Range $= s_x$ when $t = \dfrac{2u \sin \alpha}{g \cos \beta}$

$$\Rightarrow R = u\left[\frac{2u \sin \alpha}{g \cos \beta}\right]\cos \alpha + \frac{1}{2}g\left[\frac{4u^2 \sin^2\alpha}{g^2 \cos^2\beta}\right]\sin \beta$$

$$\Rightarrow R = \frac{2u^2 \sin \alpha \cos \alpha}{g \cos \beta} + \frac{2u^2 \sin^2 \alpha \sin \beta}{g \cos^2\beta}$$

$$\Rightarrow R = \frac{2u^2 \sin \alpha}{g \cos \beta}\left[\cos \alpha + \frac{\sin \alpha \sin \beta}{\cos \beta}\right]$$

$$\Rightarrow R = \frac{2u^2 \sin \alpha}{g \cos \beta}\left[\frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\cos \beta}\right]$$

$\ldots \cos A \cos B + \sin A \sin B = \cos(A - B)$

$$\Rightarrow R = \frac{2u^2}{g \cos^2 \beta}[\sin \alpha \cos(\alpha - \beta)]$$

$\ldots 2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$$\Rightarrow \sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\Rightarrow R = \frac{\not{2}u^2}{g \cos^2 \beta}\left\{\frac{1}{\not{2}}[\sin(2\alpha - \beta) + \sin \beta]\right\}$$

$$\Rightarrow R = \frac{u^2}{g \cos^2 \beta}[\sin(2\alpha - \beta) + \sin \beta]$$

... as required

In this equation, $u$, $g$ and $\beta$ are fixed. Only $\alpha$, the angle at which the projectile is fired, can vary.

The maximum value for the above expression is where $\sin(2\alpha - \beta) = 1$

$$\Rightarrow 2\alpha - \beta = 90°$$

$$\Rightarrow \alpha = \frac{\beta + 90°}{2}$$

Also, if $\sin(2\alpha - \beta) = 1$, then

$$R_{max} = \frac{u^2}{g \cos^2 \beta}[1 + \sin \beta]$$

$$\Rightarrow R_{max} = \frac{u^2}{g(1 - \sin^2 \beta)}[1 + \sin \beta]$$

$$\Rightarrow R_{max} = \frac{u^2}{g(1 - \sin \beta)\cancel{(1 + \sin \beta)}}\cancel{[1 + \sin \beta]}$$

$$\Rightarrow R_{max} = \frac{u^2}{g(1 - \sin \beta)}$$

(i) $\beta = 20° \Rightarrow \alpha = \dfrac{20° + 90°}{2} = 55°$

(ii) $\beta = 0° \Rightarrow \alpha = \dfrac{0° + 90°}{2} = 45°$

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**Q. 10.** $v_x = u \cos \theta - gt \sin \alpha$

$v_y = u \sin \theta - gt \cos \alpha$

$s_x = ut \cos \theta - \frac{1}{2}gt^2 \sin \alpha$

$s_y = ut \sin \theta - \frac{1}{2}gt^2 \cos \alpha$

(i) For time of flight, let $s_y = 0$

$$\Rightarrow ut \sin \theta - \frac{1}{2}gt^2 \cos \alpha = 0$$

$$\Rightarrow 2ut \sin \theta - gt^2 \cos \alpha = 0$$

$$\Rightarrow t(2u \sin \theta - gt \cos \alpha) = 0$$

$$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}} \qquad \underbrace{t = \frac{2u \sin \theta}{g \cos \alpha}}_{\text{Time of Flight}}$$

(ii) Range: $s_x$ when $t = \dfrac{2u \sin \theta}{g \cos \alpha}$

$$\Rightarrow \text{Range} = u\left[\frac{2u \sin \theta}{g \cos \alpha}\right]\cos \theta - \frac{1}{2}g\left[\frac{4u^2 \sin^2\theta}{g^2 \cos^2\alpha}\right]\sin \alpha$$

$$\Rightarrow \text{Range} = \frac{2u^2 \sin \theta \cos \theta}{g \cos \alpha} - \frac{2u^2 \sin^2\theta \sin \alpha}{g \cos^2\alpha}$$

$$\Rightarrow \text{Range} = \frac{2u^2 \sin \theta \cos \theta \cos \alpha - 2u^2 \sin^2\theta \sin \alpha}{g \cos^2\alpha}$$

$$\Rightarrow \text{Range} = \frac{2u^2 \sin \theta[\cos \theta \cos \alpha - \sin \theta \sin \alpha]}{g \cos^2\alpha}$$

$$\Rightarrow \text{Range} = \frac{2u^2 \sin \theta[\cos(\theta + \alpha)]}{g \cos^2\alpha}$$