

**Q. 15.** Before (Mass) After

$$\begin{array}{ccc} u_1 \vec{i} & 3 & v_1 \vec{i} \\ u_2 \vec{j} & 4 & q \vec{j} \end{array}$$

$$\begin{aligned} 1: 3u_1 + 4u_2 &= 3v_1 + 4q \\ \Rightarrow q &= \frac{1}{4}(3u_1 + 4u_2 - 3v_1) \end{aligned}$$

$$2: \frac{v_1 - q}{u_1 - u_2} = -e$$

$$\Rightarrow v_1 - q = -eu_1 + eu_2$$

$$\text{But } q = \frac{1}{4}(3u_1 + 4u_2 - 3v_1)$$

$$\therefore v_1 - \frac{1}{4}(3u_1 + 4u_2 - 3v_1) = -eu_1 + eu_2$$

$$\Rightarrow 4v_1 - 3u_1 - 4u_2 + 3v_1 = -4eu_1 + 4eu_2$$

$$\Rightarrow 7v_1 = u_1(3 - 4e) + 4u_2(1 + e) \quad \text{QED}$$

$$I = 3v_1 - 3u_1. \text{ But } v_1 = \frac{1}{7}(u_1(3 - 4e) + 4u_2(1 + e))$$

$$\begin{aligned} \therefore I &= \frac{3}{7}(u_1(3 - 4e) + 4u_2(1 + e)) - 3u_1 \\ &= \frac{3}{7}(3u_1 - 4eu_1 + 4u_2 + 4eu_2 - 7u_1) \\ &= \frac{3}{7}(-4u_1 - 4eu_1 + 4u_2 + 4eu_2) \\ &= \frac{12}{7}(-u_1 - eu_1 + u_2 + eu_2) \\ &= \frac{12}{7}(-u_1(1 + e) + u_2(1 + e)) \\ &= \frac{12}{7}(1 + e)(u_2 - u_1) \quad \text{QED} \end{aligned}$$

**Q. 16.** (i) Before (Mass) After

$$P: \quad 4u \quad 3m \quad 2u$$

$$Q: \quad 2u \quad 5m \quad q$$

$$3m(4u) + 5m(2u) = 3m(2u) + 5m(q)$$

... divide by  $m$

$$\Rightarrow 12u + 10u = 6u + 5q$$

$$\Rightarrow 5q = 16u$$

$$\Rightarrow q = \frac{16u}{5}$$

$$\frac{2u - q}{4u - 2u} = -e$$

$$\Rightarrow \frac{2u - q}{2u} = -e$$

$$\Rightarrow 2u - q = -2eu$$

$$\Rightarrow q = 2u(1 + e) \quad \dots \text{ but } q = \frac{16u}{5}$$

$$\Rightarrow 2u(1 + e) = \frac{16u}{5} \quad \dots \text{ multiply by } \frac{5}{2u}$$

$$\Rightarrow 5(1 + e) = 8$$

$$\Rightarrow 1 + e = 1.6$$

$$\Rightarrow e = 0.6$$

(ii) Let the velocity of  $P$  after impact be  $p$ .

$$3m(4u) + 5m(2u) = 3m(p) + 5m(q)$$

$$\Rightarrow 3p + 5q = 22u \quad \dots \text{ Equation 1}$$

$$\frac{p - q}{4u - 2u} = -e$$

$$\Rightarrow p - q = -2eu \quad \dots \text{ multiply by } -3$$

$$\Rightarrow -3p + 3q = 6eu \quad \dots \text{ Equation 2}$$

Adding equations 1 and 2 we get

$$8q = 22u + 6eu$$

$$\Rightarrow 8q = 2u(11 + 3e)$$

$$\Rightarrow q = \frac{u}{4}(11 + 3e) \quad \dots \text{ minimum value occurs when } e = 0$$

$$\Rightarrow q = \frac{u}{4}(11) = 2.75u$$