

$$\Rightarrow \frac{d}{d\alpha}(\tan \theta) = \frac{2 \cos \alpha - \cos^2 \alpha - \sin^2 \alpha}{(2 - \cos \alpha)^2}$$

$$\Rightarrow \frac{d}{d\alpha}(\tan \theta) = \frac{2 \cos \alpha - (\cos^2 \alpha + \sin^2 \alpha)}{(2 - \cos \alpha)^2} \quad \dots \cos^2 \alpha + \sin^2 \alpha = 1$$

$$\Rightarrow \frac{d}{d\alpha}(\tan \theta) = \frac{2 \cos \alpha - 1}{(2 - \cos \alpha)^2}$$

Putting  $\frac{d}{d\alpha}(\tan \theta) = 0$  gives

$$\frac{2 \cos \alpha - 1}{(2 - \cos \alpha)^2} = 0$$

$$\Rightarrow 2 \cos \alpha - 1 = 0$$

$$\Rightarrow \cos \alpha = \frac{1}{2}$$

$$\Rightarrow \alpha = 60^\circ$$

Shortest path will occur when  $\alpha = 60^\circ$

$$\Rightarrow \vec{v}_M = (2 - \cos 60^\circ)\vec{i} + \sin 60^\circ\vec{j}$$

$$= \left(2 - \frac{1}{2}\right)\vec{i} + \frac{\sqrt{3}}{2}\vec{j} = \frac{3}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j}$$

$$\text{Time across} = \frac{\text{distance across}}{\text{speed across}}$$

$$= \frac{36}{\frac{\sqrt{3}}{2}}$$

$$= \frac{72}{\sqrt{3}}$$

$$= 24\sqrt{3} \text{ s}$$

**Q. 26.** (a)  $\cos A = \sqrt{1 - \sin^2 A}$

(b)  $\vec{v}_{BC} = 5\vec{i} - 2\vec{j}$

$$\vec{v}_C = 5 \cos \alpha \vec{i} + 5 \sin \alpha \vec{j}$$

$$\vec{v}_B = (5 + 5 \cos \alpha)\vec{i} + (-2 + 5 \sin \alpha)\vec{j}$$

This is in a N.E. direction

$$\therefore \frac{-2 + 5 \sin \alpha}{5 + 5 \cos \alpha} = \tan 45^\circ = 1$$

$$\Rightarrow -2 + 5 \sin \alpha = 5 + 5 \cos \alpha$$

$$\Rightarrow -2 + 5 \sin \alpha = 5 + 5 \sqrt{1 - \sin^2 \alpha}$$

$$\Rightarrow -7 + 5 \sin \alpha = 5\sqrt{1 - \sin^2 \alpha}$$

$$\Rightarrow 49 - 70 \sin \alpha + 25 \sin^2 \alpha = 25(1 - \sin^2 \alpha)$$

$$\Rightarrow 50 \sin^2 \alpha - 70 \sin \alpha + 24 = 0$$

$$\Rightarrow 25 \sin^2 \alpha - 35 \sin \alpha + 12 = 0$$

$$\Rightarrow (5 \sin \alpha - 3)(5 \sin \alpha - 4) = 0$$

$$\Rightarrow \sin \alpha = \frac{3}{5} \quad \text{OR} \quad \sin \alpha = \frac{4}{5}$$

$$\Rightarrow \cos \alpha = \pm \frac{4}{5} \quad \text{OR} \quad \cos \alpha = \pm \frac{3}{5}$$

