(iii) The time of the collision is given by

$$t = \frac{\text{relative distance}}{\text{relative speed}}$$
$$= \frac{40}{5}$$
$$= 8 \text{ hours later}$$

Q. 12. (i) $\vec{v_A} = 12\vec{i} + 4\vec{j}$

$\vec{v_B} = 4\vec{j}$

$\vec{v_{BA}} = \vec{v_B} - \vec{v_A} = 4\vec{j} - (12\vec{i} + 4\vec{j})$

$= -12\vec{i}$

(ii) The position of B relative to A is $\vec{r_{BA}} = 60\vec{i}$ km

$\Rightarrow \vec{v_{BA}} = -\frac{1}{5}(\vec{r_{BA}})$

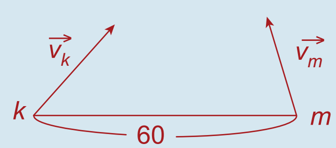
Since $\vec{v_{BA}} = -k(\vec{r_{BA}})$ where k is a positive constant, they must be on a collision course.

(iii) The time of the collision is given by

$$t = \frac{\text{relative distance}}{\text{relative speed}}$$
$$= \frac{60}{12} = 5 \text{ hours later}$$

Q. 13. (a) $\sqrt{t^2 + 9} = 5 \Rightarrow t = 4$

(b) (i)



$\vec{v_m} = -2\vec{i} + 3\vec{j}$

$\vec{v_k} = t\vec{i} + 3\vec{j}$

But $|t\vec{i} + 3\vec{j}| = 5$

$t = 4$, as before

$\therefore \vec{v_k} = 4\vec{i} + 3\vec{j}$

(ii) $\vec{v_{mk}} = (-2\vec{i} + 3\vec{j}) - (4\vec{i} + 3\vec{j})$

$= -6\vec{i}$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$
$$= \frac{60}{6} = 10 \text{ hours}$$

Q. 14. (i)

$\vec{v_P} = 5\vec{i} + 5\vec{j}$ m/s

(ii) $\vec{v_Q} = K\vec{i} + 5\vec{j}$

But $|K\vec{i} + 5\vec{j}| = 13$

$\Rightarrow K = -12$ (it must be negative so that Q approaches P)

$\therefore \vec{v_Q} = -12\vec{i} + 5\vec{j}$

(iii) $\vec{v_{QP}} = (-12\vec{i} + 5\vec{j}) - (5\vec{i} + 5\vec{j})$

$= -17\vec{j}$

$\therefore |\vec{v_{QP}}| = 17$ km/h

$$\text{Time} = \frac{3{,}400}{17} = 200 \text{ s}$$

Q. 15. (i) $\vec{v_K} = 12\vec{i} + 6\vec{j}$

For collision to occur, $\vec{j}$-velocities must match. Therefore, the minimum velocity at which H must travel in order for a collision to occur is $6\vec{j}$ m/s i.e. a minimum speed of 6 m/s due north.

(ii) Let $\vec{v_H} = a\vec{i} + 6\vec{j}$ m/s, $\quad a \in R$

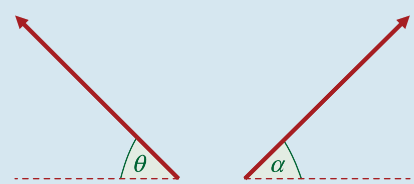
$\sqrt{a^2 + 6^2} = 10$ … we are told that the speed of H is 10 m/s

$\Rightarrow a^2 + 36 = 100$

$\Rightarrow a^2 = 64 \Rightarrow a = \pm 8$

$\Rightarrow$ Two possibilities for $\vec{v_H}$ are

$\vec{v_H} = -8\vec{i} + 6\vec{j}$ and $\vec{v_H} = 8\vec{i} + 6\vec{j}$



$\tan\theta = \tan\alpha = \frac{6}{8} = \frac{3}{4}$

$\Rightarrow \theta = \alpha = 36.87°$

$\Rightarrow$ Possible directions for H are 36.87° N of W and 36.87° N of E.

Using $\vec{v_H} = -8\vec{i} + 6\vec{j}$ gives

$\vec{v_{KH}} = \vec{v_K} - \vec{v_H} = 20\vec{i}$ m/s

$$\text{Time of Interception} = \frac{\text{relative distance}}{\text{relative speed}}$$
$$= \frac{3{,}000}{20}$$
$$= 150 \text{ s}$$

Using $\vec{v_H} = 8\vec{i} + 6\vec{j}$ gives

$\vec{v_{KH}} = \vec{v_K} - \vec{v_H} = 4\vec{i}$ m/s

$$\text{Time of Interception} = \frac{\text{relative distance}}{\text{relative speed}}$$
$$= \frac{3{,}000}{4}$$
$$= 750 \text{ s}$$