Q. 20. Let the angle of projection $= \alpha$

$u_x = u\cos\alpha \qquad u_y = u\sin\alpha$

$v_x = u\cos\alpha$

$v_y = u\sin\alpha - gt$

$s_x = ut\cos\alpha$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2$

Greatest height: s_y when $v_y = 0$

$v_y = 0$

$\Rightarrow u\sin\alpha - gt = 0$

$\Rightarrow t = \frac{u\sin\alpha}{g}$

$\Rightarrow$ Greatest height $= u\left(\frac{u\sin\alpha}{g}\right)\sin\alpha - \frac{1}{2}g\left(\frac{u\sin\alpha}{g}\right)^2$

$\Rightarrow$ Greatest height $= \frac{u^2\sin^2\alpha}{g} - \frac{u^2\sin^2\alpha}{2g}$

$\Rightarrow$ Greatest height $= \frac{2u^2\sin^2\alpha - u^2\sin^2\alpha}{2g}$

$\Rightarrow$ Greatest height $= \frac{u^2\sin^2\alpha}{2g}$

Range: s_x when $s_y = 0$

$s_y = 0$

$\Rightarrow ut\sin\alpha - \frac{1}{2}gt^2 = 0$

$\Rightarrow 2ut\sin\alpha - gt^2 = 0$

$\Rightarrow t(2u\sin\alpha - gt) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{2u\sin\alpha}{g}}_{\text{Time of Flight}}$

$\Rightarrow$ Range $= u\left(\frac{2u\sin\alpha}{g}\right)\cos\alpha$

$\Rightarrow$ Range $= \frac{2u^2\sin\alpha\cos\alpha}{g}$

$\frac{\text{Greatest height}}{\text{Range}} = \frac{2}{7}$

$\Rightarrow 2(\text{Range}) = 7(\text{Greatest height})$

$\Rightarrow 2\left(\frac{2u^2\sin\alpha\cos\alpha}{g}\right) = 7\left(\frac{u^2\sin^2\alpha}{2g}\right)$ … multiply by $\frac{2g}{u^2}$

$\Rightarrow 8\sin\alpha\cos\alpha = 7\sin^2\alpha$

$\Rightarrow \sin\alpha(8\cos\alpha - 7\sin\alpha) = 0$

$\Rightarrow \cancel{\sin\alpha = 0} \qquad 8\cos\alpha - 7\sin\alpha = 0$ … divide by $\cos\alpha$

$\Rightarrow 8 - 7\tan\alpha = 0$

$\Rightarrow \tan\alpha = \frac{8}{7}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{8}{7}\right)$

$= 49°$