Chapter 13 Exercise 13A

Q. 1. $F = k(l - l_0)$

where l = current length

l_0 = original length

k = spring constant

(i) $F = 10(6 - 5)$

$\Rightarrow F = 10$ N

(ii) $F = 10(10 - 5)$

$\Rightarrow F = 50$ N

(iii) $F = 10(5.2 - 5)$

$\Rightarrow F = 2$ N

Q. 2. $l_0 = 2, k = 9$

(i) $F = 9(3 - 2)$

$\Rightarrow F = 9$ N

(ii) $F = 9(5 - 2)$

$\Rightarrow F = 27$ N

(iii) $F = 9\left(\frac{10}{3} - 2\right)$

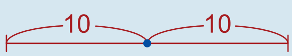
$\Rightarrow F = 12$ N

$kl = F + kl_0$

$\Rightarrow 9l = 54 + 9(2)$

$\Rightarrow l = 8$ m

Q. 3. (i)

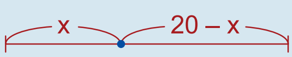


$F_l = 2(10 - 1) = 18$ N

$F_r = 4(10 - 1) = 36$ N

$\therefore F = F_r - F_l = 36 - 18 = 18$ N

(ii)



$F_l = 2(x - 1) = 2x - 2$

$F_r = 4(20 - x - 1) = 76 - 4x$

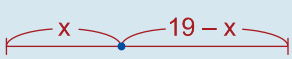
$F_l = F_r$

$\Rightarrow 2x - 2 = 76 - 4x$

$\Rightarrow x = 13$ m

from left hand wall (LHW)

Q. 4. (i)



$F_l = 5(x - 1) = 5x - 5$

$F_r = 3(19 - x - 2) = 51 - 3x$

$F_l = F_r$

$\Rightarrow 5x - 5 = 51 - 3x$

$\Rightarrow x = 7$ m from LHW

(ii) $F_l = 5x - 5$

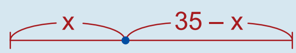
$F_r = 51 - 3x$

$F_r - F_l = 16$

$\Rightarrow (51 - 3x) - (5x - 5) = 16$

$\Rightarrow x = 5$ m from LHW

Q. 5. (i)



$F_l = 7(x - 2) = 7x - 14$

$F_r = 3(35 - x - 3) = 96 - 3x$

$F_l = F_r$

$\Rightarrow 7x - 14 = 96 - 3x$

$\Rightarrow x = 11$ m from LHW

(ii) $F_l = 7x - 14, \quad F_r = 96 - 3x$

If the force is 40 N to the right, then $F_r - F_l = 40$

$\Rightarrow (96 - 3x) - (7x - 14) = 40$

$\Rightarrow 110 - 10x = 40$

$\Rightarrow x = 7$

$\Rightarrow$ 7 metres from LHW

If the force is 40 N to the left, then $F_l - F_r = 40$

$\Rightarrow (7x - 14) - (96 - 3x) = 40$

$\Rightarrow 10x - 110 = 40$

$\Rightarrow x = 15$ m from LHW

Q. 6. $F = k(l - l_0)$

$= 50(2 - 1)$

$= 50$ N

This is the centripetal force and must equal $m\omega^2 r$

$\therefore m\omega^2 r = 50$

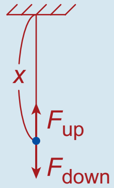
$\Rightarrow 1(\omega)^2(2) = 50$

$\Rightarrow \omega = 5$ rad/s

Q. 7. $F_{up} = k(l - l_0)$

$= 49(x - 1)$

$= 49x - 49$ N



$F_{down} = mg$

$= 10(9.8)$

$= 98$ N

But $F_{up} = F_{down}$ (in equilibrium)

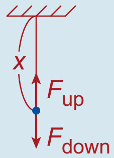
$\therefore\ 49x - 49 = 98$

$\Rightarrow x = 3$ m

Q. 8. $F_{up} = k(l - l_0)$

$= 7(x - 2)$

$= 7x - 14$ N



$F_{down} = mg$

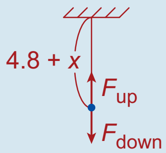
$= 2(9.8)$

$= 19.6$

$F_{up} = F_{down}$

$\Rightarrow 7x - 14 = 19.6$

$\Rightarrow x = 4.8$ m



$F_{up} = k(l - 0)$

$= 7(4.8 + x - 2)$

$= 19.6 + 7x$

$F_{down} = mg = 2(9.8) = 19.6$

Nett force $= F_{up} - F_{down}$

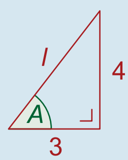
$= 19.6 + 7x - 19.6 = 7x$

Q. 9. (i) $l^2 = 4^2 + 3^2$

$\Rightarrow l = 5$ m

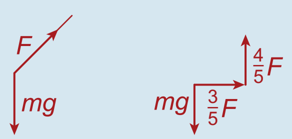
$\therefore$ Extension $= 5 - 3$

$= 2$ m



$\left(\text{also } \cos A = \frac{3}{5}, \sin A = \frac{4}{5}\right)$

(ii) **Forces** **Resolved**



$F = k(l - l_0) = k(5 - 3) = 2k$

1. $F_{up} = F_{down} \Rightarrow \frac{4}{5}F = mg$

$\Rightarrow \frac{4}{5}(2k) = mg \Rightarrow k = \frac{5mg}{8}$

(iii) 2. Centripetal force $= m\omega^2 r$

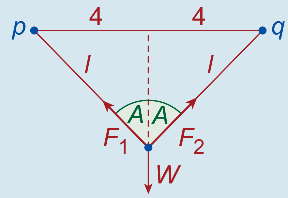
$\Rightarrow \frac{3}{5}F = m\omega^2(3)$

But $F = 2k = 2\left(\frac{5mg}{8}\right) = \frac{5mg}{4}$

$\therefore \frac{3}{5}\left(\frac{5mg}{4}\right) = m\omega^2(3)$

$\Rightarrow \omega^2 = \frac{g}{4} \Rightarrow \omega = \sqrt{\frac{g}{4}}$ rads/s

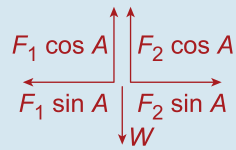
Q. 10. (i)



$F_1 = k(l - l_0) = 10(l - 2) = (10l - 20)$ N

$F_2 = k(l - l_0) = 7(l - 1) = (7l - 7)$ N

Forces **(Resolved)**



1. $F_{left} = F_{right} \Rightarrow F_1 \sin A = F_2 \sin A$

$\Rightarrow F_1 = F_2 \Rightarrow 10l - 20 = 7l - 7$

$\Rightarrow l = \frac{13}{3}$

Now $\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{4}{\frac{13}{3}} = \frac{12}{13}$

(ii) $\therefore \cos A = \frac{5}{13}$

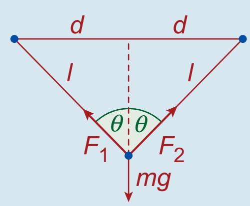
2. $F_1 = 10l - 20 = \frac{130}{3} - 20 = \frac{70}{3}$ N

$F_2 = F_1 = \frac{70}{3}$ N

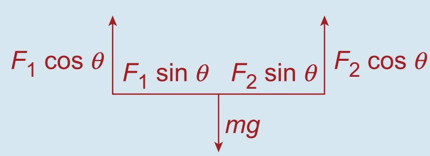
$F_{up} = F_{down} \Rightarrow F_1 \cos A + F_2 \cos A = W$

