29^2 - 20^2} = 21 \text{ cm}$$

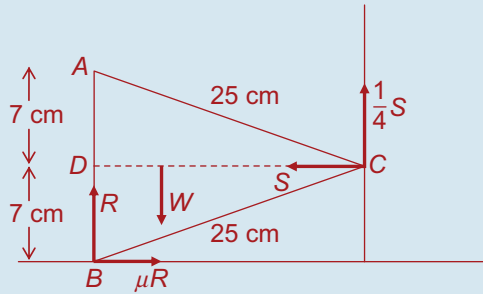
$[CD]$ is a median of the triangle. The centroid therefore lies $\frac{2}{3}$ of the way along $[CD]$.

$$\frac{2}{3}(21) = 14$$

⇒ The centre of gravity is 14 cm from C.

- ① $R = W$
- ② $\mu R = S$
- ③ $W(7) = S(20)$
 $\Rightarrow S = \frac{7}{20}W$
- ② $\mu W = \frac{7}{20}W$
 $\Rightarrow \mu = \frac{7}{20}$

Q. 4.



(i) Using Pythagoras' Theorem:

$$|CD|^2 + 7^2 = 25^2$$

$$\Rightarrow |CD| = \sqrt{25^2 - 7^2}$$

$$= 24 \text{ cm}$$

[CD] is the median of the triangle. The centroid therefore lies $\frac{2}{3}$ of the way along [CD].

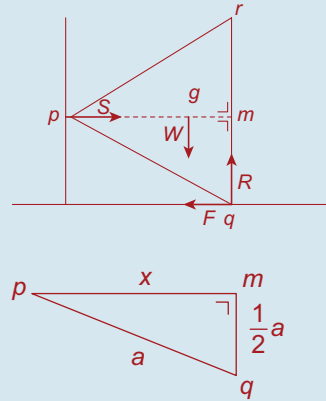
$$\frac{2}{3}(24) = 16$$

⇒ The centre of gravity is 16 cm from C.

- ① $R + \frac{1}{4}S = W$
 $\Rightarrow R = W - \frac{1}{4}S$
- ② $\mu R = S$
- ③ $W(8) = S(7) + \frac{1}{4}S(24)$
 $\Rightarrow 8W = 13S$
 $\Rightarrow S = \frac{8}{13}W$
 $\Rightarrow R = \frac{11}{13}W$
- (ii) ② $\mu\left(\frac{11}{13}W\right) = \frac{8}{13}W$
 $\Rightarrow \mu = \frac{8}{11}$

Q. 5. (a) See text

(b)



Let $|pm| = x$

$$x^2 + \left(\frac{1}{3}a\right)^2 = a^2 \Rightarrow x = \sqrt{\frac{2}{3}}a$$

$$\therefore |pg| = \frac{2}{3}\left(\sqrt{\frac{3}{2}}a\right) = \frac{1}{\sqrt{3}}a$$

$$|gm| = \frac{1}{3}\left(\frac{\sqrt{3}}{2}a\right) = \frac{1}{2\sqrt{3}}a$$

$$\textcircled{1} R = W$$

$$\textcircled{2} F = S$$

$$\textcircled{3} S\left(\frac{1}{2}a\right) = W\left(\frac{1}{2\sqrt{3}}a\right)$$

$$\Rightarrow S = \frac{1}{\sqrt{3}}W$$

The reactions are $\frac{1}{\sqrt{3}}W, W$

If it is on the point of slipping, then $F = \mu R$.

Equation 2

$$\Rightarrow \mu R = S$$

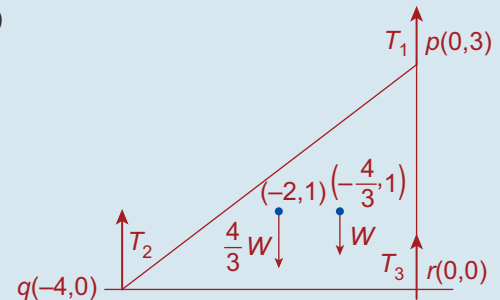
$$\Rightarrow \mu W = \frac{1}{\sqrt{3}}W$$

$$\Rightarrow \mu = \frac{1}{\sqrt{3}}$$

∴ The least value of μ is $\frac{1}{\sqrt{3}}$.

Q. 6. (a) See text

(b)



Centroid is at

$$\left(\frac{0+0+4}{3}, \frac{0+3+0}{3}\right) = \left(-\frac{4}{3}, 1\right)$$

$$\textcircled{1} T_1 + T_2 + T_3 = \frac{4}{3}W + W$$

$$\Rightarrow T_1 + T_2 + T_3 = \frac{7}{3}W$$

$\textcircled{2}$ Taking moments about the y-axis:

$$T_2(4) + T_1(0) + T_3(0) = \frac{4}{3}W(2) + W\left(\frac{4}{3}\right)$$

$$\Rightarrow 4T_2 = 4W$$

$$\Rightarrow T_2 = W$$

$\textcircled{3}$ Taking moments about the x-axis:

$$T_1(3) + T_2(0) + T_3(0) = \frac{4}{3}W(1) + W(1)$$

$$\Rightarrow T_1 = \frac{7}{9}W$$

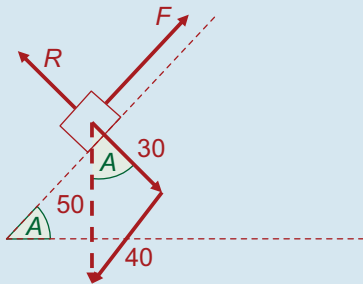
$$\text{But } T_1 + T_2 + T_3 = \frac{7}{3}W$$

$$\Rightarrow T_3 = \frac{5}{9}W$$

$$\text{Answer: } \frac{7}{9}W, W, \frac{5}{9}W$$

Exercise 8H

Q. 1.



$$\tan A = \frac{4}{3}$$

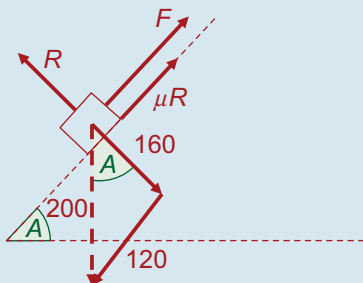
$$\Rightarrow \cos A = \frac{3}{5} \text{ and } \sin A = \frac{4}{5}$$

Component of weight acting down the

$$\text{plane} = 50 \sin A = 50\left(\frac{4}{5}\right) = 40 \text{ N}$$

$$\Rightarrow F = 40 \text{ N}$$

Q. 2.



$$\tan A = \frac{3}{4}$$

$$\Rightarrow \cos A = \frac{4}{5} \text{ and } \sin A = \frac{3}{5}$$

Component of weight acting down the
plane = $200 \sin A = 200\left(\frac{3}{5}\right) = 120 \text{ N}$

Component of weight acting perpendicular
to the plane = $200 \cos A = 200\left(\frac{4}{5}\right) = 160 \text{ N}$

$$\Rightarrow R = 160 \text{ N}$$

$$\Rightarrow \text{Friction} = \mu R$$

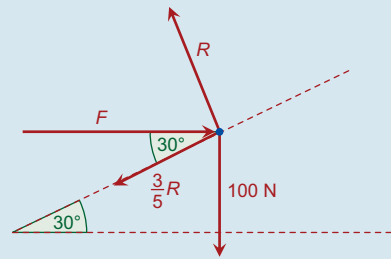
$$= \frac{1}{2}(160)$$

$$= 80 \text{ N}$$

$$\Rightarrow F + 80 = 120$$

$$\Rightarrow F = 40 \text{ N}$$

Q. 3. Here is a diagram of all the forces acting on the particle



Component of weight acting down

$$\text{the plane} = 100 \sin 30^\circ = 100\left(\frac{1}{2}\right) = 50 \text{ N}$$

Component of weight acting perpendicular

$$\text{to the plane} = 100 \cos 30^\circ = 100\left(\frac{\sqrt{3}}{2}\right) = 50\sqrt{3} \text{ N}$$

