Rod AC:

Taking moments around C we get:

$W(l \cos \beta) = X_1(2l \sin \beta) + Y_1 (2l \cos \beta)$

... divide by $l \cos \beta$

$\Rightarrow W = 2X_1 \tan \beta + 2Y_1$

$\Rightarrow W - 2Y_1 = 2X_1 \tan \beta$ ... Equation ②

$2W + 2Y_1 = 2X_1 \tan \beta$ ... Equation ① $(\times 2)$

Add: $3W = 4X_1, \tan \beta$

$\Rightarrow X_1 = \dfrac{3W}{4 \tan \beta}$ ... substitute into Equation ②

$\Rightarrow W - 2Y_1 = 2\left[\dfrac{3W}{4 \tan \beta}\right] \tan \beta$

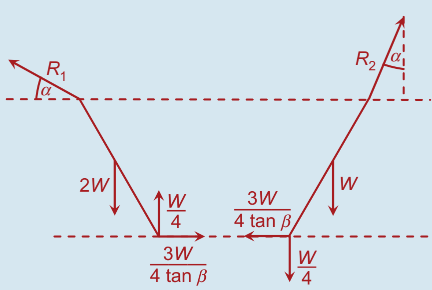
... multiply by 4

$\Rightarrow 4W - 8Y_1 = 6W$

$\Rightarrow 8Y_1 = -2W$

$\Rightarrow Y_1 = -\dfrac{W}{4}$ ... the minus sign indicates that the actual direction of Y_1 is opposite to the direction indicated in the diagram.

(ii) Looking again at the two rods separately, we have the following:



① $R_1 \sin \alpha + \dfrac{W}{4} = 2W$

$\Rightarrow R_1 \sin \alpha = \dfrac{7W}{4}$

② $R_1 \cos \alpha = \dfrac{3W}{4 \tan \beta}$

Dividing Equation ① by Equation ② gives:

$\tan \alpha = \left(\dfrac{7W}{4}\right)\left(\dfrac{4 \tan \beta}{3W}\right)$

$\Rightarrow \tan \alpha = \dfrac{7 \tan \beta}{3}$

③ $R_2 \sin \alpha = \dfrac{3W}{4 \tan \beta}$

④ $R_2 \cos \alpha = W + \dfrac{W}{4}$

$\Rightarrow R_2 \cos \alpha = \dfrac{5W}{4}$

Dividing Equation ③ by Equation ④ gives:

$\tan \alpha = \left(\dfrac{3W}{4 \tan \beta}\right)\left(\dfrac{4}{5W}\right)$

$\Rightarrow \tan \alpha = \dfrac{3}{5 \tan \beta}$

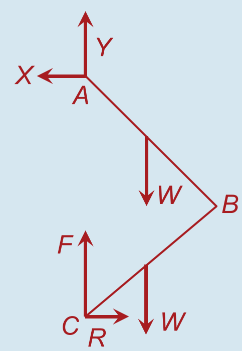
$\Rightarrow \dfrac{7 \tan \beta}{3} = \dfrac{3}{5 \tan \beta}$

$\Rightarrow 35 \tan^2 \beta = 9$

$\Rightarrow \tan^2 \beta = \dfrac{9}{35}$

$\Rightarrow \tan \beta = \dfrac{3}{\sqrt{35}}$

Q. 9. (i) Force diagram for the system including the friction force at C.



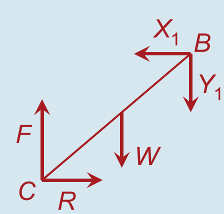
Take moments about A:

$2W\left(\dfrac{l}{2} \sin \phi\right) = R(2l \cos \phi)$

... divide by $l \cos \phi$

$\Rightarrow W \tan \phi = 2R \Rightarrow \quad R = \dfrac{1}{2} W \tan \phi$

Now, we look at the forces on the rod BC in isolation:



Taking moments around B we get:

$F(l \sin \phi) = R(l \cos \phi) + W\left(\dfrac{1}{2} \sin \phi\right)$

... divide by $l \cos \phi$

$\Rightarrow F \tan \phi = R + \dfrac{1}{2} W \tan \phi$

... but $R = \dfrac{1}{2} W \tan \phi$

$\Rightarrow F \tan \phi = \dfrac{1}{2} W \tan \phi + \dfrac{1}{2} W \tan \phi$

... divide by $\tan \phi$