$\Rightarrow W = \left(\frac{70}{3}\right)\left(\frac{5}{13}\right) + \left(\frac{70}{3}\right)\left(\frac{5}{13}\right) = \frac{700}{39}$ N

Q. 11. **Forces**



Resolved



$F_{\text{left}} = F_{\text{right}} \Rightarrow F_1 \sin\theta = F_2 \sin\theta \Rightarrow F_1 = F_2$

$\Rightarrow k_1(l - l_1) = k_2(l - l_2)$

$\Rightarrow k_1 l - k_1 l_1 = k_2 l - k_2 l_2$

$\Rightarrow l(k_1 - k_2) = k_1 l_1 - k_2 l_2$

$\Rightarrow l = \dfrac{k_1 l_1 - k_2 l_2}{k_1 - k_2}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{d}{l} = \dfrac{d(k_1 - k_2)}{k_1 l_1 - k_2 l_2}$ **QED**

Exercise 13B

Q. 1. (i) $\dfrac{2\pi}{7} = \dfrac{2\pi}{\omega} \Rightarrow \omega = 7$

Max. velocity $= \omega A = 7(5) = 35$ m/s

(ii) Max. acceleration $= \omega^2 A = (7)^2(5)$

$= 245$ m/s^2

(iii) Total distance covered $= 4A = 20$ m

Time taken $= \dfrac{2\pi}{7}$

Average speed $= \dfrac{\text{Distance}}{\text{Time}}$

$= \dfrac{20}{\frac{2\pi}{7}}$

$= \dfrac{140}{2\pi}$

$= \dfrac{70}{\pi}$ m/s

Q. 2. $v^2 = \omega^2(A^2 - x^2)$

$x = \sqrt{7}, v = 9$

$\Rightarrow 81 = \omega^2(A^2 - 7)$ …… **Equation 1**

$x = 2, v = 6\sqrt{3}$

$\Rightarrow 108 = \omega^2(A^2 - 4)$ ….. **Equation 2**

Dividing equation **1** by **2** gives

$\dfrac{81}{108} = \dfrac{\omega^2(A^2 - 7)}{\omega^2(A^2 - 4)} \Rightarrow \dfrac{A^2 - 7}{A^2 - 4} = \dfrac{3}{4}$

$\Rightarrow 3A^2 - 12 = 4A^2 - 28 \Rightarrow A = 4$

Putting this result into equation **1** gives:

$81 = \omega^2(4^2 - 7) \Rightarrow \omega^2 = 9 \Rightarrow \omega = 3$

Periodic time $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{3}$ s

Q. 3. Max. velocity $= \omega A = 6$ …. **Equation 1**

Max. acceleration $= \omega^2 A$

$= 12$ …. **Equation 2**

Dividing equation **2** by equation **1** gives $\omega = 2$

Therefore, $A = 3$

Periodic time $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \pi$ s

To find a when $v = 2\sqrt{5}$:

Step 1: Find x when $v = 2\sqrt{5}$:

$v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow 20 = 2^2(3^2 - x^2)$

$\Rightarrow x = \pm 2$

Step 2: Find a when $x = \pm 2$

$a = -\omega^2 x = -(2)^2(\pm 2) = \pm 8$ m/s^2

The magnitude of the acceleration is 8 m/s^2

Q. 4. (i) $v^2 = \omega^2(A^2 - x^2)$

$v = 8$ when $x = 1$

$\Rightarrow 64 = \omega^2(A^2 - 1)$ …. **Equation 1**

$v = 4$ when $x = 7$

$\Rightarrow 16 = \omega^2(A^2 - 49)$ …. **Equation 2**

Dividing **1** by **2** gives

$\dfrac{64}{16} = \dfrac{\omega^2(A^2 - 1)}{\omega^2(A^2 - 49)}$

$\Rightarrow \dfrac{A^2 - 1}{A^2 - 49} = \dfrac{4}{1}$

$\Rightarrow 4A^2 - 196 = A^2 - 1$

$\Rightarrow A = \sqrt{65}$

(ii) Putting this result into equation **1** gives

$64 = \omega^2(65 - 1)$

$\Rightarrow \omega = 1$

$\therefore T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{1} = 2\pi$ s

(iii) When $x = 0$, $v^2 = \omega^2(A^2 - x^2)$

$= 1(65 - 0) = 65$

$\therefore v = \sqrt{65}$ m/s

Q. 5. When $x = 1$, $v = 3$, $a = 3$

So $v = \omega\sqrt{A^2 - x^2}$ …(1)

and $a = \omega^2 x$ …(2)

$\Rightarrow 3 = \omega^2(1)$

$\Rightarrow \omega = \sqrt{3}$

$\therefore$ From (1), $3 = \sqrt{3}\sqrt{A^2 - 1}$

$\Rightarrow A = 2$

From (2), $a_{MAX} = \omega^2 A$

$\Rightarrow a_{MAX} = 3(2)$

$\Rightarrow a_{MAX} = 6$ m/s^2

Q. 6. $v^2 = \omega^2(A^2 - x^2)$. But $v = 24$ when $x = 5$.

$\therefore\ 576 = \omega^2(A^2 - 25)$ …. **Equation 1**

Also $a = -\omega^2 x$. But $a = -20$ when $x = 5$.
(a and x are always of opposite sign)

$\therefore\ -20 = -\omega^2(5) \Rightarrow \omega = 2$

Putting this into equation **1** gives:

$576 = 4(A^2 - 25) \Rightarrow A^2 - 25 = 144$

$\Rightarrow A = 13$

(i) Amplitude $= A = 13$.

(ii) Periodic time $= \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \pi$ s

(iii) In π seconds it performs 1 oscillation.

In 1 second it performs $\dfrac{1}{\pi}$ oscillations.

In 60 seconds it performs $\dfrac{60}{\pi} = 19.1$ oscillations.

Answer: 19 complete oscillations.

Q. 7. When $x = \sqrt{2}$, $a = -4\sqrt{2}$, $\therefore\ a = -\omega^2 x$

$\Rightarrow -4\sqrt{2} = -\omega^2\sqrt{2} \Rightarrow \omega = 2$.

When $x = \sqrt{2}$, $v = 2$, and $\omega = 2$,
$\therefore\ v^2 = \omega^2(A^2 - x^2) \Rightarrow 4 = 4(A^2 - 2)$
$\Rightarrow A = \sqrt{3}$

Start the clock in the centre $\Rightarrow x = A \sin \omega t$
i.e. $x = \sqrt{3} \sin 2t$

To find t when $x = 1.5$: $1.5 = \sqrt{3} \sin 2t$
$\Rightarrow 3 = 2\sqrt{3} \sin 2t$

$\Rightarrow \sin 2t = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow 2t = \dfrac{\pi}{3} \Rightarrow t = \dfrac{\pi}{6}$ s

$F = ma$. But $a = -\omega^2 x = -4(1.5) = -6$
and $m = 2$ kg.

$\therefore\ F = (2)(-6) = -12$ N

The force is of magnitude 12 N.

