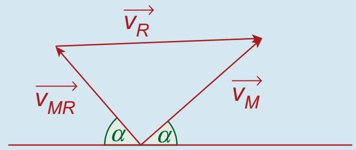
Q. 24.



$\overrightarrow{v_{MR}} = -5\cos\alpha\vec{i} + 5\sin\alpha\vec{j}$

$\overrightarrow{v_R} = 13\vec{i}$

$\overrightarrow{v_M} = (13 - 5\cos\alpha)\vec{i} + 5\sin\alpha\vec{j}$

$$\tan\theta = \frac{j\text{-component}}{i\text{-component}} = \frac{5\sin\alpha}{13 - 5\cos\alpha}$$

$$\frac{d(\tan\theta)}{d\alpha} = \frac{(13 - 5\cos\alpha)(5\cos\alpha) - 5\sin\alpha\,(5\sin\alpha)}{(13 - 5\cos\alpha)^2} = 0$$

$\Rightarrow 65\cos\alpha - 25\cos^2\alpha - 25\sin^2\alpha = 0$

$\Rightarrow 65\cos\alpha - 25(\cos^2\alpha + \sin^2\alpha) = 0$

$\Rightarrow 65\cos\alpha - 25 = 0$

$\Rightarrow \cos\alpha = \frac{5}{13}$

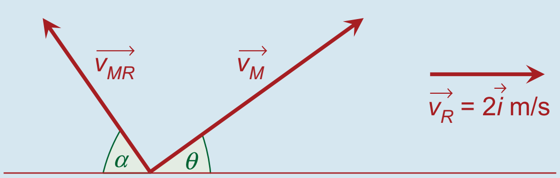
The shortest path is where θ is a maximum and therefore where $\tan\theta$ is a maximum, since $\tan\theta$ is an increasing function in θ. That is to say that the shortest path is where $\cos\alpha = \frac{5}{13}$, and hence $\sin\alpha = \frac{12}{13}$

In this case $\overrightarrow{v_M} = \left(13 - 5\left(\frac{5}{13}\right)\right)\vec{i} + 5\left(\frac{12}{13}\right)\vec{j}$

$= \frac{144}{13}\vec{i} + \frac{60}{13}\vec{j}$

Crossing time $= \frac{60}{\frac{60}{13}} = 13$ s

Q. 25.



$\overrightarrow{v_{MR}} = -\cos\alpha\vec{i} + \sin\alpha\vec{j}$

$\overrightarrow{v_R} = 2\vec{i}$

$\overrightarrow{v_{MR}} = \overrightarrow{v_M} - \overrightarrow{v_R}$

$\Rightarrow \overrightarrow{v_M} = \overrightarrow{v_{MR}} + \overrightarrow{v_R}$

$= (2 - \cos\alpha)\vec{i} + \sin\alpha\vec{j}$

$\Rightarrow \tan\theta = \frac{\sin\alpha}{2 - \cos\alpha}$

$\tan\theta$ will have a maximum value when $\frac{d}{d\alpha}(\tan\theta) = 0$

$$\frac{d}{d\alpha}(\tan\theta) = \frac{(2 - \cos\alpha)(\cos\alpha) - (\sin\alpha)(\sin\alpha)}{(2 - \cos\alpha)^2}$$ … using the Quotient Rule