Exercise 7C

Q. 1. (i)

Before	(Mass)	After
$4\vec{i} + 3\vec{j}$	M	$p\vec{i} + 3\vec{j}$
$\vec{i} + 2\vec{j}$	M	$q\vec{i} + 2\vec{j}$

$M(4) + M(1) = M(p) + M(q)$

$\Rightarrow p + q = 5$

$$\frac{p - q}{4 - 1} = -\frac{1}{3}$$

$\Rightarrow p - q = -1$

Solving these gives $p = 2, q = 3$.

The new velocities are $2\vec{i} + 3\vec{j}$ and $3\vec{i} + 2\vec{j}$

(ii) $\text{K.E.}_{\text{before}} = \frac{1}{2}(M)(4^2 + 3^2) + \frac{1}{2}(M)(1^2 + 2^2)$

$= 15M$ J

$\text{K.E.}_{\text{after}} = \frac{1}{2}M(2^2 + 3^2) + \frac{1}{2}M(3^2 + 2^2)$

$= 13M$ J

Loss $= 15M - 13M = 2M$ J

Q. 2.

Before	(Mass)	After
$3\vec{i} + 4\vec{j}$	2	$p\vec{i} + 4\vec{j}$
$-4\vec{i} + 3\vec{j}$	3	$q\vec{i} + 3\vec{j}$

$2(3) + 3(-4) = 2(p) + 3(q)$

$\Rightarrow 2p + 3q = -6$

$$\frac{p - q}{3 + 4} = -\frac{3}{7}$$

$\Rightarrow p - q = -3$

Solving these gives $p = -3, q = 0$

(i) Their velocities are $-3\vec{i} + 4\vec{j}$, $0\vec{i} + 3\vec{j}$

(ii) $\text{K.E.}_{\text{before}} =$

$\frac{1}{2}(2)(3^2 + 4^2) + \frac{1}{2}(3)((-4)^2 + 3^2)$

$= 62\frac{1}{2}$ J

$\text{K.E.}_{\text{after}} =$

$\frac{1}{2}(2)((-3)^2 + 4^2) + \frac{1}{2}(3)(0^2 + 3^2)$

$= 38\frac{1}{2}$ J

Loss $= 62\frac{1}{2} - 38\frac{1}{2}$

$= 24$ J

(iii) $\vec{I_1} = M\vec{v} - M\vec{u}$

$= 2(-3\vec{i} + 4\vec{j}) - 2(3\vec{i} + 4\vec{j})$

$= -12\vec{i}$ Ns

The magnitude of the impulse is 12 Ns.

Q. 3.

Before	(Mass)	After
$6\vec{i} + \vec{j}$	M	$0\vec{i} + \vec{j}$
$-2\vec{i} - 5\vec{j}$	$2M$	$q\vec{i} - 5\vec{j}$

$M(6) + 2M(-2) = M(0) + 2Mq$

$\Rightarrow q = 1$

(i) Its velocity is $\vec{i} - 5\vec{j}$.

(ii) $\frac{0 - 1}{6 + 2} = -e \Rightarrow e = \frac{1}{8}$

Q. 4.

Before	(Mass)	After
$5\vec{i} + 5\vec{j}$	$2M$	$p\vec{i} + 5\vec{j}$
$0\vec{i} + 0\vec{j}$	M	$q\vec{i} + 0\vec{j}$

$2M(5) + M(0) = 2Mp + Mq$

$\Rightarrow 2p + q = 10$

$$\frac{p - q}{5 - 0} = -\frac{1}{2}$$

$\Rightarrow 2p - 2q = -5$

Solving these gives $p = \frac{21}{2}, q = 5$

(i) Their velocities are $2\frac{1}{2}\vec{i} + 5\vec{j}$; $5\vec{i} + 0\vec{j}$

(ii) $\vec{I_1} = M\vec{v} - M\vec{u}$

$= 2M\left(2\frac{1}{2}\vec{i} + 5\vec{j}\right) - 2M(5\vec{i} + 5\vec{j})$

$= -5m\vec{i}$ Ns

$\vec{I_2} = M(5\vec{i} + 0\vec{j}) - M(0\vec{i} + 0\vec{j})$

$= 5M\vec{i}$ Ns

(iii) $\text{K.E.}_{\text{before}} =$

$\frac{1}{2}(2M)(5^2 + 5^2) + \frac{1}{2}(M)(0^2 + 0^2)$

$= 50M$ J

$\text{K.E.}_{\text{after}} =$

$\frac{1}{2}(2M)\left(\left(\frac{5}{2}\right)^2 + 5^2\right) + \frac{1}{2}(M)(5^2 + 0^2)$

$= 43\frac{3}{4}M$ J

Percentage Loss $= \frac{6\frac{1}{4}M}{50M} \times \frac{100}{1}$

$= 12\frac{1}{2}\%$