Q. 8. Amplitude $= \dfrac{\text{Highest} - \text{Lowest}}{2}$

(i) $\Rightarrow A = \dfrac{13 - 3}{2}$

$\Rightarrow A = 5$ m

(ii) $\dfrac{T}{2} = \dfrac{11}{2}$

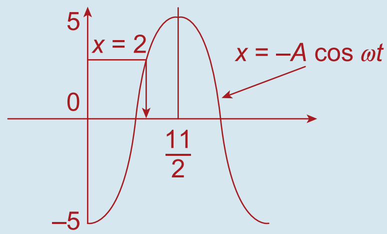
$\Rightarrow T = 11$ hrs

(iii) $T = \dfrac{2\pi}{\omega}$

$\Rightarrow \omega = \dfrac{2\pi}{T}$

$\Rightarrow \omega = \dfrac{2\pi}{11}$ rad/s

Graph:

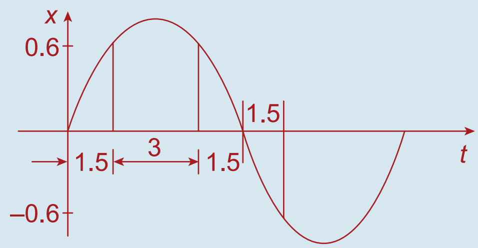


We have $x = -5 \cos \dfrac{2\pi}{11} t$

$\Rightarrow 2 = -5 \cos \dfrac{2\pi}{11} t$

$\Rightarrow t = 3.47$ hrs **OR** 3:29 PM

Q. 9. (i) Consider the Sine Curve:



$x = A \sin \omega t$

From the graph $\dfrac{T}{2} = 6$ s

$\Rightarrow T = 12$ s

(ii) But $T = \dfrac{2\pi}{\omega}$

$\Rightarrow \omega = \dfrac{2\pi}{12}$

$\Rightarrow \omega = \dfrac{\pi}{6}$

$0.6 = A \sin \dfrac{\pi}{6}(1.5)$

$\Rightarrow \dfrac{3}{5} = A \sin \dfrac{\pi}{4}$

$\Rightarrow \dfrac{3}{5} = A \dfrac{1}{\sqrt{2}}$

$\Rightarrow A = \dfrac{3\sqrt{2}}{5}$ m

Q. 10. **Periodic Time:**

$\dfrac{T}{2} = 6{:}58 - 0{:}58$

$\Rightarrow \dfrac{T}{2} = 6$ hrs

$\Rightarrow T = 12$ hrs

But $T = \dfrac{2\pi}{\omega}$

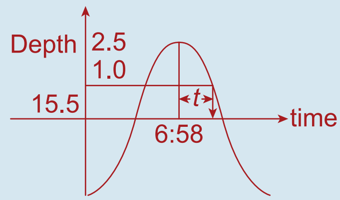
$\Rightarrow \omega = \dfrac{2\pi}{12}$

$\Rightarrow \omega = \dfrac{\pi}{6}$ rad/hr

Amplitude:

$A = \frac{18 - 13}{2} \Rightarrow A = 2.5$ m

$\therefore$ Mean level (Equilibrium Position)
$= 15.5$ m



6:58 + t is latest time ship can leave.

$x = A \cos \omega t$

$1 = 2.5 \cos \frac{\pi t}{6}$

$\Rightarrow t = 2.214$ hrs

$\Rightarrow t = 2$ hrs $12' \, 50.62''$

so 6:58 + 2:12′ 50.62″

= 9:10′ 50.62″ **OR** 9:10 PM

Exercise 13C

Q. 1. $x = 3 \sin 5t. \therefore \frac{dx}{dt} = 15 \cos 5t.$

$\therefore \frac{d^2x}{dt^2} = -75 \sin 5t$

$= -25(3 \sin 5t) = -25x$

Since the acceleration is proportional to x but in the opposite direction, it will perform SHM.

$x = 3 \sin 5t \Rightarrow 1.5 = 3 \sin 5t$

$\Rightarrow \sin 5t = \frac{1}{2} \Rightarrow 5t, \frac{\pi}{6} \Rightarrow t = \frac{\pi}{30}$ s

Q. 2. (i) $x = 4 \cos 2t. \therefore \frac{dx}{dt} = -8 \sin 2t$

$\therefore \frac{d^2x}{dt^2} = -16 \cos 2t = -4(4 \cos 2t)$

$= -4x$

Since the acceleration is proportional to the distance from p, but in the opposite direction, it will perform SHM ($A = 4$, $\omega = 2$)

(ii) Greatest distance $= A = 4$ m

(iii) Its velocity is zero at the extreme point. Since the clock starts when the particle is at an extreme point, use $x = 4 \cos 2t$.

$x = 4 \cos 2t \Rightarrow 2.5 = 4 \cos 2t$

$\Rightarrow \cos 2t = 0.625$

$\Rightarrow 2t = \cos^{-1}(0.625) \Rightarrow 2t = 0.8956$

$\Rightarrow t = 0.4478$ s

Q. 3. $x = 9 \cos 3t$

(i) For $t = 0$, $x = 9 \cos 0 \Rightarrow x = 9$

(ii) **Note:** x is measured from equilibrium.

So, when particle has travelled 2 metres, $x = 7$

$x = 9 \cos 3t$

$\Rightarrow 7 = 9 \cos 3t$

$\Rightarrow \cos 3t = \frac{7}{9}$

$\Rightarrow 3t = \cos^{-1} \frac{7}{9}$

$\Rightarrow t = 0.227$ s

Q. 4. (i) $x = 13 \sin(\omega t + \varepsilon)$,

when $t = 0$, $x = 5 \Rightarrow 5 = 13 \sin \varepsilon$

$\sin \varepsilon = \frac{5}{13} = 0.3846$

$\Rightarrow \varepsilon = \sin^{-1}(0.3846) = 0.3948$

(ii) $v^2 = \omega^2(A^2 - x^2)$, $v = 24$ when $x = 5$.
Also $A = 13$

$\therefore (24)^2 = \omega^2(13^2 - 5^2)$

$\Rightarrow 576 = \omega^2(144) \Rightarrow \omega = 2$

(iii) $x = 0 \Rightarrow 13 \sin(\omega t + \varepsilon) = 0$

$\Rightarrow \omega t + \varepsilon = 0$ **OR** π **OR** 2π etc.

$\Rightarrow 2t + 0.3948$

$= 0$ **OR** 3.1416 **OR** 6.2832 etc.