Component of F acting along

$$\text{the plane} = F \cos 30^\circ = \frac{\sqrt{3}}{2}F$$

Component of F acting perpendicular to

$$\text{the plane} = F \sin 30^\circ = \frac{1}{2}F$$

$$\Rightarrow R = 50\sqrt{3} + \frac{1}{2}F$$

$$\text{But, } \frac{3}{5}R + 50 = \frac{\sqrt{3}}{2}F$$

$$\Rightarrow \frac{3}{5}\left[50\sqrt{3} + \frac{1}{2}F\right] + 50 = \frac{\sqrt{3}}{2}F$$

$$\Rightarrow 30\sqrt{3} + \frac{3}{10}F + 50 = \frac{\sqrt{3}}{2}F$$

$$\Rightarrow 300\sqrt{3} + 3F + 500 = 5\sqrt{3}F$$

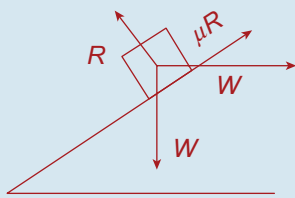
$$\Rightarrow F(5\sqrt{3} - 3) = 300\sqrt{3} + 500$$

$$\Rightarrow F = \frac{100(3\sqrt{3} + 5)}{5\sqrt{3} - 3} \approx 180 \text{ N}$$

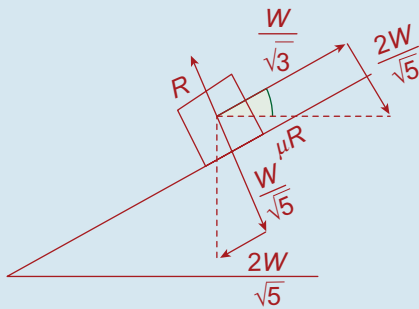
Q. 4. $\tan \alpha = 2$

$$\Rightarrow \sin \alpha = \frac{2}{\sqrt{5}} \quad \cos \alpha = \frac{1}{\sqrt{5}}$$

Forces



Resolved



$$\textcircled{1} R = \frac{2W}{\sqrt{5}} + \frac{W}{\sqrt{5}} = \frac{3W}{\sqrt{5}}$$

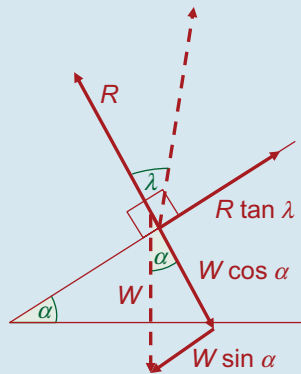
$$\textcircled{2} \mu R + \frac{W}{\sqrt{5}} = \frac{2W}{\sqrt{5}}$$

$$\Rightarrow \mu R = \frac{W}{\sqrt{5}}$$

Dividing $\textcircled{1}$ by $\textcircled{2}$

$$\Rightarrow \mu = \frac{1}{3}$$

Q. 5.



The diagram shows the forces on the particle. Since $\mu = \tan \lambda$, the friction force is $R \tan \lambda$ as shown.

Particle will slip down the plane if

$$W \sin \alpha > R \tan \lambda \quad \dots \text{ but } R = W \cos \alpha$$

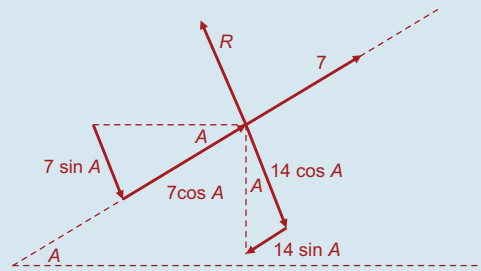
$$\Leftrightarrow W \sin \alpha > W \cos \alpha \tan \lambda$$

... divide by $W \cos \alpha$

$$\Leftrightarrow \tan \alpha > \tan \lambda$$

$$\Leftrightarrow \alpha > \lambda$$

Q. 6. Here is a diagram of the resolved forces acting on the body:



$$7 \cos A + 7 = 14 \sin A$$

$$\Rightarrow \cos A + 1 = 2 \sin A$$

$$\Rightarrow \cos A + 1 = 2\sqrt{1 - \cos^2 A}$$

... square both sides

$$\Rightarrow \cos^2 A + 2 \cos A + 1 = 4(1 - \cos^2 A)$$

$$\Rightarrow \cos^2 A + 2 \cos A + 1 = 4 - 4 \cos^2 A$$

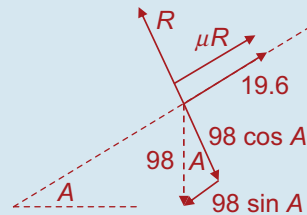
$$\Rightarrow 5 \cos^2 A + 2 \cos A - 3 = 0$$

$$\Rightarrow (5 \cos A - 3)(\cos A + 1) = 0$$

$$\Rightarrow \cos A = \frac{3}{5} \quad \text{OR} \quad \cos A = -1$$

$\cos A = -1$ is excluded because A is an acute angle.

Q. 7. (i) Particle about to slip down the plane



$$R = 98 \cos A$$

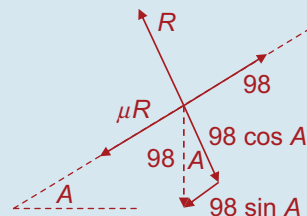
$$19.6 + \mu R = 98 \sin A$$

$$\Rightarrow 19.6 + 98 \mu \cos A = 98 \sin A$$

$$\Rightarrow 2 + 10 \mu \cos A = 10 \sin A$$

$$\Rightarrow 10 \sin A - 10 \mu \cos A = 2 \quad \dots \textcircled{1}$$

Particle about to slip up the plane



$$R = 98 \cos A$$

$$98 = \mu R + 98 \sin A$$

$$\Rightarrow 98 = 98 \mu \cos A + 98 \sin A$$

$$\Rightarrow 10 = 10 \mu \cos A + 10 \sin A$$

$$\Rightarrow 10 \sin A + 10 \mu \cos A = 10 \dots \textcircled{2}$$

Adding equations $\textcircled{1}$ and $\textcircled{2}$ we get

$$20 \sin A = 12$$

$$\Rightarrow \sin A = \frac{3}{5}$$

$$\Rightarrow A = \sin^{-1}\left(\frac{3}{5}\right) = 36.87^\circ$$

(ii) From equation $\textcircled{2}$ we get

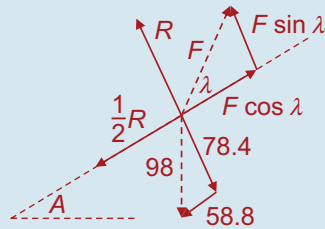
$$10\left(\frac{3}{5}\right) + 10\mu\left(\frac{4}{5}\right) = 10$$

$$\Rightarrow 6 + 8\mu = 10$$

$$\Rightarrow 8\mu = 4$$

$$\Rightarrow \mu = \frac{1}{2}$$

(iii) The force diagram therefore looks like this:



$$R + F \sin \lambda = 78.4$$

$$\Rightarrow R = 78.4 - F \sin \lambda$$

$$\Rightarrow 78.4 - F \sin \lambda = 2F \cos \lambda - 117.6$$

$$\Rightarrow F(2 \cos \lambda + \sin \lambda) = 196$$

$$\Rightarrow F = \frac{196}{2 \cos \lambda + \sin \lambda}$$

$$= 196(2 \cos \lambda + \sin \lambda)^{-1}$$

$$\Rightarrow \frac{dF}{d\lambda} = -196(2 \cos \lambda + \sin \lambda)^{-2} (-2 \sin \lambda + \cos \lambda)$$

$$\Rightarrow \frac{dF}{d\lambda} = \frac{196(\cos \lambda - 2 \sin \lambda)}{(2 \cos \lambda + \sin \lambda)^2}$$

