Q. 12. (i) $v_x = u\cos\alpha$

$v_y = u\sin\alpha - gt$

$s_x = ut\cos\alpha$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2$

Range: s_x when $s_y = 0$

$ut\sin\alpha - \frac{1}{2}gt^2 = 0$

$\Rightarrow 2ut\sin\alpha - gt^2 = 0$

$\Rightarrow t(2u\sin\alpha - gt) = 0$

$$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{2u\sin\alpha}{g}}_{\text{Time of Flight}}$$

$\Rightarrow \text{Range} = u\left[\frac{2u\sin\alpha}{g}\right]\cos\alpha = \frac{2u^2\sin\alpha\,\cos\alpha}{g}$

(ii) Passes through $3\vec{i} + \vec{j}$

$\Rightarrow s_x = 3$ when $s_y = 1$

$\Rightarrow ut\cos\alpha = 3$ when $ut\sin\alpha - \frac{1}{2}gt^2 = 1$

$\Rightarrow t = \frac{3}{u\cos\alpha} \quad \Rightarrow \quad u\left[\frac{3}{u\cos\alpha}\right]\sin\alpha - \frac{1}{2}g\left[\frac{9}{u^2\cos^2\alpha}\right] = 1$

$\Rightarrow 3\tan\alpha - \frac{9g}{2u^2}(1 + \tan^2\alpha) = 1$

$\Rightarrow 6u^2\tan\alpha - 9g(1 + \tan^2\alpha) = 2u^2$

$\Rightarrow 2u^2(3\tan\alpha - 1) = 9g(1 + \tan^2\alpha)$

$\Rightarrow 2u^2 = \frac{9g(1 + \tan^2\alpha)}{3\tan\alpha - 1}$

Passes through $\vec{i} + 3\vec{j} \quad \Rightarrow \quad s_x = 1$ when $s_y = 3$

$\Rightarrow ut\cos\alpha = 1$ when $ut\sin\alpha - \frac{1}{2}gt^2 = 3$

$\Rightarrow t = \frac{1}{u\cos\alpha} \quad \Rightarrow \quad u\left[\frac{1}{u\cos\alpha}\right]\sin\alpha - \frac{1}{2}g\left[\frac{1}{u^2\cos^2\alpha}\right] = 3$

$\Rightarrow \tan\alpha - \frac{g}{2u^2}(1 + \tan^2\alpha) = 3 \quad \ldots\ 2u^2 = \frac{9g(1 + \tan^2\alpha)}{3\tan\alpha - 1}$

$\Rightarrow \tan\alpha - g\left[\frac{3\tan\alpha - 1}{9g(1 + \tan^2\alpha)}\right](1 + \tan^2\alpha) = 3$

$\Rightarrow \tan\alpha - \frac{3\tan\alpha - 1}{9} = 3 \quad$ ... multiply by 3

$\Rightarrow 9\tan\alpha - 3\tan\alpha + 1 = 27$

$\Rightarrow 6\tan\alpha = 26$

$\Rightarrow \tan\alpha = \frac{13}{3} \quad$ ... as required