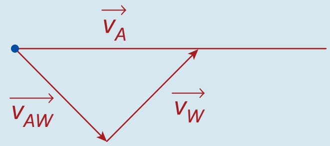
Q. 23. (i)



$$\overrightarrow{v_W} = \frac{v}{\sqrt{2}}\vec{i} + \frac{v}{\sqrt{2}}\vec{j}$$

$$\overrightarrow{v_{AW}} = x\cos\alpha\vec{i} - x\sin\alpha\vec{j}$$

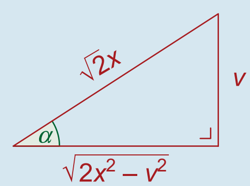
$$\therefore \overrightarrow{v_A} = \left(\frac{v}{\sqrt{2}} + x\cos\alpha\right)\vec{i} + \left(\frac{v}{\sqrt{2}} - x\sin\alpha\right)\vec{j}$$

j-component $= 0$

$$\Rightarrow \frac{\vec{v}}{2} - x\sin\alpha = 0$$

$$\Rightarrow \sin\alpha = \frac{v}{\sqrt{2}}x$$

$$\therefore \cos\alpha = \frac{\sqrt{2x^2 - v^2}}{\sqrt{2}x}$$



$$\therefore \overrightarrow{v_A} = \left(\frac{v}{2} + \frac{x\sqrt{2x^2 - v^2}}{\sqrt{2}x}\right)\vec{i}$$

$$= \left(\frac{v}{\sqrt{2}} + \frac{\sqrt{2x^2 - v^2}}{\sqrt{2}}\right)\vec{i}$$

$$\therefore \left|\overrightarrow{v_A}\right| = \frac{v + \sqrt{2x^2 - v^2}}{\sqrt{2}} = U_1$$

Similarily $U_2 = \dfrac{\sqrt{2x^2 - v^2} - v}{\sqrt{2}}$

$$\therefore U_1 - U_2 = \frac{2v}{\sqrt{2}} = \sqrt{2}v$$ **QED**

(ii) $$U_1 U_2 = \left(\frac{\sqrt{2x^2 - v^2} + v}{\sqrt{2}}\right)\left(\frac{\sqrt{2x^2 - v^2} - v}{\sqrt{2}}\right)$$

$$= \frac{2x^2 - 2v^2}{2}$$

$$= x^2 - v^2$$ **QED**

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{d}{\frac{\sqrt{2x^2 - v^2} + v}{\sqrt{2}}} + \frac{d}{\frac{(\sqrt{2x^2 - v^2} - v)}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}d}{\sqrt{2x^2 - v^2} + v} + \frac{\sqrt{2}d}{\sqrt{2x^2 - v^2} - v}$$

$$= \frac{\sqrt{2}d\,(\sqrt{2x^2 - v^2} - v) + \sqrt{2}d\,\sqrt{2x^2 - v^2} + v}{(\sqrt{2x^2 - v^2} + v)(\sqrt{2x^2 - v^2} - v)}$$

$$= \frac{2\sqrt{4x^2 - 2v^2}d}{2x^2 - 2v^2}$$

$$= \frac{\sqrt{4x^2 - 2v^2}\,d}{x^2 - v^2}$$