The first time ($t > 0$) will be when
$2t + 0.3948 = 3.1416$

$\Rightarrow 2t = 2.7468$

$\Rightarrow t = 1.3734$ s

Q. 5. (i) $x = 3 \cos 2t + 4 \sin 2t$

$A = \sqrt{3^2 + 4^2} = 5$

$T = \frac{2\pi}{2} = \pi$

(ii) $x = 8 \cos 4t + 6 \sin 4t$

$A = \sqrt{8^2 + 6^2} = 10$

$T = \frac{2\pi}{4} = \frac{\pi}{2}$

(iii) $x = 12\cos t + 5\sin t$

$A = \sqrt{12^2 + 5^2} = 13$

$T = 2\pi$

(iv) $x = 3\cos \pi t + \sqrt{7}\sin \pi t$

$A = \sqrt{3^2 + \sqrt{7}^2} = 4$

$T = \dfrac{2\pi}{\pi} = 2$

(v) $x = \sin 3t + \cos 3t$

$A = \sqrt{1^2 + 1^2} = \sqrt{2}$

$T = \dfrac{2\pi}{3}$

(vi) $x = 21\sin 2\pi t + 20\cos 2\pi t$

$A = \sqrt{21^2 + 20^2} = 29$

$T = \dfrac{2\pi}{2\pi} = 1$

(vii) $x = \sqrt{3}\sin 5t - \cos 5t$

$A = \sqrt{\sqrt{3}^2 + 1} = 2$

$T = \dfrac{2\pi}{5}$

(viii) $x = 2\sin\dfrac{t}{2} + 3\cos\dfrac{t}{2}$

$A = \sqrt{2^2 + 3^2} = \sqrt{13}$

$T = \dfrac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$

(ix) $x = 24\sin\dfrac{t}{4} - 7\cos\dfrac{t}{4}$

$A = \sqrt{24^2 + 7^2} = 25$

$T = \dfrac{2\pi}{\left(\frac{1}{4}\right)} = 8\pi$

(x) $x = 2\sin\dfrac{t}{3} + \cos\dfrac{t}{3}$

$A = \sqrt{2^2 + 1^2} = \sqrt{5}$

$T = \dfrac{2\pi}{\left(\frac{1}{3}\right)} = 6\pi$

Q. 6. $x = 12\cos t + 35\sin t$

$\Rightarrow \dot{x} = -12(1)\sin t + 35(1)\cos t$

$\Rightarrow \ddot{x} = -12(1)^2\cos t - 35(1)^2\sin t$

$\Rightarrow \ddot{x} = -12\cos t - 35\sin t$

(i) $\Rightarrow \ddot{x} = -x$

$\Rightarrow$ SHM with $\omega = 1$ rad/s

(ii) $T = \dfrac{2\pi}{\omega}$

$\Rightarrow T = \dfrac{2\pi}{1}$

$\Rightarrow T = 2\pi$ s

(iii) $A = \sqrt{12^2 + 35^2}$

$\Rightarrow A = 37$ m

(iv) For $x = 0$;

$12\cos t + 35\sin t = 0$

$\Rightarrow 35\sin t = -12\cos t$

$\Rightarrow \tan t = \dfrac{-12}{35}$

$\Rightarrow t = -0.3303, -0.3303 + \pi$, etc.

$\Rightarrow t = -0.3303 + \pi$ for first positive value.

$\Rightarrow t = 2.811$ s

Q. 7. $x = 12\sin 2t + 5\cos 2t$

$\dot{x} = 12(2)\cos 2t - 5(2)\sin 2t$

$\ddot{x} = -12(2)^2\sin 2t - 5(2)^2\cos 2t$

$\Rightarrow \ddot{x} = -48\sin 2t - 20\cos 2t$

(i) $\Rightarrow \ddot{x} = -4x$

$\Rightarrow$ SHM with $\omega = 2$ rad/s

(ii) $T = \dfrac{2\pi}{\omega}$

$\Rightarrow T = \dfrac{2\pi}{2}$

$\Rightarrow T = \pi$ s

(iii) $A = \sqrt{12^2 + 5^2}$

$\Rightarrow A = 13$ m

(iv) For $x = 0$

$12\sin 2t + 5\cos 2t = 0$

$\Rightarrow \sin 2t = \dfrac{-5}{12}\cos 2t$

$\Rightarrow \tan 2t = \dfrac{-5}{12}$

$\Rightarrow 2t = \tan^{-1}\dfrac{-5}{12} + n\pi$ for all solutions

$\Rightarrow 2t = -0.3948 + n\pi$

$\Rightarrow t = -0.1974 + \dfrac{n\pi}{2}$

$\Rightarrow n = 1$ gives $t_1 = 1.373$ s

(v) $n = 2$ gives $t_2 = 2.944$ s

Q. 8. (i) $x = A\cos(\omega t + \alpha)$

$\Rightarrow \dfrac{dx}{dt} = -\omega A\sin(\omega t + \alpha)$

$\Rightarrow \dfrac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \alpha) = -\omega^2 x$

Since the acceleration is proportional to x but in the opposite direction, it will perform SHM.

$v = -2A$ when $x = \frac{3A}{5}$ and $A = A$

$v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow 4A^2 = \omega^2\left(A^2 - \frac{9A^2}{25}\right)$

$\Rightarrow 4A^2 = \omega^2\left(\frac{16A^2}{25}\right)$

$\Rightarrow \omega^2 = \frac{25}{4}$

$\Rightarrow \omega = \frac{5}{2}$

Also $x = \frac{34}{5}$ when $t = 0$

$\therefore x = A\cos(\omega t + \alpha)$

$\Rightarrow \frac{3A}{5} = A\cos(\alpha)$

$\Rightarrow \cos\alpha = \frac{3}{5} = 0.6$

$\Rightarrow \alpha = 0.9273$ radians

(ii) Now, $A\cos(\omega t + \alpha) = 0$

$\Rightarrow \omega t + \alpha$
$= \frac{\pi}{2}$ **OR** $\frac{3\pi}{2}$ **OR** $\frac{5\pi}{2}$ etc.

$\Rightarrow \frac{5}{2}t + 0.9273 = \frac{\pi}{2} = 1.571$

$\Rightarrow t = 0.2575$ s

Q. 9. Maximum acceleration must be not greater than g if the bodies are to stay on the platform.

$\Rightarrow \omega^2 A \leqslant 9.8$

$\Rightarrow \omega^2(0.2) \leqslant 9.8$

$\Rightarrow \omega \leqslant 7.$

Taking ω at its maximum value, 7.

$T = \frac{2\pi}{\omega} = \frac{2\pi}{7} = 0.8977$ s

Number of oscillations per minute

$= \frac{60}{0.8977} = 66$ complete oscillations

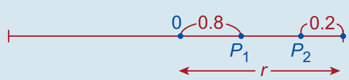
Q. 10. (a) $x = r\cos\omega t$

$\therefore \frac{dx}{dt} = -r\omega\sin\omega t$

$\therefore \frac{d^2x}{dt^2} = -r\omega^2\cos\omega t = -\omega^2 x$

$\therefore a = -\omega^2 x \therefore$ SHM

(b) (i)



When $x = 0.8$, $v = 6$. When $x = r - 0.2$, $a = -24$ (because a must be negative if x is positive)

$v^2 = \omega^2(A^2 - x^2).$

Put $x = 0.8$, $v = 6$, $A = r$

$\Rightarrow 36 = \omega^2(r^2 - 0.64)$

.... **Equation 1**

$a = -\omega^2 x.$

Put $x = t - 0.2$, $a = -24$

$\Rightarrow -24 = -\omega^2(r - 0.2)$

$\Rightarrow 24 = \omega^2(r - 0.2)$ … **Equation 2**

Dividing equation **2** by equations **1** gives:

$\frac{24}{36} = \frac{\omega^2(r - 0.2)}{\omega^2(r^2 - 0.64)}$

$\Rightarrow \frac{r - 0.2}{r^2 - 0.64} = \frac{2}{3}$

(ii) $2r^2 - 1.28 = 3r - 0.6$

$\Rightarrow 2r^2 - 3r - 0.68 = 0.$

$\Rightarrow 200r^2 - 300r - 68 = 0$

$\Rightarrow 50r^2 - 75r - 17 = 0$

$\Rightarrow (5r + 1)(10r - 17) = 0$

$\Rightarrow r = -0.2$ **OR** $r = 1.7$

$r = -0.2$ has no meaning, so $r = 1.7$

Putting this result into equation **2** gives: $24 = \omega^2(1.7 - 0.2)$

$\Rightarrow 24 = \omega^2(1.5) \Rightarrow \omega^2 = 16$

$\Rightarrow \omega = 4$

$\therefore$ Period $= \frac{2\pi}{4} = \frac{\pi}{2}$ s

(iii) Start clock at centre: $x = A\sin\omega t$
i.e. $x = 1.7\sin 4t$

At P_1, $x = 0.8$

$\Rightarrow 0.8 = 1.7\sin 4t$

$\Rightarrow \sin 4t = \frac{8}{17} = 0.4706$

$\Rightarrow 4t = \sin^{-1}(0.4706) = 0.4900$

$\Rightarrow t = 0.1225$ s

At P_2, $x = 1.5$

$\Rightarrow 1.5 = 1.7\sin 4t$

$\Rightarrow \sin 4t = \frac{15}{17} = 0.8824$

$\Rightarrow 4t = \sin^{-1}(0.8824) = 1.0809$

$\Rightarrow t = 0.2702$ s

Time to travel from P_1 to P_2

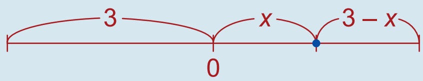
$= 0.2702 - 0.1225$

$= 0.1477$ s

$= 0.15$ s

Exercise 13D

Q. 1. (i)



$F_r = k(l - l_0)$
$\quad = 2(3 - x - 1)$
$\quad = 4 - 2x$

$F_l = k(l - l_0)$
$\quad = 2(3 + x - 1)$
$\quad = 4 + 2x$

$F = F_r - F_l$
$\quad = 4 - 2x - 4 - 2x$
$\quad = -4x$

$F = ma \Rightarrow -4x = 1(a) \Rightarrow a = -4x$

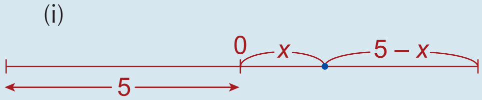
This is SHM with $\omega = 2$.

(ii) Periodic time $= \dfrac{2\pi}{2} = \pi$ s

(iii) A = original distance from $0 = 1$.

Midway between the walls $\Rightarrow x = 0$
$\Rightarrow v^2 = \omega^2(A^2 - x^2) \Rightarrow v^2 = 4(1 - 0)$
$\Rightarrow v = 2$ m/s

Q. 2. (i)



$F_r = k(l - l_0)$
$\quad = 9(5 - x - 1)$
$\quad = 36 - 9x$

$F_l = 9(5 + x - 1)$
$\quad = 36 + 9x$

$F_t = F_r - F_l$
$\quad = 36 - 9x - 36 - 9x$
$\quad = -18x$

$F = ma$
$\Rightarrow -18x = \dfrac{1}{2}a$
$\Rightarrow a = -36x$

It will perform SHM with $\omega = 6$. When it is released, $x = 2$. Therefore $A = 2$.

(ii) It starts from an extreme point.
$\therefore x = a\cos\omega t$ i.e. $x = 2\cos 6t$.

We want to find t when $x = 1$.

$\therefore l = 2\cos 6t$

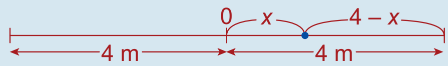
$\Rightarrow \cos 6t = \dfrac{1}{2}$

$\Rightarrow 6t = \dfrac{\pi}{3}$

$\Rightarrow t = \dfrac{\pi}{18}$ s

(iii) $v^2 = \omega^2(A^2 - x^2)$
$\Rightarrow v^2 = 6^2(2^2 - 1^2) = 108$
$\Rightarrow v = \sqrt{108} = 6\sqrt{3}$ m/s

Q. 3. (i)



$F_r = 20(4 - x - 1)$
$\quad = 60 - 20x$

$F_l = 20(4 + x - 1) = 60 + 20x$

$F = F_r - F_l$
$\quad = 60 - 20x - 60 - 20x$
$\quad = -40x$

$F = ma \Rightarrow -40x = 5a \Rightarrow a = -8x$

It will perform SHM with $\omega = \sqrt{8}$. When it is released, $x = 3$, therefore $A = 3$.

(ii) Maximum speed $= \omega A = \sqrt{8}\,(3)$
$\quad = 3\sqrt{8}$ m/s.

(iii) We want to find x when $v = \sqrt{8}$ m/s.

$v^2 = \omega^2(A^2 - x^2)$
$\Rightarrow 8 = 8(9 - x^2)$
$\Rightarrow 9 - x^2 = 1$
$\Rightarrow x = \sqrt{8}$ m

It starts from an extreme point.
$\therefore x = A\cos\omega t \Rightarrow x = 3\cos\sqrt{8}t$

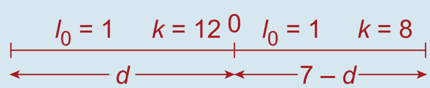
(iv) To find t when $x = \sqrt{8}$
$\Rightarrow \sqrt{8} = 3\cos\sqrt{8}t$

$\Rightarrow \cos\sqrt{8}t = \dfrac{\sqrt{8}}{3} = \dfrac{2.828}{3} = 0.9427$

$\Rightarrow \sqrt{8}t = \cos^{-1}(0.9427) = 0.34$

$\Rightarrow t = \dfrac{0.34}{\sqrt{8}} = 0.12$ s

Q. 4. (i) Let 0 be the position of equilibrium



$F_r = k(l - l_0)$
$\quad = 8(7 - d - 1)$
$\quad = 48 - 8d$

$F_l = k(l - l_0)$
$\quad = 12(d - 1)$
$\quad = 12d - 12$

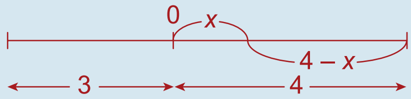
$F_r = F_l \Rightarrow 48 - 8d$

$= 12d - 12$

$\Rightarrow d = 3$

Answer: 3 metres from left hand wall.

(ii)



$F_r = k(l - l_0)$

$= 8(4 - x - 1)$

$= 24 - 8x$

$F_l = k(l - l_0)$

$= 12(3 + x - 1)$

$= 24 + 12x$

$F = F_r - F_l$

$= 24 - 8x - 24 - 12x$

$= -20x$

$F = ma$

$\Rightarrow -20x = 5a$

$\Rightarrow a = -4x$

It will perform SHM with $\omega = 2$.

(iii) When it is released its displacement, x, from 0 is $\frac{1}{2}$ m.

Therefore $A = \frac{1}{2}$.

Periodic time $= \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ s

Maximum velocity $= \omega A$

$= 2\left(\frac{1}{2}\right)$

$= 1$ m/s

(iv) Firstly, find x when $v = \frac{\sqrt{3}}{2}$:

$v^2 = \omega^2(A^2 - x^2)$

$\Rightarrow \frac{3}{4} = 4\left(\frac{1}{2} - x^2\right)$

$\Rightarrow x = \frac{1}{4}$

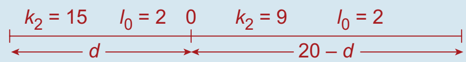
When $x = \frac{1}{4}$

$a = \omega^2 x = -(4)\left(\frac{1}{4}\right) = -1$ m/s^2

The acceleration is of magnitude 1 m/s^2.

$F = ma \Rightarrow F = (5)(1) = 5$ N

Q. 5. (i)



$F_r = k(l - l_0)$

$= 9(20 - d - 2)$

$= 162 - 9d$

$F_l = k(l - l_0)$

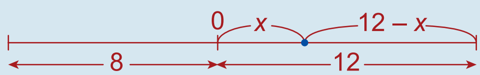
$= 15(d - 2)$

$= 15d - 30$

$F_r = F_l$

$\Rightarrow 162 - 9d = 15d - 30$

$\Rightarrow d = 8$



$F_r = k(l - l_0)$

$= 9(12 - x - 2)$

$= 90 - 9x$

$F_l = k(l - l_0)$

$= 15(8 + x - 2)$

$= 90 + 15x$

$F = F_r - F_l$

$= 90 - 9x - 90 - 15x$

$= -24x$

$F = ma$

$\Rightarrow -24x = \frac{1}{6}a$

$\Rightarrow a = -144x$

It will perform SHM with $\omega = 12$

(ii) When it was released its displacement, x, from 0 was 1 m.