Putting $\frac{dF}{d\lambda} = 0$ we get

$$\cos \lambda - 2 \sin \lambda = 0$$

$$\Rightarrow 1 - 2 \tan \lambda = 0$$

$$\Rightarrow \tan \lambda = \frac{1}{2}$$

$$\Rightarrow \sin \lambda = \frac{1}{\sqrt{5}} \text{ and } \cos \lambda = \frac{2}{\sqrt{5}}$$

$$F = \frac{196}{2 \cos \lambda + \sin \lambda}$$

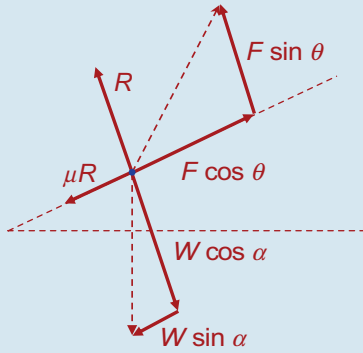
$$= \frac{196}{\frac{4}{\sqrt{5}} + \frac{1}{\sqrt{5}}}$$

$$= \frac{196}{\frac{5}{\sqrt{5}}}$$

$$= \frac{196}{\sqrt{5}} \text{ N}$$

Q. 8. $\mu = \tan \lambda$

The diagram shows resolved forces acting on the particle.



$$R + F \sin \theta = W \cos \alpha$$

$$\mu R + W \sin \alpha = F \cos \theta$$

$$\Rightarrow R = W \cos \alpha - F \sin \theta$$

$$\Rightarrow R \tan \lambda = F \cos \theta - W \sin \alpha$$

$$\Rightarrow R = \frac{F \cos \theta - W \sin \alpha}{\tan \lambda}$$

$$\Rightarrow W \cos \alpha - F \sin \theta = \frac{F \cos \theta - W \sin \alpha}{\tan \lambda}$$

$$\Rightarrow W \cos \alpha - F \sin \theta = (F \cos \theta - W \sin \alpha) \left(\frac{\cos \lambda}{\sin \lambda} \right) \dots \text{multiply by } \sin \lambda$$

$$\Rightarrow W \cos \alpha \sin \lambda - F \sin \theta \sin \lambda = F \cos \theta \cos \lambda - W \sin \alpha \cos \lambda$$

$$\Rightarrow F(\cos \theta \cos \lambda + \sin \theta \sin \lambda) = W(\cos \alpha \sin \lambda + \sin \alpha \cos \lambda)$$

$$\Rightarrow F \cos(\theta - \lambda) = W \sin(\alpha + \lambda)$$

$$\Rightarrow F = \frac{W \sin(\alpha + \lambda)}{\cos(\theta - \lambda)}$$

(i) Force acting up along the plane $\Rightarrow \theta = 0$

$$\Rightarrow F = \frac{W \sin(\alpha + \lambda)}{\cos(-\lambda)} \dots \cos(-\lambda) = \cos \lambda$$

$$\Rightarrow F = \frac{W \sin(\alpha + \lambda)}{\cos \lambda}$$

(ii) Horizontal force $\Rightarrow \theta = -\alpha$

$$\Rightarrow F = \frac{W \sin(\alpha + \lambda)}{\cos(-\alpha - \lambda)}$$

$$= \frac{W \sin(\alpha + \lambda)}{\cos[-(\alpha + \lambda)]}$$

$$= \frac{W \sin(\alpha + \lambda)}{\cos(\alpha + \lambda)}$$

$$= W \tan(\alpha + \lambda)$$

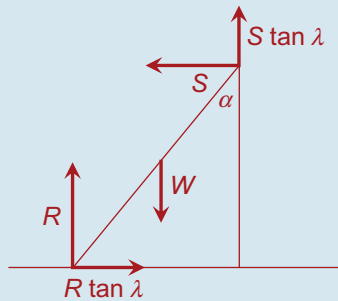
(iii) $F = \frac{W \sin(\alpha + \lambda)}{\cos(\theta - \lambda)}$

Minimum force will occur when $\cos(\theta - \lambda)$ is at its maximum value, i.e. $\cos(\theta - \lambda) = 1$

$$\Rightarrow F_{\text{MIN}} = W \sin(\alpha + \lambda).$$

- Q. 9. (i) The normal reaction, R , and the limiting friction, F , acting on a body which is either moving or on the point of moving, can be added to form a resultant. The angle between this resultant and the normal reaction is the **angle of friction**.

(ii)



Let the length of the ladder be $2l$

$$\textcircled{1} \quad R + S \tan \lambda = W$$

$$\Rightarrow R = W - S \tan \lambda$$

$$\textcircled{2} \quad R \tan \lambda = S$$

$$\textcircled{3} \quad W \sin \alpha = S(2l \cos \alpha) + S \tan \lambda(2l \sin \alpha)$$

$$\Rightarrow S(2 \cos \alpha + 2 \tan \lambda \sin \alpha) = W \sin \alpha$$

$$\Rightarrow S = \frac{W \sin \alpha}{2(\cos \alpha + \tan \lambda \sin \alpha)}$$

$$\textcircled{2} \quad (W - S \tan \lambda) \tan \lambda = S$$

$$\Rightarrow W \tan \lambda - S \tan^2 \lambda = S$$

$$\Rightarrow S(1 + \tan^2 \lambda) = W \tan \lambda$$

$$\Rightarrow S = \frac{W \tan \lambda}{1 + \tan^2 \lambda}$$

$$\Rightarrow \frac{W \sin \alpha}{2(\cos \alpha + \tan \lambda \sin \alpha)} = \frac{W \tan \lambda}{1 + \tan^2 \lambda}$$

$$\Rightarrow (1 + \tan^2 \lambda) \sin \alpha = 2 \tan \lambda (\cos \alpha + \tan \lambda \sin \alpha)$$

$$\Rightarrow \sin \alpha + \tan^2 \lambda \sin \alpha = 2 \tan \lambda \cos \alpha + 2 \tan^2 \lambda \sin \alpha$$

$$\Rightarrow \tan^2 \lambda \sin \alpha + 2 \tan \lambda \cos \alpha = \sin \alpha \quad \dots \text{ divide by } \cos \alpha$$

$$\Rightarrow \tan^2 \lambda \tan \alpha + 2 \tan \lambda = \tan \alpha$$

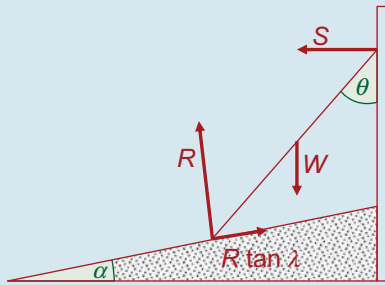
$$\Rightarrow \tan \alpha (1 - \tan^2 \lambda) = 2 \tan \lambda$$

$$\Rightarrow \tan \alpha = \frac{2 \tan \lambda}{1 - \tan^2 \lambda}$$

$$\Rightarrow \tan \alpha = \tan 2\lambda$$

$$\Rightarrow \alpha = 2\lambda$$

Q. 10.



Let the length of the ladder be $2l$.

We must firstly resolve R and $R \tan \lambda$ into horizontal and vertical components:

The horizontal component of R is $R \sin \alpha$.

The vertical component of R is $R \cos \alpha$.

The horizontal component of $R \tan \lambda$ is $R \tan \lambda \cos \alpha$.

