

Q. 13.  $\frac{u^2 \sin^2 \alpha}{2g} = \frac{2u^2 \sin \alpha \cos \alpha}{g}$   
 $\Rightarrow \tan \alpha = 4$   
 $\Rightarrow \alpha = 76^\circ$

Q. 14. Let the speed of projection =  $u$  and the angle of projection =  $\alpha$

$$u_x = u \cos \alpha \quad u_y = u \sin \alpha$$

$$v_x = u \cos \alpha$$

$$v_y = u \sin \alpha - gt$$

$$s_x = ut \cos \alpha$$

$$s_y = ut \sin \alpha - \frac{1}{2}gt^2$$

Maximum height:  $s_y$  when  $v_y = 0$

$$v_y = 0$$

$$\Rightarrow u \sin \alpha - gt = 0$$

$$\Rightarrow t = \frac{u \sin \alpha}{g}$$

$$\Rightarrow \text{Maximum height} = u \left( \frac{u \sin \alpha}{g} \right) \sin \alpha - \frac{1}{2}g \left( \frac{u \sin \alpha}{g} \right)^2$$

$$\Rightarrow \text{Maximum height} = \frac{u^2 \sin^2 \alpha}{g} - \frac{u^2 \sin^2 \alpha}{2g}$$

$$\Rightarrow \text{Maximum height} = \frac{2u^2 \sin^2 \alpha - u^2 \sin^2 \alpha}{2g}$$

$$\Rightarrow \text{Maximum height} = \frac{u^2 \sin^2 \alpha}{2g}$$

Range:  $s_x$  when  $s_y = 0$

$$s_y = 0$$

$$\Rightarrow ut \sin \alpha - \frac{1}{2}gt^2 = 0$$

$$\Rightarrow 2ut \sin \alpha - gt^2 = 0$$

$$\Rightarrow t(2u \sin \alpha - gt) = 0$$

$$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \quad \underbrace{t = \frac{2u \sin \alpha}{g}}_{\text{Time of Flight}}$$

$$\Rightarrow \text{Range} = u \left( \frac{2u \sin \alpha}{g} \right) \cos \alpha$$

$$\Rightarrow \text{Range} = \frac{2u^2 \sin \alpha \cos \alpha}{g}$$

Maximum Height = 2(Range)

$$\Rightarrow \frac{u^2 \sin^2 \alpha}{2g} = 2 \left( \frac{2u^2 \sin \alpha \cos \alpha}{g} \right)$$

$$\Rightarrow \sin^2 \alpha = 8 \sin \alpha \cos \alpha$$

$$\Rightarrow \sin \alpha (\sin \alpha - 8 \cos \alpha) = 0$$

$$\Rightarrow \cancel{\sin \alpha} = 0 \quad \sin \alpha - 8 \cos \alpha = 0 \quad \dots \text{divide by } \cos \alpha$$

$$\Rightarrow \tan \alpha - 8 = 0$$

$$\Rightarrow \tan \alpha = 8$$

$$\Rightarrow \alpha = \tan^{-1} 8$$

$$= 83^\circ$$

OR

$$v_y^2 = u_y^2 + 2a_y s_y$$

$$\therefore 0 = (u \sin \alpha)^2 + 2(-g)H$$

$$\therefore H = \frac{u^2 \sin^2 \alpha}{2g}$$