Therefore $A = 1$.

Maximum acceleration $= \omega^2 A$

$= 144(1)$

$= 144$ m/s^2

(iii) $\frac{3}{5}$ (Max. acceleration) $= \frac{3}{5}(144)$

$= \frac{432}{5}$ m/s^2

When $a = \frac{432}{5}$, what is x?, $a = -\omega^2 x$

$\Rightarrow \frac{432}{5} = -144x$

$\Rightarrow x = \frac{-3}{5}$

When $x = -\frac{3}{5}$, what is v?

$v^2 = \omega^2(A^2 - x^2)$

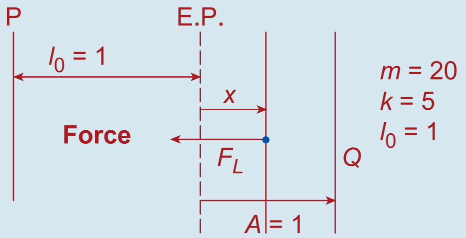
$\Rightarrow v^2 = 144\left(1 - \frac{9}{25}\right)$

$\Rightarrow v^2 = 144\left(\frac{16}{25}\right)$

$\Rightarrow v = 12\left(\frac{4}{5}\right)$

$= \frac{48}{5} = 9.6$ m/s

Q. 6.



(i) **Hooke's Law:** $F = k\,[l - l_0]$

where, k = spring constant

l = current length

l_0 = original length.

At x, NZL: $\Sigma F = ma$

$\overset{+}{\rightarrow}\ F_L = ma$

$\Rightarrow -k(l_0 + x - l_0) = ma$

$\Rightarrow -5(x) = 20\,a$

$\Rightarrow a = -\frac{x}{4}$

$\Rightarrow$ SHM with $\omega = \frac{1}{2}$

(ii) $\Rightarrow T = \frac{2\pi}{\omega} = 4\pi$

Note: Particle travels from Q to E.P. with SHM. It then travels to P with uniform velocity. (String has gone slack)

t_1; from Q to E.P., is $\frac{T}{4} = \left(\frac{1}{4} \text{ of cycle}\right)$
$= \pi$

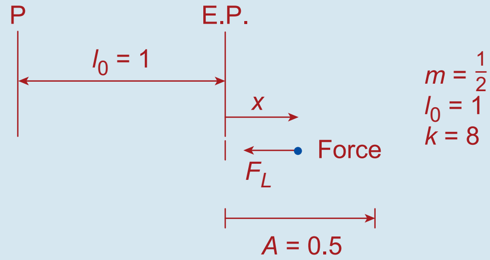
t_2; from E.P. to P, $t = \frac{\text{distance}}{\text{speed}}$

where $v = \omega A$ at E.P.

$\Rightarrow t_2 = \frac{l_0}{\left(\frac{1}{2}\right)(1)} = 2$

so Total Time Taken $= t_1 + t_2 = \pi + 2$, **QED**

Q. 7.



$F = k[l - l_0]$

At position x, NZL: $\Sigma F = ma$

$\overset{+}{\rightarrow}\ -F_L = ma$

$\Rightarrow -k(l - l_0) = ma$

$\Rightarrow -8[\cancel{1} + x - \cancel{1}] = \frac{1}{2}a$

(i) $\Rightarrow a = -16x$

$\Rightarrow$ SHM, $\omega = 4$

$\omega = 4,\quad A = 0.5,\quad T = \frac{2\pi}{\omega} = \frac{\pi}{2}$

(ii) Max $v = \omega A$

$\Rightarrow v_{MAX} = 4\left(\frac{1}{2}\right)$

$= 2$ m/s

(iii) $x = A\cos\omega t$ (Starting at Extreme Point)

Note, when particle has travelled 0.2 m, $x = 0.3$.

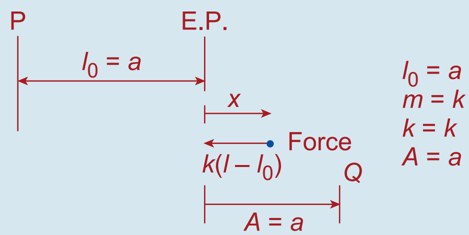
$\Rightarrow 0.3 = 0.5\cos 4t$

$\Rightarrow \cos 4t = \frac{3}{5}$

$\Rightarrow 4t = \cos^{-1}\left(\frac{3}{5}\right)$

$\Rightarrow t = 0.23$ s

Q. 8.



(i) At x, NZL: $\Sigma F = ma$

$\overset{+}{\rightarrow}\ -k(l - l_0) = m\ddot{x}$

Note: We use $\ddot{x}$, instead of the more usual a, for acceleration, to avoid confusion. Here A is amplitude, a is original length.

So $-k\,(\cancel{a} + x - \cancel{a}) = k\ddot{x}$

$\Rightarrow \ddot{x} = -x$

$\Rightarrow$ SHM with $\omega = 1$ rad/s

(ii) t_1, from Q to E.P. $= \frac{T}{4} = \frac{2\pi}{4\omega} = \frac{\pi}{2}$ s

t_2, from E.P. to P, $t_2 = \frac{\text{distance}}{\text{speed}}$

where $V = \omega A$ at E.P.

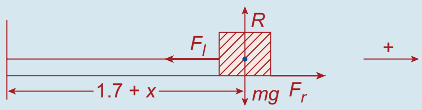
$\Rightarrow t_2 = \frac{a}{\omega A} = \frac{a}{1(a)} = 1$ s

$\therefore$ Total time $= t_1 + t_2$

$= \frac{\pi}{2} + 1$

$= 2.57$ s

Q. 9. (i)



(ii) 1. $R = mg = (1)(9.8) = 9.8$

2. $F_r = \mu R = \frac{1}{2}(9.8) = 4.9$

3. $F_l = k(l - l_0)$

$= 7(1.7 + x - 1)$

$= 4.9 + 7x$

$F = F_r - F_l$

$= 4.9 - 4.9 - 7x$

$= -7x$

$F = ma$

$\Rightarrow -7x = la$

$\Rightarrow a = -7x$

It will perform SHM with $w = \sqrt{7}$.

Periodic time $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{7}}$

(iii) The centre of oscillation, where $x = 0$, is 1.7 m from p. It was released where $x = 2$. Therefore $A = 2$.

It starts from an extreme point

$\therefore x = A\cos\omega t \Rightarrow x = 2\cos\sqrt{7}t$

To find t when $x = 0.3$ (i.e. $2 - 1.7$)

$0.3 = 2\cos\sqrt{7}t$

$\Rightarrow \cos\sqrt{7}t = 0.15$

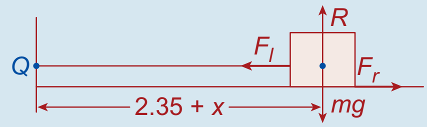
$\Rightarrow \sqrt{7}t = \cos^{-1}(0.15)$

$= 1.4202$

$\Rightarrow t = \frac{1.4201}{\sqrt{7}}$

$= 0.54$ s

Q. 10. (i)



1. $R = mg = 5(9.8) = 49$

2. $F_r = R = (1)(49) = 49$

3. $F_l = k(l - l_0)$

$= 140(2.35 + x - 2)$

$= 49 + 140x$

$F = F_r - F_l$

$= 49 - 49 - 140x$

$= -140x$

$F = ma$

$\Rightarrow -140x = 5a$

$\Rightarrow a = -28x$

This is SHM with $\omega = \sqrt{28} = 2\sqrt{7}$.

