

(ii) **Whole rectangle:**

$$\begin{aligned} \text{Area} &= 90 \times 115 \\ &= 10,350 \end{aligned}$$

Centre of gravity is at (45, 75.5)

Larger Section:

$$\begin{aligned} \text{Area} &= 80 \times 70 \\ &= 5,600 \end{aligned}$$

Centre of gravity is at (45, 40)

Smaller Section:

$$\begin{aligned} \text{Area} &= 80 \times 30 \\ &= 2,400 \end{aligned}$$

Centre of gravity is at (45, 95)

Remainder:

$$\begin{aligned} \text{Area} &= 10,350 - 5,600 - 2,400 \\ &= 2,350 \end{aligned}$$

Centre of gravity is at (x, y)

$$\left. \begin{array}{l} 5,600 \text{ at } (45, 40) \\ 2,400 \text{ at } (45, 95) \\ 2,350 \text{ at } (x, y) \end{array} \right\} = \begin{array}{l} 10,350 \\ \text{at } (45, 57.5) \end{array}$$

Taking moments about the x-axis:

$$\begin{aligned} 5,600(40) + 2,400(95) + 2,350(y) \\ = 10,350(57.5) \end{aligned}$$

$$\Rightarrow y = 60.9 \text{ cm}$$

Q. 11. (i) $\text{Area } \Delta OPQ = \frac{1}{2}(\text{base})(\text{height})$
... take [OP] as the base

$$\begin{aligned} \Rightarrow \text{Area } \Delta OPQ &= \frac{1}{2}(12)(18) \\ &= 108 \text{ square units} \end{aligned}$$

$\text{Area } \Delta OQR = \frac{1}{2}(\text{base})(\text{height})$
... take [OR] as the base

$$\begin{aligned} \Rightarrow \text{Area } \Delta OQR &= \frac{1}{2}(30)(18) \\ &= 270 \text{ square units} \end{aligned}$$

(ii) Centre of Gravity of ΔOPQ

$$\begin{aligned} &= \left(\frac{0 + 0 + 18}{3}, \frac{0 + 12 + 18}{3} \right) \\ &= (6, 10) \end{aligned}$$

Centre of Gravity of ΔOQR

$$\begin{aligned} &= \left(\frac{0 + 18 + 30}{3}, \frac{0 + 18 + 0}{3} \right) \\ &= (16, 6) \end{aligned}$$

$$\left. \begin{array}{l} \text{(iii) } 108 \text{ at } (6, 10) \\ 270 \text{ at } (16, 6) \end{array} \right\} = 378 \text{ at } (x, y)$$

Taking moments around the y-axis:

$$\begin{aligned} (108)(6) + (270)(16) &= (378)(x) \\ \Rightarrow 378x &= 4,968 \\ \Rightarrow x &= 13.14 \end{aligned}$$

Taking moments around the x-axis:

$$\begin{aligned} (108)(10) + (270)(6) &= (378)(y) \\ \Rightarrow 378y &= 2,700 \\ \Rightarrow y &= 7.14 \end{aligned}$$

\Rightarrow Centre of Gravity of lamina is at (13.14, 7.41)

Q. 12. (i) $\text{Area } \Delta OPQ = \frac{1}{2} |x_1y_2 - x_2y_1|$
 $= \frac{1}{2} |(18)(6) - (-6)(18)|$
 $= 108$

$\text{Area } \Delta OQR = \frac{1}{2}(\text{base})(\text{height})$
... take [OR] as the base

$$\begin{aligned} \Rightarrow \text{Area } \Delta OQR &= \frac{1}{2}(36)(18) \\ &= 324 \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{Area } \Delta OQR : \text{Area } \Delta OPQ &= 324 : 108 \\ &= 3 : 1 \end{aligned}$$

(ii) Centre of Gravity of ΔOPQ

$$\begin{aligned} &= \left(\frac{0 - 6 + 18}{3}, \frac{0 + 6 + 18}{3} \right) \\ &= (4, 8) \end{aligned}$$

Centre of Gravity of ΔOQR

$$\begin{aligned} &= \left(\frac{0 + 18 + 36}{3}, \frac{0 + 18 + 0}{3} \right) \\ &= (18, 6) \end{aligned}$$

$$\left. \begin{array}{l} \text{(iii) } 1 \text{ at } (4, 8) \\ 3 \text{ at } (18, 6) \end{array} \right\} = 4 \text{ at } (x, y)$$

Taking moments around the y-axis:

$$\begin{aligned} (1)(4) + (3)(18) &= (4)(x) \\ \Rightarrow 4x &= 58 \\ \Rightarrow x &= 14.5 \end{aligned}$$

Taking moments around the x-axis:

$$\begin{aligned} (1)(8) + (3)(6) &= (4)(y) \\ 4y &= 26 \\ \Rightarrow y &= 6.5 \end{aligned}$$

\Rightarrow Centre of Gravity of lamina is at (14.5, 6.5)