At $t = \frac{40}{\sqrt{6g}}$,

$$s_x = 10\left(\frac{1}{\sqrt{2}}\right)\left(\frac{40}{\sqrt{6g}}\right) - \frac{1}{2}\left(\frac{1}{2}\right)g\left(\frac{1{,}600}{6g}\right)$$

$$= \frac{200}{\sqrt{3g}} - \frac{200}{3g}$$

$$= \frac{200(\sqrt{3} - 1)}{3g}$$

(ii) If $\alpha = 60°$,

$$s_y = 10 \sin 30° \, t - \frac{1}{2} \cos 30° \, gt^2 = 0$$

$$\Rightarrow \frac{10}{\sqrt{2}} t - \frac{\sqrt{3}}{4} gt^2 = 0$$

$\Rightarrow t = 0$ **OR** $t = \frac{20}{\sqrt{3g}}$

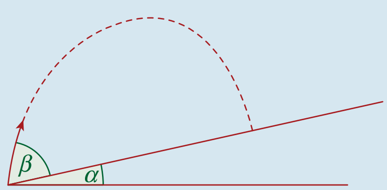
At $t = \frac{20}{\sqrt{3g}}$,

$$s_x = 10\left(\frac{\sqrt{3}}{2}\right)\left(\frac{20}{\sqrt{3g}}\right) - \frac{1}{2}\left(\frac{1}{2}\right)g\left(\frac{400}{3g}\right)$$

$$= \frac{10{,}000}{g^2} - \frac{100}{3g} = \frac{200}{3g}$$

Q. 5. $\tan^{-1} 2 = \cos^{-1} \frac{1}{\sqrt{5}} = \sin^{-1} \frac{2}{\sqrt{5}}$,

$$\vec{u} = 7\vec{i} + 14\vec{j}$$



Taking the horizontal as the x-axis.

$v_y = 0 \Rightarrow 14 - \frac{1}{2}gt = 0$

$\Rightarrow t = \frac{28}{g}$ seconds

= time to reach greatest height above the horizontal.

Now taking the line of greatest slope as the x-axis, $\vec{u} = u\cos(\beta - \alpha)\vec{i} + u\sin(\beta - \alpha)\vec{j}$

But, $\cos(\beta - \alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$

$$= \frac{1}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}} + \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{4}{5}$$

Therefore, $\sin(\beta - \alpha) = \frac{3}{5}$

$v_y = 0 \Rightarrow u\sin(\beta - \alpha) - \frac{1}{2}g\cos\alpha \, t = 0$

$$\Rightarrow 7\sqrt{5}\left(\frac{3}{5}\right) - \frac{1}{2}g\left(\frac{2}{\sqrt{5}}\right)t = 0$$

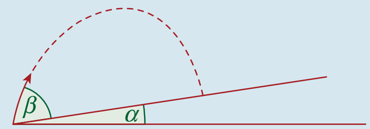
$$\Rightarrow \frac{21\sqrt{5}}{5} - \frac{\sqrt{5}}{5} gt = 0$$

$\Rightarrow t = \frac{21}{g}$ seconds

The ratio $t_1 : t_2 = \frac{28}{g} : \frac{21}{g} = 4 : 3$

Q. 6. $\sin\alpha = \frac{5}{13}$; $\cos\alpha = \frac{12}{13}$

$\sin\beta = \frac{4}{5}$; $\cos\beta = \frac{3}{5}$



$$s_x = u\cos\beta \, t - \frac{1}{2}g\sin\alpha \, t^2$$

$$= 10\left(\frac{3}{5}\right)t - \frac{1}{2}g\left(\frac{5}{13}\right)t^2$$

$$= 6t - \frac{5}{26}gt^2$$

$$s_y = u\sin\beta \, t - \frac{1}{2}g\cos\alpha \, t^2$$

$$= 10\left(\frac{4}{5}\right)t - \frac{1}{2}g\left(\frac{12}{13}\right)t^2$$

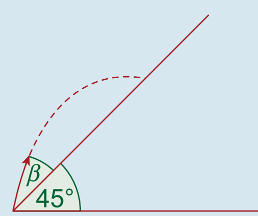
$$= 8t - \frac{6}{13}gt^2$$

$$s_x = 2s_y \Rightarrow 6t - \frac{5}{26}gt^2 = 2\left(8t - \frac{6}{13}gt^2\right)$$

$$\Rightarrow \frac{19}{26}gt^2 - 10t = 0$$

$\Rightarrow t = 0$ **OR** $t = \frac{260}{19g}$

Q. 7. $\sin\beta = \frac{1}{\sqrt{5}}$; $\cos\beta = \frac{2}{\sqrt{5}}$



$$s_y = 0 \Rightarrow u\sin\beta \, t - \frac{1}{2}g\cos 45° \, t^2 = 0$$

$$\Rightarrow 4t - \frac{1}{2\sqrt{2}}gt^2 = 0$$

$\Rightarrow t = 0$ **OR** $t = \frac{8\sqrt{2}}{g}$ = time of flight

$$v_x = u\cos\beta - g\sin 45° \, t = 8 - \frac{g}{\sqrt{2}}t$$

At $t = \frac{8\sqrt{2}}{g}$, $v_x = 8 - \frac{g}{\sqrt{2}}\left(\frac{8\sqrt{2}}{g}\right)$

$= 8 - 8 = 0$ m/s

$$v_y = u\sin\beta - g\cos 45° \, t = 4 - \frac{g}{\sqrt{2}} t$$

At $t = \frac{8\sqrt{2}}{g}$, $v_y = 4 - \frac{g}{\sqrt{2}}\left(\frac{8\sqrt{2}}{g}\right)$

$= 4 - 8 = -4$ m/s

$$v = \sqrt{v_x^{\,2} + v_y^{\,2}} = \sqrt{0 + 16} = 4 \text{ m/s}$$