(ii) $|QO| = 2.35$

(iii) It starts when $|qp| = 3$

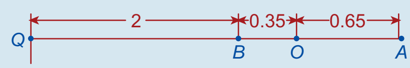
$\therefore 2.35 + x = 3$

$\Rightarrow x = 0.65$

The amplitude is, therefore, 0.65

(iv) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{7}}$

(v)



The journey from A to B can be divided into two parts: A to O and O to B.

A to O: $\frac{1}{4}$ of a full cycle $t = \frac{1}{4}\left(\frac{\pi}{\sqrt{7}}\right)$

$= 0.2968$

O to B: $x = A\sin\omega t$ (starts at centre)

$0.35 = 0.65\sin 2\sqrt{7}t$

$\Rightarrow \sin 2\sqrt{7}t = \frac{35}{65} = \frac{7}{13} = 0.5385$

$\Rightarrow 2\sqrt{7}t = \sin^{-1}(0.5385) = 0.5687$

$\Rightarrow t = 0.1075$

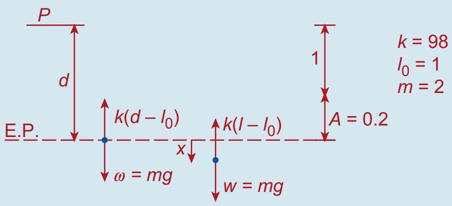
Total time $= 0.2968 + 0.1075$

$= 0.4043$

$= 0.404$ s

Exercise 13E

Q. 1.



(i) At Equilibrium Position (E.P.): $\uparrow = \downarrow$

so $k[d - l_0] = 2g$

$\Rightarrow 98[d - 1] = 19.6$

$\Rightarrow d = 1.2$ m

(ii) NZL: $\Sigma F = ma$

$\downarrow mg - k[d + x - l_0] = ma$

$\Rightarrow 19.6 - 98[0.2 + x] = 2a$

$\Rightarrow a = -49x \Rightarrow$ SHM with $\omega = 7$

(iii) $\Rightarrow T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{7}$ s

(iv) $v_{MAX} = \omega A \Rightarrow v_{MAX} = 7(0.2)$

$\Rightarrow v_{MAX} = 1.4$ m

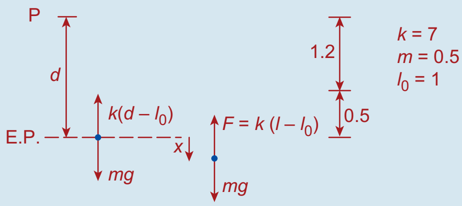
(v) Falls 0.15 metres $\Rightarrow x = 0.2 - 0.15$

$\Rightarrow x = 0.05$

We have $x = A\cos\omega t$, (Particle released from extrems position)

So $0.05 = 0.2\cos 7t \Rightarrow t = 0.188$ s

Q. 2.



At E.P. $\uparrow = \downarrow$

$\Rightarrow mg = k(d - l_0)$ ... 1

$\Rightarrow 4.9 = 7(d - 1)$

$\Rightarrow d = 1.7, A = 1.7 - 1.2 = 0.5$

NZL: $\Sigma F = ma$

$\downarrow mg - k[l - l_0] = ma$

$mg - k[d + x - l_0] = ma$

From 1 $\cancel{k(d - lo)} - \cancel{k(d - lo)} - kx = ma$

(i) $\Rightarrow a = -\frac{7}{0.5}x$

$\Rightarrow a = -14x$

$\Rightarrow$ SHM

$\Rightarrow \omega = \sqrt{14}$

(ii) $a_{MAX} = \omega^2 A = 14(0.5)$

$\Rightarrow a_{MAX} = 7$ m/s^2

$F_{MAX} = ma_{MAX} = 0.5(7)$

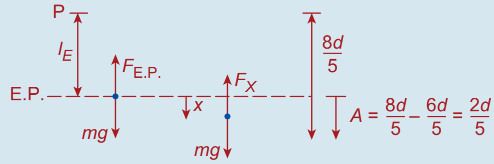
$\Rightarrow F_{MAX} = 3.5$ N

(iii) When 2 metres below P, $x = -0.3$ with $x = A\cos\omega t$

so $-0.3 = 0.5\cos\sqrt{14}t$

$\Rightarrow t = 0.59$ s

Q. 3.



(i) At E.P., $F_{E.P.} = mg$

$\Rightarrow k[l_E - l_0] = mg$ ... 1

$\Rightarrow \frac{\cancel{49}^{5}\cancel{m}}{d}[l_E - d] = \cancel{mg}$

$\Rightarrow l_E = \frac{6d}{5}$

At x, NZL: $\Sigma F = ma$

$\downarrow mg - Fx = ma$

$\Rightarrow mg - k[l_E + x - l_0] = ma$

$\Rightarrow mg - mg - kx = ma$ From 1

$\Rightarrow a = -\frac{k}{m}x$

$\Rightarrow a = -\frac{49}{d}x$

$\Rightarrow$ SHM

$\omega = \frac{7}{\sqrt{d}}$

(ii) String becomes slack: $x = -\frac{d}{5}$

We have $x = A\cos\omega t$

$\Rightarrow -\frac{d}{5} = \frac{2d}{5}\cos\frac{7}{\sqrt{d}}t$

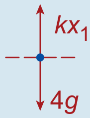
$\Rightarrow \cos\frac{7t}{\sqrt{d}} = -\frac{1}{2}$

$\Rightarrow \frac{7t}{\sqrt{d}} = \frac{2\pi}{3}$

$\Rightarrow t = \frac{2\pi\sqrt{d}}{21}$

Note: Hooke's Law: $F = kx$, x = extension.

Q. 4. **For the 4 kg mass**

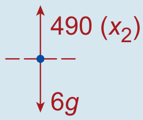


For equilibrium $\uparrow = \downarrow$

$\Rightarrow kx_1 = 4g$ where x_1 = extension

$\Rightarrow k = \dfrac{4g}{0.08} \Rightarrow k = 490$ N/m

For the combined (6 kg) mass



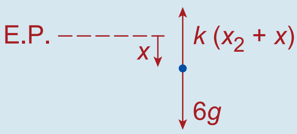
For equilibrium $\uparrow = \downarrow$

$\Rightarrow 490\,x_2 = 6g$ ①

$\Rightarrow x_2 = 0.12$

Now, the Amplitude, A is $x_2 - x_1 = 0.04$ m

Combined mass at displacement x from equilibrium:



NZL: $\Sigma F = ma$

$\Rightarrow 6g - k(x_2 + x) = 6a$

But $6g = 490x_2$ from ①

(i) $\Rightarrow 490x_2 - 490(x_2 + x) = 6a$

$\Rightarrow a = -\dfrac{245}{3}x$

So the motion is Simple Harmonic with

$\omega = \sqrt{\dfrac{245}{3}}$

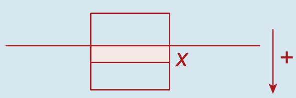
(ii) $\Rightarrow T = \dfrac{2\pi}{\omega}$

$= 2\pi\sqrt{\dfrac{3}{245}}$ **OR** $2\pi\sqrt{\dfrac{3}{25}g}$ s

(iii) $V_{MAX} = \omega A = \sqrt{\dfrac{245}{3}}\,(0.04)$

$= \dfrac{7\sqrt{15}}{75}$ **OR** $\dfrac{1}{5}\sqrt{\dfrac{9}{3}}$ m/s

Q. 5. (i) Mass $= V_p = h^3(1000\,s) = 1000\,h^3s$



When it is depressed a distance x, the extra bouyancy B' is given by (note the downward is positive)

$B' = -$weight of displaced liquid

$= -V_{Pg} = (h^2x)(1000)g = -1000\,h^2xg$

$F = ma$

$\Rightarrow -1000\,h^2xg = (1000\,h^3s)\,a$

$\Rightarrow a = -\dfrac{x}{hs}$

$\therefore$ It will perform SHM with $\omega = \sqrt{\dfrac{g}{hs}}$.

