Q. 16. (i) $\overrightarrow{v_Y} = 10\vec{j}$ km/h

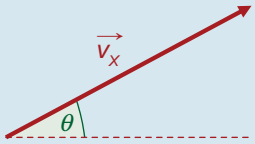
$\overrightarrow{v_X} = a\vec{i} + 10\vec{j}, a \in R,$

where $\sqrt{a^2 + 10^2} = 20$

$\Rightarrow a^2 + 100 = 400$

$\Rightarrow a = \sqrt{300} = 10\sqrt{3}$

$\Rightarrow \overrightarrow{v_X} = 10\sqrt{3}\vec{i} + 10\vec{j}$



$\tan\theta = \dfrac{10}{10\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta = \tan^{-1}\dfrac{1}{\sqrt{3}} = 30°$

$\Rightarrow$ Captain must steer in a direction 30° N of E.

(ii) $\overrightarrow{v_{XY}} = \overrightarrow{v_X} - \overrightarrow{v_Y}$

$= 10\sqrt{3}\vec{i} + 10\vec{j} - 10\vec{j} = 10\sqrt{3}\vec{i}$

Time to interception $= \dfrac{\text{relative distance}}{\text{relative speed}}$

$= \dfrac{40}{10\sqrt{3}}$

$\simeq 2.309$ hours

$\simeq 2$ hours 19 mins

Q. 17. (i) $\overrightarrow{r_{BA}} = \overrightarrow{r_B} - \overrightarrow{r_A}$

$= 37\vec{i} + 25\vec{j} - (2\vec{i} - 3\vec{j})$

$= 35\vec{i} + 28\vec{j}$

$\overrightarrow{v_{BA}} = \overrightarrow{v_B} - \overrightarrow{v_A}$

$= -2\vec{i} - 3\vec{j} - (3\vec{i} + \vec{j})$

$= -5\vec{i} - 4\vec{j}$

$\Rightarrow \overrightarrow{v_{BA}} = -\dfrac{1}{7}(\overrightarrow{r_{BA}})$

Since $\overrightarrow{v_{BA}} = -k(\overrightarrow{r_{BA}})$ where k is a positive constant, they must be on a collision course.

(ii) Time to collision $= \dfrac{\text{relative distance}}{\text{relative speed}}$

$= \dfrac{\sqrt{35^2 + 28^2}}{\sqrt{(-5)^2 + (-4)^2}}$

$= \dfrac{7\sqrt{41}}{\sqrt{41}} = 7$ hours

$\Rightarrow$ Collision occurs at 17.00 hours.

Q. 18. (i) $\overrightarrow{r_A} = -8\vec{i} + 4\vec{j}$

$\overrightarrow{r_B} = 24\vec{i} - 12\vec{j}$

$\overrightarrow{r_{BA}} = \overrightarrow{r_B} - \overrightarrow{r_A}$

$= 24\vec{i} - 12\vec{j} - (-8\vec{i} + 4j)$

$= 32\vec{i} - 16\vec{j}$

$\overrightarrow{v_A} = 3\vec{i} + \vec{j}$

$\overrightarrow{v_B} = \vec{i} + 2\vec{j}$

$\overrightarrow{v_{BA}} = \overrightarrow{v_B} - \overrightarrow{v_A} = \vec{i} + 2\vec{j} - (3\vec{i} + \vec{j})$

$= -2\vec{i} + \vec{j}$

$\overrightarrow{v_{BA}} = -\dfrac{1}{8}(\overrightarrow{r_{BA}})$

Since $\overrightarrow{v_{BA}} = -k(\overrightarrow{r_{BA}})$ where k is a positive constant, they must be on a collision course.

(ii) Time to collision $= \dfrac{\text{relative distance}}{\text{relative speed}}$

$= \dfrac{\sqrt{32^2 + (-16)^2}}{\sqrt{(-2)^2 + 1^2}}$

$= \dfrac{16\sqrt{5}}{\sqrt{5}} = 16$ hours

$\Rightarrow$ Collision will occur at 16.00 hours.

Q. 19. (i) $\overrightarrow{r_X} = 10\vec{i} - 4\vec{j}$

$\overrightarrow{r_Y} = 37\vec{i} + k\vec{j}$

$\overrightarrow{r_{YX}} = \overrightarrow{r_Y} - \overrightarrow{r_X} = 27\vec{i} + (k + 4)\vec{j}$

$\overrightarrow{v_X} = 3\vec{i} + \vec{j}$

$\overrightarrow{v_Y} = -\vec{j}$

$\overrightarrow{v_{YX}} = \overrightarrow{v_Y} - \overrightarrow{v_X} = -3\vec{i} - 2\vec{j}$

$\dfrac{27}{-3} = \dfrac{k + 4}{-2}$ …collision course $\Rightarrow \overrightarrow{v_{YX}}$ is a scalar multiple of $\overrightarrow{r_{YX}}$

$\Rightarrow 3k + 12 = 54$

$\Rightarrow 3k = 42$

$\Rightarrow k = 14$

(ii) $\overrightarrow{r_{YX}} = 27\vec{i} + 18\vec{j}$

$\overrightarrow{v_{YX}} = -3\vec{i} - 2\vec{j}$

Time to collision $= \dfrac{\text{relative distance}}{\text{relative speed}}$

$= \dfrac{\sqrt{27^2 + 18^2}}{\sqrt{(-3)^2 + (-2)^2}}$

$= \dfrac{9\sqrt{13}}{\sqrt{13}}$

$= 9$ hours

Collision occurs at 10.00 hours.