The vertical component of $R \tan \lambda$ is $R \tan \lambda \sin \alpha$

$$\textcircled{1} \quad S + R \sin \alpha = R \tan \lambda \cos \alpha$$

$$\Rightarrow R = \frac{S}{\tan \lambda \cos \alpha - \sin \alpha}$$

$$\textcircled{2} \quad W = R \cos \alpha + R \tan \lambda \sin \alpha$$

$$\Rightarrow W = R(\tan \lambda \sin \alpha + \cos \alpha)$$

$$\textcircled{3} \quad W(l \sin \theta) = S(2l \cos \theta)$$

$$\Rightarrow S = \frac{1}{2}W \tan \theta \quad \dots \text{ as required}$$

$$\textcircled{2} \quad W = \left(\frac{S}{\tan \lambda \cos \alpha - \sin \alpha} \right) (\tan \lambda \sin \alpha + \cos \alpha)$$

$$\Rightarrow W = \frac{1}{2}W \tan \theta \left(\frac{\tan \lambda \sin \alpha + \cos \alpha}{\tan \lambda \cos \alpha - \sin \alpha} \right) \quad \dots \text{ divide top and bottom by } \cos \alpha$$

$$\Rightarrow 1 = \frac{1}{2} \tan \theta \left(\frac{\tan \lambda \tan \alpha + 1}{\tan \lambda - \tan \alpha} \right)$$

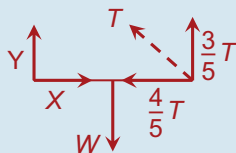
$$\Rightarrow \tan \theta (\tan \lambda \tan \alpha + 1) = 2(\tan \lambda - \tan \alpha)$$

$$\Rightarrow \tan \theta = 2 \left(\frac{\tan \lambda - \tan \alpha}{1 + \tan \lambda \tan \alpha} \right)$$

$$\Rightarrow \tan \theta = 2 \tan(\lambda - \alpha) \quad \dots \text{ as required.}$$

Exercise 8I

Q. 1. Here is a diagram of the forces acting on the rod PQ :



$$\textcircled{1} \quad Y = \frac{3}{5}T = W \quad \dots \text{ Equation 1}$$

$$\textcircled{2} \quad X + \frac{4}{5}T = W \quad \dots \text{ Equation 2}$$

$\textcircled{3}$ Take moments about P

$$W(2) = \frac{3}{5}T(4) \quad \dots \text{ Equation 2}$$

$$\text{Equation 3:} \quad 5W = 6T$$

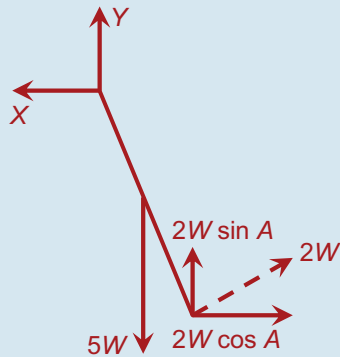
$$\Rightarrow T = \frac{5}{6}W$$

Equation 1: $Y + \frac{3}{5} \left[\frac{5}{6} W \right] = W$
 $\Rightarrow 2Y + W = 2W$
 $\Rightarrow 2Y = W$
 $\Rightarrow Y = \frac{1}{2} W$

Equation 2: $X = \frac{4}{5} \left[\frac{5}{6} W \right]$
 $\Rightarrow X = \frac{2}{3} W$

Answer: The horizontal and vertical reactions at the hinge are $\frac{2}{3}W$ and $\frac{1}{2}W$. The tension in the string is $\frac{5}{6}W$.

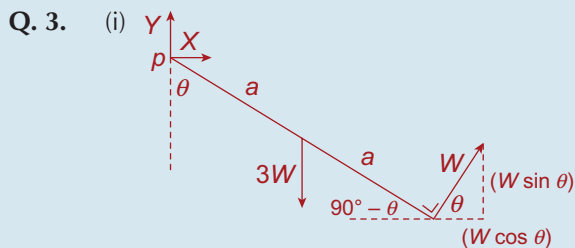
Q. 2. (i) Here is a diagram of the forces acting on the rod:



① $Y + 2W \sin A = 5W$
 ② $X = 2W \cos A$
 ③ Take moments about P
 $5W(l \sin A) = 2W(2l)$
 $\Rightarrow \sin A = \frac{4}{5} \Rightarrow \cos A = \frac{3}{5}$

① $Y + \frac{8}{5}W = 5W \Rightarrow Y = \frac{17}{5}W$
 ② $X = \frac{6}{5}W$

(ii) $A = \sin^{-1} \frac{4}{5} = 53^\circ$



(ii) ① $Y + W \sin \theta = 3W$
 ② $X + W \cos \theta = 0$

③ Taking moments about p, using unresolved forces:

$3W(a \sin \theta) = W(2a)$
 $\Rightarrow \sin \theta = \frac{2}{3}$
 $= 0.6667$

$\Rightarrow \theta = 41^\circ 49'$

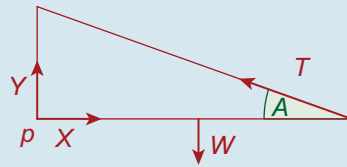
(iii) $X = -W \cos \theta$
 $= -W \left(\frac{\sqrt{5}}{3} \right)$
 $= -\frac{\sqrt{5}}{3} W$
 $Y = 3W - W \sin \theta$
 $= 3W - W \left(\frac{2}{3} \right)$
 $= \frac{7W}{3}$

Q. 4. Since $\tan A = \frac{5}{12}$

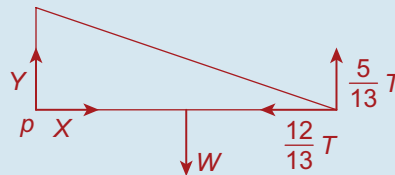
$\cos A = \frac{12}{13}$

$\sin A = \frac{5}{13}$

Forces



Resolved



① $Y + \frac{5}{13} T = W$

② $X = \frac{12}{13} T$

③ Taking moments about p :

$$W(6) = \frac{5}{13}T(12)$$

$$\Rightarrow T = \frac{13}{10}W$$

$$\therefore X = \frac{12}{13}\left(\frac{13}{10}W\right)$$

$$= \frac{6}{5}W$$

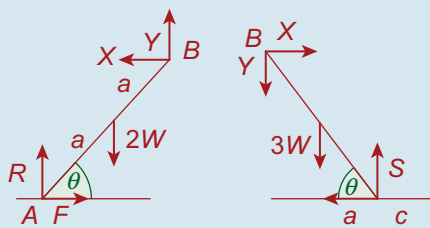
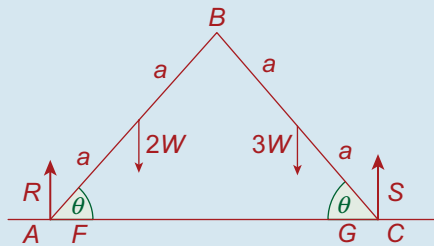
$$Y + \frac{5}{13}\left(\frac{13}{10}T\right) = W$$

$$\Rightarrow Y = \frac{1}{2}W$$

Answer: (i) $\frac{6}{5}W, \frac{1}{2}W$

(ii) $\frac{13}{10}W$

Q. 5. (i)



(ii) **From system ABC**

① $R + S = 2W + 3W$

$$\Rightarrow R + S = 5W$$

② $F = G$

③ Taking moments about a .

$$2W(a \cos \theta) + 3W(3a \cos \theta) = S(4a \cos \theta)$$

$$\Rightarrow 4S = 11W$$

$$\Rightarrow S = 2\frac{3}{4}W$$

$$\therefore R = 2\frac{1}{4}W, \text{ from Equation 1.}$$

(iii) Since $R < S, \mu R < \mu S$

\therefore slipping will occur at A first.

