(iii) Draw a circle of radius 40 km with centre at P.

Ships will be within range of each other while the relative path, $\overrightarrow{v_{QP}}$, is inside this circle.

This will be for a relative distance of $2x$.

$x^2 + d^2 = 40^2$ … but $d = 11.24$

$\Rightarrow x = \sqrt{40^2 - 11.24^2} = 38.39$ km

$\Rightarrow$ Ships will be within range of each other for a relative distance of $2(38.39) = 76.78$ km.

$$\text{Time} = \frac{\text{relative distance}}{\text{relative speed}}$$
$$= \frac{76.78}{35.76}$$
$$= 2.15 \text{ hours}$$
$$= 2 \text{ hours } 9 \text{ mins}$$

From the diagram,

$(x+y)^2 + d^2 = 75^2$ … but $d = 11.24$

$\Rightarrow x + y = \sqrt{75^2 - 11.24^2}$

$= 74.15$ … but $x = 38.39$

$\Rightarrow y = 74.15 - 38.39$

$= 35.76$

Time before coming into range:

$$\frac{\text{relative distance}}{\text{relative speed}} = \frac{35.76}{35.76} = 1 \text{ hour}$$

$\Rightarrow$ Ships will come into range at 13.00 hours.

Ships stay within range for 2 hours and 9 minutes.

$\Rightarrow$ Ships will lose sight of each other at 15.09 hours.

Exercise 4C

Q. 1. (i) $\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{60}{12} = 5$ s

(ii) Distance travelled by Q = speed × time = $5 \times 5 = 25$ m

$\Rightarrow$ Distance from $O = 51 - 25 = 26$ m

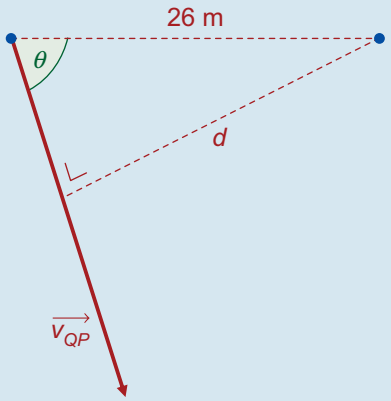
(iii) $\overrightarrow{v_P} = 12\vec{j}$

$\overrightarrow{v_Q} = 5\vec{i}$

$\overrightarrow{v_{QP}} = \overrightarrow{v_Q} - \overrightarrow{v_P}$

$= 5\vec{i} - 12\vec{j}$

(iv)



$\tan\theta = \frac{12}{5}$

$\Rightarrow \sin\theta = \frac{12}{13}$

But, $\sin\theta = \frac{d}{26}$

$\Rightarrow \frac{d}{26} = \frac{12}{13}$

$\Rightarrow d = 24$ m

Q. 2. (i) $\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{20}{8} = 2.5$ s

(ii) Distance travelled by M = speed × time = $6 \times 2.5 = 15$ m

$\Rightarrow$ Distance from $O = 65 - 15 = 50$ m

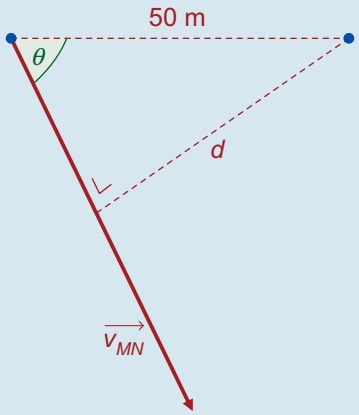
(iii) $\overrightarrow{v_M} = 6\vec{i}$

$\overrightarrow{v_N} = 8\vec{j}$

$\overrightarrow{v_{MN}} = \overrightarrow{v_M} - \overrightarrow{v_N}$

$= 6\vec{i} - 8\vec{j}$ m/s

(iv)



$\tan\theta = \frac{8}{6} = \frac{4}{3}$

$\Rightarrow \sin\theta = \frac{4}{5}$

But, $\sin\theta = \frac{d}{50}$

$\Rightarrow \frac{d}{50} = \frac{4}{5}$

$\Rightarrow d = 40$ m