Q. 8. (i) $s_y = 0 \Rightarrow \sqrt{26}\sin\alpha\, t - \frac{1}{2}g\left(\frac{5}{\sqrt{26}}\right)t^2 = 0$

$\Rightarrow t = 0$ **OR** $t = \frac{52\sin\alpha}{5g}$

(ii) $s_x = \sqrt{26}\cos\alpha\, t - \frac{1}{2}g\left(\frac{1}{\sqrt{26}}\right)t^2$

At $t = \frac{52\sin\alpha}{5g}$,

$$s_x = \sqrt{26}\cos\alpha\left(\frac{52\sin\alpha}{5g}\right) - \frac{1}{2}g\left(\frac{1}{\sqrt{26}}\right)\left(\frac{2{,}704\sin^2\alpha}{25g^2}\right)$$

$$= \frac{52\sqrt{26}\sin\alpha\cos\alpha}{5g} - \frac{52\sqrt{26}\sin^2\alpha}{25g}$$

$$= \frac{52\sqrt{26}\sin\alpha}{25g}(5\cos\alpha - \sin\alpha) = \text{the range}$$

(iii) If $\alpha = \beta$ then $\sin\alpha = \sin\beta = \frac{1}{\sqrt{26}}$; $\cos\alpha = \frac{5}{\sqrt{26}}$.

$$\therefore \text{The range} = \frac{52\sqrt{26}\left(\frac{1}{\sqrt{26}}\right)}{25g}\left(\frac{25}{\sqrt{26}} - \frac{1}{\sqrt{26}}\right)$$

$$= \frac{48\sqrt{26}}{25g}$$

(iv) If $\alpha = 2\beta$, then $\sin\alpha = \sin 2\beta$

$$= 2\sin\beta\cos\beta$$

$$= 2\left(\frac{1}{\sqrt{26}}\right)\left(\frac{5}{\sqrt{26}}\right)$$

$$= \frac{5}{13}$$

Also, $\cos\alpha = \cos 2\beta = \cos^2\beta - \sin^2\beta$

$$= \frac{25}{26} - \frac{1}{26}$$

$$= \frac{12}{13}$$

$$\therefore \text{The range} = \frac{52\sqrt{26}\left(\frac{5}{13}\right)}{25g}\left(\frac{60}{13} - \frac{5}{13}\right)$$

$$= \frac{44\sqrt{26}}{13g}$$

Q. 9. $v_x = u\cos\alpha + gt\sin\beta$

$v_y = u\sin\alpha - gt\cos\beta$

$s_x = ut\cos\alpha + \frac{1}{2}gt^2\sin\beta$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2\cos\beta$

Range: s_x when $s_y = 0$

$s_y = 0$

$\Rightarrow ut\sin\alpha - \frac{1}{2}gt^2\cos\beta = 0$ … multiply by 2

$\Rightarrow 2ut\sin\alpha - gt^2\cos\beta = 0$

$\Rightarrow t(2u\sin\alpha - gt\cos\beta) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \quad \underbrace{t = \frac{2u\sin\alpha}{g\cos\beta}}_{\text{Time of Flight}}$