$$\vec{v_B} = 20\cos 45°\vec{i} - 20\sin 45°\vec{j}$$
$$= 20\left(\frac{1}{\sqrt{2}}\right)\vec{i} - 20\left(\frac{1}{\sqrt{2}}\right)\vec{j}$$
$$= 10\sqrt{2}\,\vec{i} - 10\sqrt{2}\,\vec{j}$$
$$\vec{v_{AB}} = \vec{v_A} - \vec{v_B}$$
$$= -2\sqrt{2}\,\vec{i} + 18\sqrt{2}\,\vec{j} \text{ km/h}$$

(ii) $\tan\theta = \dfrac{2\sqrt{2}}{18\sqrt{2}} = \dfrac{1}{9}$

$\Rightarrow \sin\theta = \dfrac{1}{\sqrt{82}}$

But, $\sin\theta = \dfrac{d}{10}$

$\Rightarrow \dfrac{d}{10} = \dfrac{1}{\sqrt{82}}$

$\Rightarrow d = \dfrac{10}{\sqrt{82}} = 1.104 \text{ km} = 1{,}104 \text{ m}$

(iii) Draw a circle of radius 2 km with its centre at B.

As long as the relative path, $\vec{v_{AB}}$, is within this circle, the ships will be in visual contact.

This will be for a relative distance of $2x$.

$x^2 + d^2 = 2^2 \quad \ldots \text{ but } d = \dfrac{10}{\sqrt{82}}$

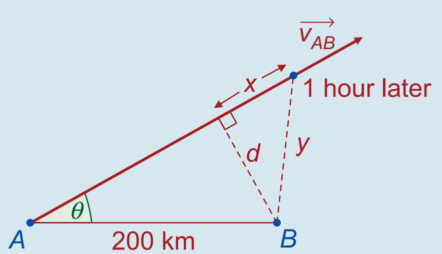
$\Rightarrow x = \sqrt{4 - \dfrac{100}{82}}$

$= 1.6675 \text{ km} = 1667.5 \text{ m}$

$\Rightarrow$ Ships will be in visual contact for a relative distance of $2(1.6675) = 3.335$ km

$$\text{Time} = \frac{\text{relative distance}}{\text{relative speed}}$$
$$= \frac{3.335}{\sqrt{(-2\sqrt{2})^2 + (18\sqrt{2})^2}}$$
$$= 0.13 \text{ h} = 7 \text{ min } 49 \text{ s}$$

Q. 10.



(i) $\vec{v_B} = 10\vec{j}$

$$\vec{v_A} = 20\cos 45°\vec{i} + 20\sin 45°\vec{j}$$
$$= 20\left(\frac{1}{\sqrt{2}}\right)\vec{i} + 20\left(\frac{1}{\sqrt{2}}\right)\vec{j}$$
$$= 10\sqrt{2}\,\vec{i} + 10\sqrt{2}\,\vec{j}$$
$$\vec{v_{AB}} = \vec{v_A} - \vec{v_B}$$
$$= 10\sqrt{2}\,\vec{i} + (10\sqrt{2} - 10)\vec{j}$$
$$= 14.14\vec{i} + 4.14\vec{j}$$

(ii) $\tan\theta = \dfrac{10(\sqrt{2} - 1)}{10\sqrt{2}} = \dfrac{\sqrt{2} - 1}{\sqrt{2}}$

$\Rightarrow \theta = 16.325°$

$\Rightarrow \sin\theta = 0.281$

But, $\sin\theta = \dfrac{d}{200}$

$\Rightarrow \dfrac{d}{200} = 0.281$

$\Rightarrow d = 56.2$ km

(iii) One hour later:

relative distance = relative speed × time

$\Rightarrow x = \sqrt{(10\sqrt{2})^2 + (10\sqrt{2} - 10)^2} \times (1)$

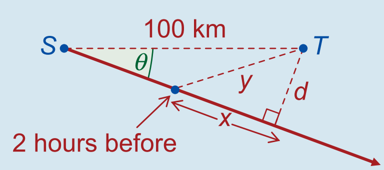
$= 14.736$ km

$y^2 = x^2 + d^2 \quad \ldots$ from diagram

$\Rightarrow y = \sqrt{14.736^2 + 56.2^2}$

$\Rightarrow y = 58.1$ km

Q. 11.



(i) $\vec{v_T} = -8\vec{j}$

$$\vec{v_S} = 20\cos 30°\vec{i} - 20\sin 30°\vec{j}$$
$$= 20\left(\frac{\sqrt{3}}{2}\right)\vec{i} - 20\left(\frac{1}{2}\right)\vec{j}$$
$$= 10\sqrt{3}\,\vec{i} - 10\vec{j}$$
$$\vec{v_{ST}} = \vec{v_S} - \vec{v_T}$$
$$= 10\sqrt{3}\,\vec{i} - 2\vec{j}$$
$$|\vec{v_{ST}}| = \sqrt{(10\sqrt{3})^2 + (-2)^2}$$
$$= 4\sqrt{19} = 17.44 \text{ km/h}$$

$$\tan\theta = \frac{2}{10\sqrt{3}}$$
$$= \frac{1}{5\sqrt{3}}$$

$\Rightarrow \theta = 6.59°$

$\Rightarrow 6.59°$ S of E