Q. 20. (i)

Before	**(Mass)**	**After**
u	m	p
0	$3m$	q

$m(u) + 3m(0) = m(p) + 3m(q)$ … divide by m

$\Rightarrow p + 3q = u$ … **Equation 1**

$$\frac{p - q}{u - 0} = -e$$

$\Rightarrow -p + q = eu$ … **Equation 2**

Adding equations **1** and **2** we get

$4q = u(1 + e)$

$\Rightarrow q = \frac{u}{4}(1 + e)$ … speed of 2nd sphere after collision

$p = q - eu$ … from **Equation 2**

$\Rightarrow p = \frac{u}{4}(1 + e) - eu$

$\Rightarrow p = \frac{u + eu - 4eu}{4} = \frac{u - 3eu}{4}$

$\Rightarrow p = \frac{u}{4}(1 - 3e)$ … speed of 1st sphere after collision

(ii) $\text{K.E.}_{\text{before}} = \frac{1}{2}mu^2$

$\text{K.E.}_{\text{after}} = \frac{1}{2}mp^2 + \frac{1}{2}(3m)q^2$

$= \frac{m}{2}\left[\frac{u^2}{16}(1 - 6e + 9e^2)\right] + \frac{3m}{2}\left[\frac{u^2}{16}(1 + 2e + e^2)\right]$

$= \frac{mu^2}{32}[1 - 6e + 9e^2 + 3 + 6e + 3e^2]$

$= \frac{mu^2}{32}[12e^2 + 4]$

$= \frac{mu^2}{8}(3e^2 + 1)$

$\text{Loss} = \frac{mu^2}{2} - \frac{mu^2}{8}(3e^2 + 1)$

$= \frac{mu^2}{2}\left[1 - \frac{3e^2}{4} - \frac{1}{4}\right]$

$= \frac{mu^2}{2}\left[\frac{4 - 3e^2 - 1}{4}\right]$

$= \frac{mu^2}{8}(3 - 3e^2)$

$= \frac{3mu^2}{8}(1 - e^2)$

(iii) Let $e = \frac{1}{4}$

$\Rightarrow \text{Loss} = \frac{3mu^2}{8}\left(\frac{15}{16}\right) = \frac{45mu^2}{128}$

$\% \text{ Loss} = \frac{\frac{45mu^2}{128}}{\frac{mu^2}{2}} \times 100$

$= \frac{45}{64} \times 100 = 70\%$

(iv) Loss in K.E. is maximised when $e = 0$

$\Rightarrow \text{Loss} = \frac{3mu^2}{8}$

$\% \text{ Loss} = \frac{\frac{3mu^2}{8}}{\frac{mu^2}{2}} \times 100$

$= \frac{3}{4} \times 100$

$= 75\%$