The periodic time $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{hs}{g}}$

(ii) In this case $B' = -(h^2x)\,(1000\,k)g$
$= 1000\,h^2xkg$

$F = ma$

$\Rightarrow -1000\,h^2xkg = 1000\,h^3sa$

$\Rightarrow a = -\dfrac{gk}{hs}x$

$\therefore$ SHM with $\omega = \sqrt{\dfrac{gk}{hs}}$

Periodic time $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{hs}{gk}}$.

Q. 6. (i) As before, 0.6 of its height will be submerged, i.e. (0.6)(80) = 48 cm.

(ii) Originally, its displacement from equilibrium is 2 cm = 0.02 m.

Therefore $A = 0.02 = \dfrac{1}{50}$.

Mass $= V_p = [(0.8)(0.5)(0.2)]\,(600) = 48$ kg

When it is displaced a distance x m below the water, the extra buoyancy B′ is given by:

B′ = weight of liquid displaced

$= V_{pg} = (0.5)(0.2)(x)(1000)(9.8) = -980x$

$F = ma$

$\Rightarrow -980x = 48a$

$\Rightarrow a = -\dfrac{980}{48}x = -\dfrac{245}{12}x$

This is SHM with $\omega = \sqrt{\dfrac{245}{12}}$

Maximum acceleration $= \omega^2 A$

$= \dfrac{245}{12} \times \dfrac{1}{50} = \dfrac{49}{120}$ m/s^2

Exercise 13F

Q. 1. $T = 2\pi\sqrt{\frac{l}{g}}$

$= 2\pi\sqrt{\frac{0.5}{9.8}}$

$= 1.419$ s

Q. 2. $T = 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow \frac{l}{g} = \frac{T^2}{4\pi^2}$

$\Rightarrow l = \frac{gT^2}{4\pi^2}$

$= \frac{9.8\,(3)^2}{4\pi^2}$

$\Rightarrow l = 2.234$ m

Q. 3. $T = 2\pi\sqrt{\frac{l}{g}}$

$= 2\pi\sqrt{\frac{4}{9.8}}$

$\Rightarrow T = 4.014$ s

Q. 4. $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = \frac{gT^2}{4\pi^2}$

$\Rightarrow l = \frac{9.8(5.5)^2}{4\pi^2} \Rightarrow l = 7.51 \text{ m} = 751 \text{ cm}$

Q. 5. $T_1 : T_2 = 2 : 1$

$\Rightarrow 2\pi\sqrt{\frac{l_1}{g}} : 2\pi\sqrt{\frac{l_2}{g}} = 2 : 1$

$\Rightarrow \sqrt{l_1} : \sqrt{l_2} = 2 : 1$

$\Rightarrow l_1 : l_2 = 4 : 1$

Q. 6. $l_1 : l_2 = 4 : 9$

$\Rightarrow \sqrt{l_1} : \sqrt{l_2} = 2 : 3$

$\Rightarrow 2\pi\sqrt{\frac{l_1}{g}} : 2\pi\sqrt{\frac{l_2}{g}} = 2 : 3$

$\Rightarrow T_1 : T_2 = 2 : 3$

Q. 7. Let g' be the acceleration due to gravity at the satellite.

$g' < g$ (by Newton's Law) $\Rightarrow \frac{l}{g'} > \frac{l}{g}$

$\Rightarrow 2\pi\sqrt{\frac{l}{g'}} > 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow$ its period of oscilliation is longer than normal

$\therefore$ it will go slow.

Q. 8. (i) Number of oscillations

$= 24 \times 60 \times 60 = 86{,}400$

(ii) The old periodic time $= 1$

$\therefore 2\pi\sqrt{\frac{l}{g}} = 1$

The new periodic time.

$T = 2\pi\sqrt{\frac{l(1.02)}{g}}$

$= \sqrt{1.02}\left(2\pi\sqrt{\frac{l}{g}}\right)$

$= \sqrt{1.02(1)}$

$= 1.01$ s

It will now perform $\frac{86{,}400}{1.01}$ oscillations in a day, i.e. 85,545 oscillations in a day

It performs $(86{,}400 - 85{,}545) = 855$ fewer.

Q. 9. Let T = the original time $= \frac{60}{30} = 2$ s

Let T' = the new time $= \frac{60}{31}$ s

$T' : T = \frac{60}{31} : 2$

$= 60 : 62$

$= 30 : 31$

$\therefore 2\pi\sqrt{\frac{l'}{g}} : 2\pi\sqrt{\frac{l}{g}} = 30 : 31$

$= l' : l = 30 : 31$

$\Rightarrow l' : l = 900 : 961$

$\Rightarrow \frac{l'}{l} = \frac{900}{961}$

$\Rightarrow l' = \frac{900}{961}l = 0.9365l$

$\therefore$ the % reduction

$= 100 - 93.65 = 6.35\%$

Q. 10. Let l', T' be the new length and periodic time respectively.

$l' : l = 2 : 1 \;\therefore\; T' : T = 2\pi\sqrt{\frac{l'}{g}} : 2\pi\sqrt{\frac{l}{g}}$

$= \sqrt{2} : \sqrt{1} = 1.414 : 1$

$\therefore T' = 1.414\,T$

There has been an increase of 41% in its periodic time.

Q. 11. Let g' = acceleration due to gravity on the moon. l', T' will be the length and periodic time of the pendulum on the moon.

$$T : T' = 2\pi\sqrt{\frac{l}{g}} : 2\pi\sqrt{\frac{l'}{g'}}$$

$$= 2\pi\sqrt{\frac{8}{l}} : 2\pi\sqrt{\frac{3}{\frac{1}{6}}}$$

$$= \sqrt{8} : \sqrt{18}$$

$$= \sqrt{4} : \sqrt{9}$$

$$= 2 : 3$$

Q. 12. (i) As in text.

(ii) $T = 2\pi\sqrt{\frac{k}{g}}$

$$\Rightarrow T^2 = 4\pi^2 \frac{k}{g}$$

$$\Rightarrow g = \frac{4\pi^2 k}{T^2} \quad \textbf{QED}$$

(iii) 39 cycles / min

$$= 1 \text{ cycle} / \frac{1}{39} \text{ min}$$

$$= 1 \text{ cycle} / \frac{60}{39} \text{ s}$$

$$\text{So, } g = \frac{4\pi^2(0.6)}{\left(\frac{60}{39}\right)^2}$$

$$\Rightarrow g = 10.0 \text{ m/s}^2$$

(iv) $\% \text{ Error} = \dfrac{(10.0 - 9.8)\,(100)}{9.8} = 2.04\%$

$= 2\%$ (to nearest percent)