Chapter 10 Exercise 10A

The answers given on page 261. No solutions necessary.

Exercise 10B

Q. 1. (i) $s_y = u \sin \alpha\, t - \frac{1}{2} g \cos \beta\, t^2 = 0$

$$\Rightarrow t = 0 \quad \textbf{OR} \quad t = \frac{2u \sin \alpha}{g \cos \beta}$$

$$= \text{time of flight}$$

(ii) $v_y = 0 \Rightarrow u \sin \alpha - g \cos \beta\, t = 0$

$$\Rightarrow t = \frac{u \sin \alpha}{g \cos \beta} = \frac{1}{2} \quad \text{(time of flight)}$$

Q. 2. $v_x = u \cos 45° - gt \sin 30° = \frac{u}{\sqrt{2}} - \frac{gt}{2}$

$v_y = u \sin 45° - gt \cos 30° = \frac{u}{\sqrt{2}} - \frac{gt\sqrt{3}}{2}$

$s_x = ut \cos 45° - \frac{1}{2} gt^2 \sin 30° = \frac{ut}{\sqrt{2}} - \frac{gt^2}{4}$

$s_y = ut \sin 45° - \frac{1}{2} gt^2 \cos 30° = \frac{ut}{\sqrt{2}} - \frac{gt^2\sqrt{3}}{4}$

Range: s_x when $s_y = 0$

$$\frac{ut}{\sqrt{2}} - \frac{gt^2\sqrt{3}}{4} = 0 \quad \ldots \text{ multiply by } 4\sqrt{2}$$

$$\Rightarrow 4ut - gt^2\sqrt{6} = 0$$

$$\Rightarrow t(4u - gt\sqrt{6}) = 0$$

$$\Rightarrow \underset{\text{Point of Projection}}{t = 0} \qquad \underbrace{t = \frac{4u}{g\sqrt{6}}}_{\text{Time of flight}} \quad \ldots \text{ substitute into } s_x$$

$$\Rightarrow \text{Range} = \frac{u}{\sqrt{2}}\left[\frac{4u}{g\sqrt{6}}\right] - \frac{g}{4}\left[\frac{16u^2}{6g^2}\right]$$

$$\Rightarrow \text{Range} = \frac{4u^2}{g\sqrt{12}} - \frac{2u^2}{3g} = \frac{2u^2}{g\sqrt{3}} - \frac{2u^2}{3g}$$

$$= \frac{2u^2\sqrt{3} - 2u^2}{3g}$$

$$\Rightarrow \text{Range} = \frac{2u^2(\sqrt{3} - 1)}{3g}$$

Maximum perpendicular height:
s_y when $v_y = 0$

$$\frac{u}{\sqrt{2}} - \frac{gt\sqrt{3}}{2} = 0 \quad \ldots \text{ multiply by } 2$$

$$\Rightarrow u\sqrt{2} - gt\sqrt{3} = 0$$

$$\Rightarrow t = \frac{u\sqrt{2}}{g\sqrt{3}} \quad \ldots \text{ substitute into } s_y$$

$\Rightarrow$ Maximum perpendicular height

$$= \frac{u}{\sqrt{2}}\left[\frac{u\sqrt{2}}{g\sqrt{3}}\right] - \frac{g\sqrt{3}}{4}\left[\frac{2u^2}{3g^2}\right]$$

$$= \frac{u^2}{g\sqrt{3}} - \frac{u^2}{2g\sqrt{3}}$$

$$= \frac{2u^2 - u^2}{2g\sqrt{3}}$$

$$= \frac{u^2}{2g\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

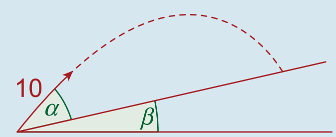
$$= \frac{\sqrt{3}u^2}{6g}$$

Q. 3. $\cos \alpha = \frac{4}{5}$

$\sin \alpha = \frac{3}{5}$

$\cos \beta = \frac{12}{13}$

$\sin \beta = \frac{5}{13}$



$s_y = 0$

$$\Rightarrow 10 \sin \alpha\, t - \frac{1}{2} g \cos \beta\, t^2 = 0$$

$$\Rightarrow 6t - \frac{6}{13} gt^2 = 0$$

$$\Rightarrow t = 0 \quad \textbf{OR} \quad t = \frac{13}{g} = \text{time of flight}$$

At $t = \frac{13}{g}$, $s_x = 10 \cos \alpha\, t - \frac{1}{2} g \sin \beta\, t^2$

$$= 10\left(\frac{4}{5}\right)\left(\frac{13}{g}\right) - \frac{1}{2} g\left(\frac{5}{13}\right)\left(\frac{169}{g^2}\right)$$

$$= \frac{143}{2g} = R, \text{ the range}$$

$\frac{2}{5}$ths of the time of flight $= \frac{2}{5} \times \frac{13}{g} = \frac{26}{5g}$

At $t = \frac{26}{5g}$, $s_x = 10\left(\frac{4}{5}\right)\left(\frac{26}{5g}\right) - \frac{1}{2} g\left(\frac{5}{13}\right)\left(\frac{676}{25g^2}\right)$

$$= \frac{208}{5g} - \frac{26}{5g} = \frac{182}{5g}$$

Now, $\frac{182}{5g} = \frac{28}{55}\left(\frac{143}{2g}\right) = \frac{28}{55} R$

Q. 4. $s_y = 10 \sin(\alpha - 30°)t - \frac{1}{2} \cos 30°\, gt^2$

$s_x = 10 \cos(\alpha - 30°)t - \frac{1}{2} \sin 30°\, gt^2$

(i) If $\alpha = 75°$,

$$s_y = 10 \sin 45°\, t - \frac{1}{2} \cos 30°\, gt^2 = 0$$

$$\Rightarrow \frac{10}{\sqrt{2}} t - \frac{\sqrt{3}}{4} gt^2 = 0$$

$$\Rightarrow t = 0 \quad \textbf{OR} \quad t = \frac{40}{\sqrt{6}g}$$