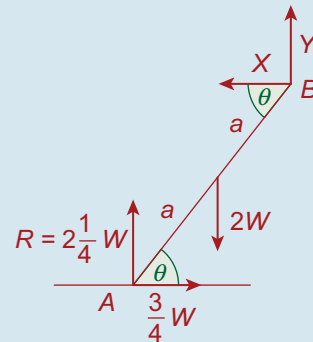
(iv) Let the rod AB be on the point of slipping.

$$\therefore F = \mu R$$

$$= \frac{1}{3}R$$

$$= \frac{1}{3}\left(2\frac{1}{4}W\right)$$

$$= \frac{3}{4}W$$



① $R + Y = 2W$

② $\frac{3}{4}W = X$

③ Taking moments about B :

$$2W(a \cos \theta) + \frac{3}{4}W(2a \sin \theta) = 2\frac{1}{4}(2a \cos \theta)$$

$$\Rightarrow 2 \cos \theta + \frac{3}{2} \sin \theta = \frac{3}{2} \cos \theta$$

$$\Rightarrow 3 \sin \theta = 5 \cos \theta$$

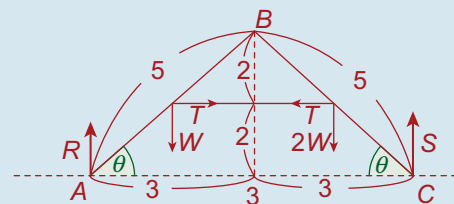
$$\Rightarrow 3 \tan \theta = 5$$

$$\Rightarrow \tan \theta = \frac{5}{3}$$

$$\Rightarrow \theta = \tan^{-1} \frac{5}{3}$$

$$= 59^\circ 2'$$

Q. 6.



Since $\cos \theta = \frac{3}{5}$, $\sin \theta = \frac{4}{5}$ and $\tan \theta = \frac{4}{3}$

The system ABC

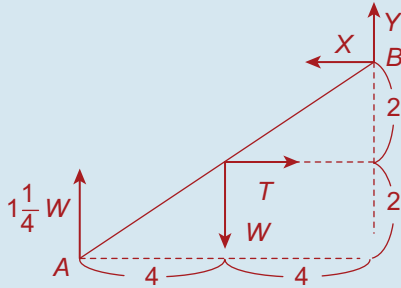
① $R + S = W + 2W$

$$\Rightarrow R + S = 3W$$

② $T = T$

③ Taking moments about A:

$$\begin{aligned} W\left(1\frac{1}{2}\right) + T(2) + 2W\left(4\frac{1}{2}\right) \\ = T(2) + S(6) \\ \Rightarrow S = 1\frac{3}{4}W \\ \therefore R = 1\frac{1}{4}W, \text{ from Equation 1} \end{aligned}$$



The rod AB

① $1\frac{1}{4}W + Y = W$
 $\Rightarrow Y = -\frac{1}{4}W$

② $T = X$

③ Taking moments about B:

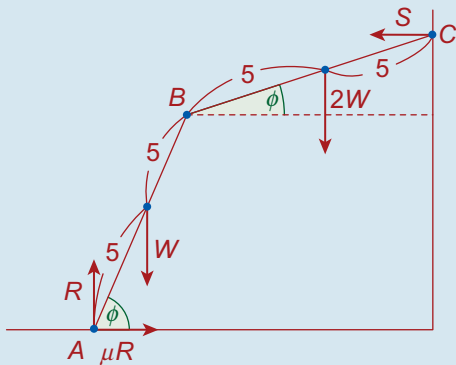
$$\begin{aligned} T(2) + W\left(1\frac{1}{2}\right) &= \left(1\frac{1}{4}\right)W(3) \\ \Rightarrow T &= 1\frac{1}{8}W \end{aligned}$$

Answer: $T = 1\frac{1}{8}W,$

$R = 1\frac{1}{4}W,$

$S = 1\frac{3}{4}W$

Q. 7.



Since $\tan \phi = \frac{4}{3},$

$\sin \phi = \frac{4}{5},$

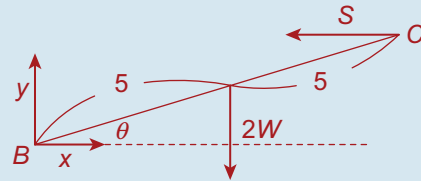
$\cos \phi = \frac{3}{5}$

① $R = W + 2W = 3W$

② $\mu R = S$

③ Taking moments about A:

$$\begin{aligned} W(5 \cos \phi) + 2W(10 \cos \phi + 5 \cos \theta) \\ = S(10 \sin \phi + 10 \sin \theta) \\ \Rightarrow 3W + 12W + 10W \cos \theta \\ = 8S + 10S \sin \theta \\ \Rightarrow 15W + 10W \cos \theta = 8S + 10S \sin \theta \end{aligned}$$



④ $Y = 2W$

⑤ $X = S$

⑥ Taking moments about B:

$$\begin{aligned} 2W(5 \cos \theta) &= S(10 \sin \theta) \\ \Rightarrow 10W \cos \theta &= 10S \sin \theta \end{aligned}$$

This means that Equation ③ reads:

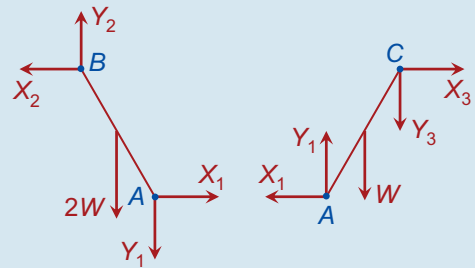
$$15W + 10S \sin \theta = 8S + 10S \sin \theta$$

$$\Rightarrow S = \frac{15}{8}W$$

Equation ② now reads:

$$\begin{aligned} \mu R &= S \\ \Rightarrow \mu(3W) &= \frac{15}{8}W \\ \Rightarrow \mu &= \frac{5}{8} \end{aligned}$$

Q. 8. (i) Let the length of each rod be $2l$. Here are the forces acting on the rods AB and AC:



Rod AB:

Taking moments around B we get:

$$\begin{aligned} 2W(l \cos \beta) + Y_1(2l \cos \beta) \\ = X_1(2l \sin \beta) \quad \dots \text{divide by } 2l \cos \beta \\ \Rightarrow W + Y_1 = X_1 \tan \beta \quad \dots \text{Equation 1} \end{aligned}$$

Rod AC:

Taking moments around C we get:

$$W(l \cos \beta) = X_1(2l \sin \beta) + Y_1(2l \cos \beta)$$

... divide by $l \cos \beta$

$$\Rightarrow W = 2X_1 \tan \beta + 2Y_1$$

$$\Rightarrow W - 2Y_1 = 2X_1 \tan \beta \quad \dots \text{Equation (2)}$$

$$\underline{2W + 2Y_1 = 2X_1 \tan \beta \quad \dots \text{Equation (1)}} \quad (\times 2)$$

Add: $3W = 4X_1, \tan \beta$

$$\Rightarrow X_1 = \frac{3W}{4 \tan \beta} \quad \dots \text{ substitute into Equation (2)}$$

$$\Rightarrow W - 2Y_1 = 2 \left[\frac{3W}{4 \tan \beta} \right] \tan \beta$$

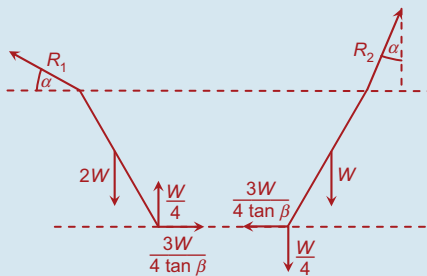
... multiply by 4

$$\Rightarrow 4W - 8Y_1 = 6W$$

$$\Rightarrow 8Y_1 = -2W$$

$$\Rightarrow Y_1 = -\frac{W}{4} \quad \dots \text{ the minus sign indicates that the actual direction of } Y_1 \text{ is opposite to the direction indicated in the diagram.}$$

(ii) Looking again at the two rods separately, we have the following:



$$\textcircled{1} R_1 \sin \alpha + \frac{W}{4} = 2W$$

$$\Rightarrow R_1 \sin \alpha = \frac{7W}{4}$$

$$\textcircled{2} R_1 \cos \alpha = \frac{3W}{4 \tan \beta}$$

Dividing Equation (1) by Equation (2) gives:

$$\tan \alpha = \left(\frac{7W}{4} \right) \left(\frac{4 \tan \beta}{3W} \right)$$

$$\Rightarrow \tan \alpha = \frac{7 \tan \beta}{3}$$

$$\textcircled{3} R_2 \sin \alpha = \frac{3W}{4 \tan \beta}$$

$$\textcircled{4} R_2 \cos \alpha = W + \frac{W}{4}$$

$$\Rightarrow R_2 \cos \alpha = \frac{5W}{4}$$

Dividing Equation (3) by Equation (4) gives:

$$\tan \alpha = \left(\frac{3W}{4 \tan \beta} \right) \left(\frac{4}{5W} \right)$$

$$\Rightarrow \tan \alpha = \frac{3}{5 \tan \beta}$$

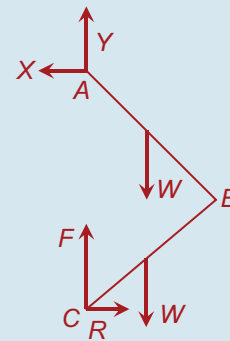
$$\Rightarrow \frac{7 \tan \beta}{3} = \frac{3}{5 \tan \beta}$$

$$\Rightarrow 35 \tan^2 \beta = 9$$

$$\Rightarrow \tan^2 \beta = \frac{9}{35}$$

$$\Rightarrow \tan \beta = \frac{3}{\sqrt{35}}$$

Q. 9. (i) Force diagram for the system including the friction force at C.



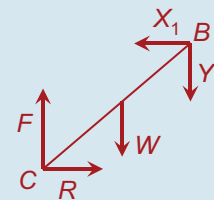
Take moments about A:

$$2W \left(\frac{l}{2} \sin \phi \right) = R(2l \cos \phi)$$

... divide by $l \cos \phi$

$$\Rightarrow W \tan \phi = 2R \Rightarrow R = \frac{1}{2} W \tan \phi$$

Now, we look at the forces on the rod BC in isolation:



Taking moments around B we get:

$$F(l \sin \phi) = R(l \cos \phi) + W \left(\frac{l}{2} \sin \phi \right)$$

... divide by $l \cos \phi$

$$\Rightarrow F \tan \phi = R + \frac{1}{2} W \tan \phi$$

... but $R = \frac{1}{2} W \tan \phi$

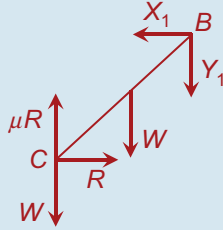
$$\Rightarrow F \tan \phi = \frac{1}{2} W \tan \phi + \frac{1}{2} W \tan \phi$$

... divide by $\tan \phi$

$$\Rightarrow F = \frac{1}{2}W + \frac{1}{2}W$$

$$\Rightarrow F = W$$

- (ii) An additional force is now applied downwards at C. The force diagram for the BC now looks like this:



Because we are now at limiting friction, the friction force is μR .

Taking moments around B we get:

$$W(l \sin \phi) + W\left(\frac{1}{2} \sin \phi\right) + R(l \cos \phi) = \mu R(l \sin \phi) \quad \dots \text{divide by } l \cos \phi$$

$$\Rightarrow W \tan \phi + \frac{1}{2}W \tan \phi + R = \mu R \tan \phi \quad \dots R = \frac{1}{2}W \tan \phi$$

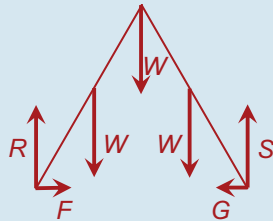
$$\Rightarrow W \tan \phi + W \tan \phi = \mu \left(\frac{1}{2}W \tan^2 \phi\right) \quad \dots \text{divide by } W \tan \phi$$

$$\Rightarrow 2 = \mu \left(\frac{1}{2} \tan \phi\right) \quad \dots \text{multiply by } 2$$

$$\Rightarrow \mu \tan \phi = 4$$

$$\Rightarrow \mu = \frac{4}{\tan \phi}$$

- Q. 10. Note:** The person will need to stand at A to maximise the chance of slipping. Because the system will then be symmetrical, slipping will occur at B and C simultaneously. Here are the forces acting on the system:



Taking moments around C we get:

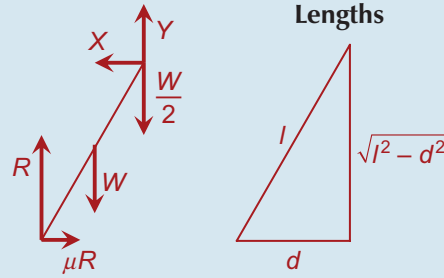
$$W\left(\frac{d}{2}\right) + W(d) + W\left(\frac{3d}{2}\right) = R(2d) \quad \dots \text{multiply by } \frac{2}{d}$$

$$\Rightarrow W + 2W + 3W = 4R$$

$$\Rightarrow 4R = 6W$$

$$\Rightarrow R = \frac{3}{2}W$$

Now, look at the ladder [AB] in isolation. Here are the forces when it is just on the point of slipping:



Taking moments around A we get:

$$R(d) = W\left(\frac{d}{2}\right) + \mu R(\sqrt{l^2 - d^2}) \quad \dots R = \frac{3}{2}W$$

$$\Rightarrow \frac{3}{2}Wd = \frac{1}{2}Wd + \mu \left(\frac{3}{2}W\right) \sqrt{l^2 - d^2} \quad \dots \text{multiply by } \frac{2}{W}$$

$$\Rightarrow 3d = d + 3\mu\sqrt{l^2 - d^2}$$

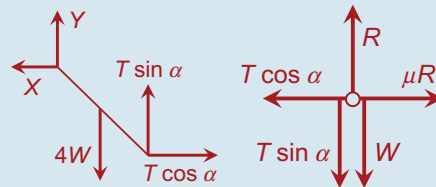
$$\Rightarrow 3\mu\sqrt{l^2 - d^2} = 2d$$

$$\Rightarrow \mu = \frac{2d}{3\sqrt{l^2 - d^2}}$$

\Rightarrow In order to avoid slipping

$$\mu \geq 2 \frac{d}{3\sqrt{l^2 - d^2}}$$

- Q. 11. (i) Rod [AB] Ring C**



Taking moments about A we get:

$$4W(l \cos \alpha) = T \cos \alpha(2l \sin \alpha) + T \sin \alpha(2l \cos \alpha)$$

\dots divide by $2l \cos \alpha$

$$\Rightarrow 2W = T \sin \alpha + T \sin \alpha$$

$$\Rightarrow 2W = 2T \sin \alpha$$

$$\Rightarrow W = T \sin \alpha$$

From the diagram of forces on the ring C we can see that

$$\mu R = T \cos \alpha \quad \dots \text{Equation (1)}$$

$$\text{and } R = T \sin \alpha + W$$

$$\dots \text{ but } W = T \sin \alpha$$

$$\Rightarrow R = 2T \sin \alpha \quad \dots \text{Equation (2)}$$

Dividing Equation (1) by Equation (2) we get:

$$\mu = \frac{1}{2 \tan \alpha}$$

$$\Rightarrow 2\mu \tan \alpha = 1$$

$$\Rightarrow \tan \alpha = \frac{1}{2\mu} \quad \dots \text{(i)}$$

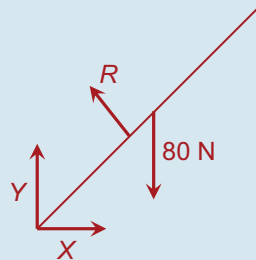
$$\text{(ii) } W = T \sin \alpha$$

$$\Rightarrow T = W / \sin \alpha \quad \dots \tan \alpha = \frac{1}{2\mu}$$

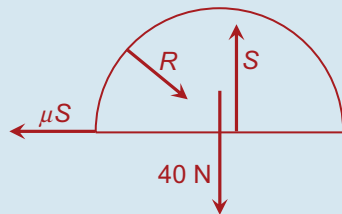
$$\Rightarrow \sin \alpha = \frac{1}{\sqrt{1 + 4\mu^2}}$$

$$\Rightarrow T = W\sqrt{1 + 4\mu^2}$$

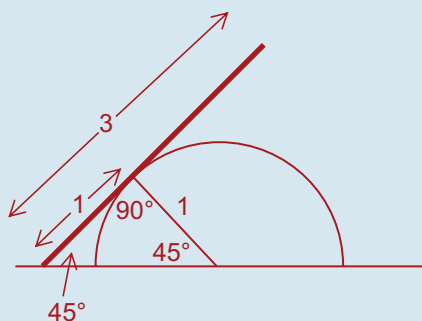
Q. 12. (i) Rod



Hemisphere



Lengths



(ii) Taking moments around the hinge we get:

$$80\left(\frac{1.5}{\sqrt{2}}\right) = R(1)$$

$$R = \frac{120}{\sqrt{2}} = 60\sqrt{2} \text{ N}$$

$$\text{(iii) } S = \frac{R}{\sqrt{2}} + 40$$

$$\mu S = \frac{R}{\sqrt{2}}$$

$$\Rightarrow \mu\left(\frac{R}{\sqrt{2}} + 40\right) = \frac{R}{\sqrt{2}}$$

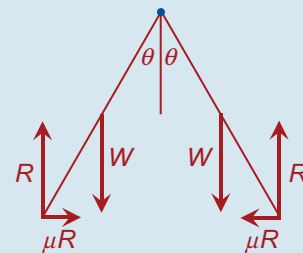
$$\dots \text{ but } R = 60\sqrt{2}$$

$$\Rightarrow \mu(60 + 40) = 60$$

$$\Rightarrow 100\mu = 60$$

$$\Rightarrow \mu = 0.6$$

Q. 13. Assume ladders are on the point of slipping. We will then be at the smallest value of μ that will prevent slipping from occurring. Here is the force diagram for the system:

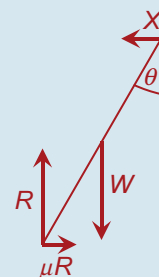


Note: Because both ladders have the same weight, both reaction forces are the same and slipping occurs simultaneously at A and C.

$$2R = 2W$$

$$\Rightarrow R = W$$

Now, we examine the forces on the ladder [AB] in isolation:



X is the supporting force from the other ladder. There is no Y component as the vectors in the Y direction already sum to zero.

Taking moments around B we get:

$$R(l \sin \theta) = \mu R(l \cos \theta) + W\left(\frac{l}{2} \sin \theta\right)$$

... multiply by $\frac{2}{l \cos \theta}$ and let $W = R$

$$\Rightarrow 2R \tan \theta = 2\mu R + R \tan \theta$$

... divide by R

$$\Rightarrow 2 \tan \theta = 2\mu + \tan \theta$$

$$\Rightarrow 2\mu = \tan \theta$$

$$\Rightarrow \mu = \frac{1}{2} \tan \theta$$

This is the minimum value of θ that will prevent slipping from occurring.

$$\Rightarrow \mu \geq \frac{1}{2} \tan \theta$$

- Q. 14.** (i) Let x equal the distance from P to where the block touches the rod.

$$\sin \alpha = \frac{0.8y}{x} \quad \dots \tan \alpha = \frac{4}{3}$$

$$\Rightarrow \sin \alpha = \frac{4}{5}$$

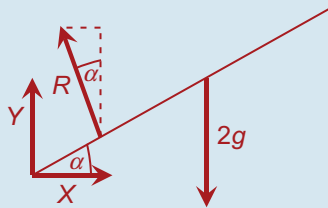
$$\Rightarrow \frac{4}{5} = \frac{0.8y}{x}$$

... multiply both sides by 5x

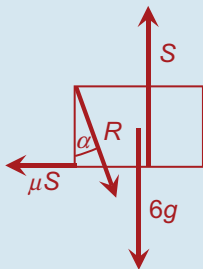
$$\Rightarrow 4x = 4y$$

$$\Rightarrow x = y$$

- (ii) **Rod**



Block



- (iii) Firstly, examine the diagram of the rod. Taking moments around the hinge we get:

$$2g(3 \cos \alpha) = R(1) \quad \dots \cos \alpha = \frac{3}{5}$$

$$\Rightarrow R = \frac{18}{5}g$$

Now, examine the diagram of the block:

$$\textcircled{1} S = R \cos \alpha + 6g$$

$$\Rightarrow S = \left(\frac{18}{5}g\right)\left(\frac{3}{5}\right) + 6g$$

$$\Rightarrow S = \frac{204}{25}g$$

$$\textcircled{2} \mu S = R \sin \alpha$$

$$\Rightarrow \mu\left(\frac{204}{25}g\right) = \left(\frac{18}{5}g\right)\left(\frac{4}{5}\right)$$

... multiply by $\frac{25}{g}$

$$\Rightarrow 204\mu = 72$$

$$\Rightarrow \mu = \frac{72}{204} = \frac{6}{17}$$

- (iv) Looking at the diagram of the rod:

$$X = R \sin \alpha$$

$$= \left(\frac{18}{5}g\right)\left(\frac{4}{5}\right)$$

$$= 28.224$$

$$Y + R \cos \alpha = 2g$$

$$\Rightarrow Y = 2g - R \cos \alpha$$

$$= 2g - \left(\frac{18}{5}g\right)\left(\frac{3}{5}\right)$$

$$= -1.568$$

... i.e. 1.568 N downwards

\Rightarrow Reaction at hinge

$$= 28.224\vec{i} - 1.568\vec{j} \text{ N}$$

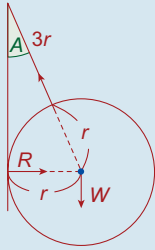
Magnitude

$$= \sqrt{(28.224)^2 + (1.568)^2}$$

$$= 28 \text{ N}$$

Exercise 8J

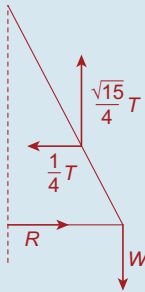
Q. 1. Forces:



$$\sin A = \frac{r}{3r + r} = \frac{1}{4}$$

$$\therefore \cos A = \frac{\sqrt{15}}{4}$$

Resolved Forces:



$$(1) \frac{\sqrt{15}}{4} T = W$$

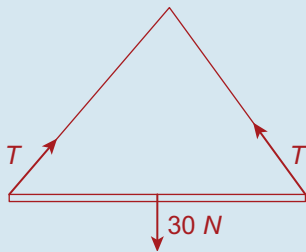
$$\Rightarrow T = \frac{4W}{\sqrt{15}}$$

$$(2) R = \frac{1}{4} T$$

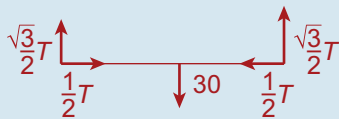
$$= \frac{1}{4} \left(\frac{4W}{\sqrt{15}} \right)$$

$$= \frac{W}{\sqrt{15}}$$

Q. 2. Forces:



Resolved Forces:

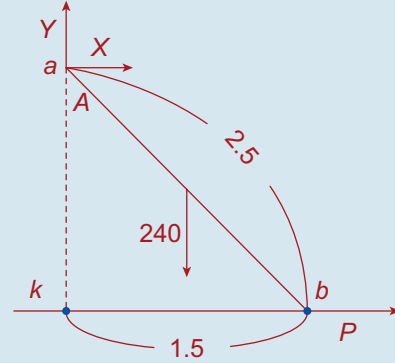


$$(1) \frac{\sqrt{3}}{2} T + \frac{\sqrt{3}}{2} T = 30$$

$$\Rightarrow T = \frac{30}{\sqrt{3}}$$

$$= 10\sqrt{3} \text{ N}$$

Q. 3. (i) "..... are concurrent"
(ii)



$$|ak|^2 + |kb|^2 = |ab|^2$$

$$\Rightarrow |ak|^2 + 2.25 = 6.25$$

$$\Rightarrow |ak| = 2$$

$$(1) Y = 240$$

$$(2) P + X = 0 \Rightarrow X = -P$$

(3) Taking moments about a:

$$240(0.75) = P(2)$$

$$\Rightarrow P = 90 \text{ N}$$

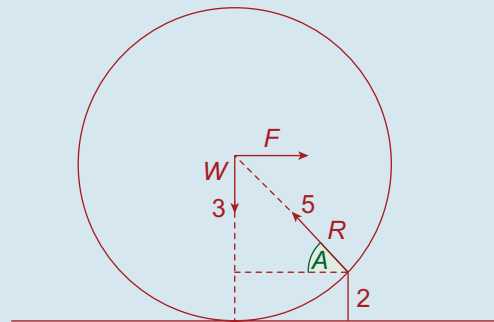
$$\vec{X} = -90\vec{i}, \vec{Y} = 240\vec{j}$$

$$\text{Resultant} = -90\vec{i} + 240\vec{j} = |\vec{R}|$$

$$= \sqrt{(-90)^2 + (240)^2}$$

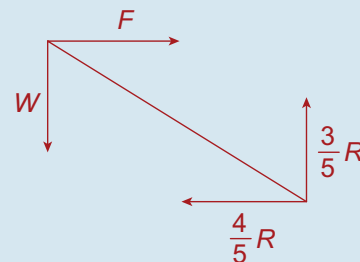
$$= 256 \text{ N}$$

Q. 4. (i) Forces:



$$\text{Since } \sin A = \frac{3}{5}, \cos A = \frac{4}{5}$$

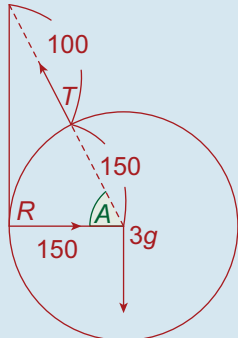
Resolved Forces:



(ii) In accordance with Theorem 8.7

(iii) (1) $\frac{3}{5}R = W \Rightarrow R = \frac{5}{3}W$
 (2) $F = \frac{4}{5}R = \frac{4}{5}(\frac{5}{3}W) = \frac{4}{3}W$

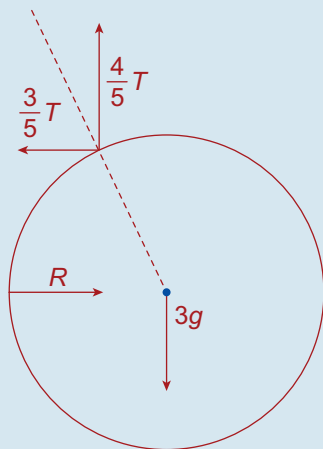
Q. 5. (a) Forces:



$\cos A = \frac{150}{250} = \frac{3}{5}$

$\therefore \sin A = \frac{4}{5}$

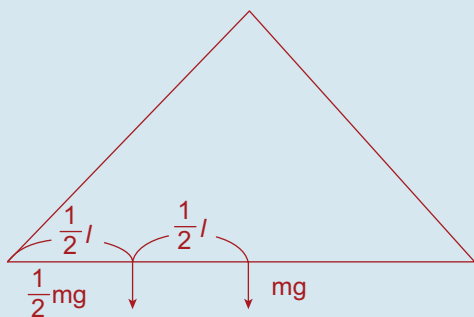
Resolved Forces:



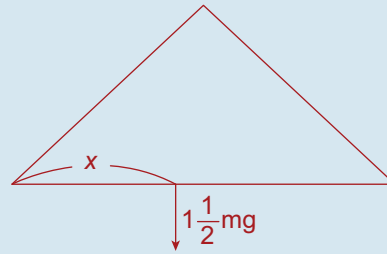
(1) $\frac{4}{5}T = 3g \Rightarrow T = \frac{15}{4}g$

Q. 6. Step 1. To find centre of gravity:

Forces:

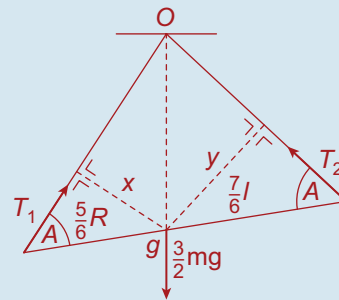


Resultant:



The sum of the moment = the moment of the sum

$\frac{1}{2}mg(\frac{1}{2}l) + mg(l) = 1\frac{1}{2}mg(x)$
 $\Rightarrow x = \frac{5}{6}l$



The resultant weight, $\frac{3}{2}mg$ acts through g , which must be below in accordance with Theorem 8.7

Taking moments about g :

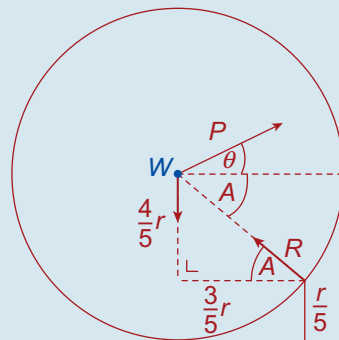
$T_1(x) = T_2(y)$

But $x = \frac{5}{6}l \sin A$, $y = \frac{7}{6}l \sin A$

$\therefore T_1(\frac{5}{6}l \sin A) = T_2(\frac{7}{6}l \sin A)$

$\Rightarrow \frac{T_1}{T_2} = \frac{7}{5}$

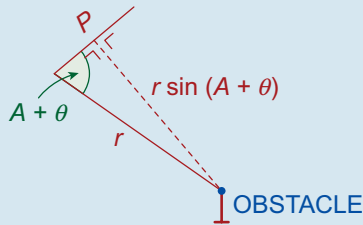
Q. 7.



$\sin A = \frac{4}{5}$

$\therefore \cos A = \frac{3}{5}$

Taking moments about the top of the obstacle:



$$W\left(\frac{3}{5}r\right) = P(r \sin(A + \theta))$$

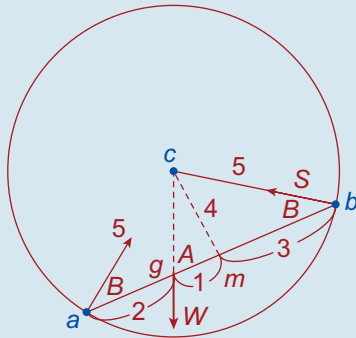
$$\Rightarrow P = \frac{3W}{5 \sin(A + \theta)}$$

(i) In this case $\theta = 0$

$$\therefore P = \frac{3W}{5 \sin A} = \frac{3W}{4}$$

(ii) In this case we want a minimum value for $\frac{3W}{5 \sin(A + \theta)}$. This value is attained when $\sin(A + \theta) = 1$, and is $\frac{3W}{5}$

Q. 8.



Let m be the midpoint $[ab]$.

$|cm| = 4$ (by Pythagoras), $|gm| = 1$ and $|gc| = \sqrt{17}$

$$\sin A = \frac{4}{\sqrt{17}} \text{ and } \sin B = \frac{4}{5}$$

Taking moments about a :

$$W(2 \sin A) = S(6 \sin B)$$

$$\Rightarrow W\left(\frac{8}{\sqrt{17}}\right) = S\left(\frac{24}{5}\right)$$

$$\Rightarrow S = \frac{5W}{3\sqrt{17}}$$

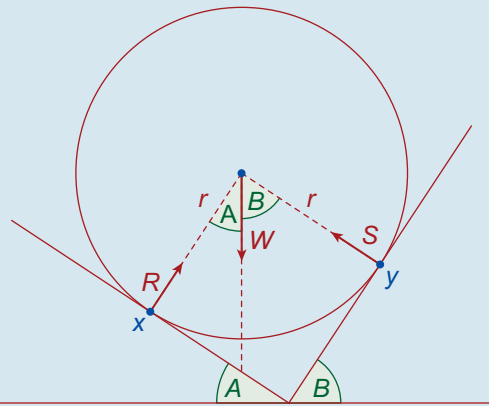
Taking moments about b :

$$W(4 \sin A) = S(6 \sin B)$$

$$\Rightarrow W\left(\frac{16}{\sqrt{17}}\right) = S\left(\frac{24}{5}\right)$$

$$\Rightarrow R = \frac{10W}{3\sqrt{17}}$$

Q. 9.



Since $\tan A = \frac{1}{2}$, $\sin A = \frac{1}{\sqrt{5}}$ and $\cos A = \frac{2}{\sqrt{5}}$

Since $\tan B = \frac{3}{4}$, $\sin B = \frac{3}{5}$ and $\cos B = \frac{4}{5}$

Also, $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{4}{5}\right) + \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{5}\right)$$

$$= \frac{10}{5\sqrt{5}}$$

$$= \frac{2}{\sqrt{5}}$$

Taking moments about x :

$$W(r \sin A) = S(r \sin(A + B))$$

$$\Rightarrow W\left(\frac{1}{\sqrt{5}}r\right) = S\left(\frac{2}{\sqrt{5}}r\right)$$

$$\Rightarrow S = \frac{1}{2}W$$

Q.E.D.