$\Rightarrow$ Range $= \dfrac{u^2[\sin(2\theta + \alpha) + \sin(-\alpha)]}{g\cos^2\alpha}$

$\Rightarrow$ Range $= \dfrac{u^2[\sin(2\theta + \alpha) - \sin\alpha]}{g\cos^2\alpha}$

Everything is fixed except θ. Therefore, maximum range occurs when $\sin(2\theta + \alpha) = 1$

$\Rightarrow 2\theta + \alpha = \dfrac{\pi}{2}$

$\Rightarrow 2\theta = \dfrac{\pi}{2} - \alpha$

$\Rightarrow \theta = \dfrac{1}{2}\left(\dfrac{\pi}{2} - \alpha\right)$

Exercise 10C

Q. 1. $s_y = 0 \Rightarrow u\sin(\alpha - \beta)t - \frac{1}{2}g\cos\beta\, t^2 = 0$

$\Rightarrow t = 0$ **OR** $t = \dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}$

Now, $v_x = u\cos(\alpha - \beta) - g\sin\beta\, t$

But $v_x = 0$ at $t = \dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}$

$\therefore u\cos(\alpha - \beta) - g\sin\beta\left(\dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}\right) = 0$

Divide by $u\cos(\alpha - \beta)$.

$= 1 - 2\tan(\alpha - \beta)\tan\beta = 0$

$\Rightarrow 2\tan(\alpha - \beta)\tan\beta = 1$

If $(\alpha - \beta) = \frac{\pi}{4}$, then $\tan(\alpha - \beta) = 1$

and hence $2\tan\beta = 1$

$\Rightarrow \tan\beta = \dfrac{1}{2}$

$\Rightarrow \sin\beta = \dfrac{1}{\sqrt{5}}$ and $\cos\beta = \dfrac{2}{\sqrt{5}}$

In this case $t = \dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}$

$\Rightarrow t = \dfrac{2u\left(\frac{1}{\sqrt{2}}\right)}{g(2/\sqrt{5})} = \dfrac{\sqrt{5}u}{\sqrt{2}g}$

$s_x = u\cos(\alpha - \beta)t - \frac{1}{3}g\sin\beta\, t^2.$

At $t = \dfrac{\sqrt{5}u}{\sqrt{2}g}$,

$s_x = u\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{\sqrt{5}u}{\sqrt{2}g}\right) - \dfrac{1}{2}g\left(\dfrac{1}{\sqrt{5}}\right)\left(\dfrac{5u^2}{2g^2}\right)$

$= \dfrac{\sqrt{5}u^2}{2g} - \dfrac{\sqrt{5}u^2}{4g}$

$= \dfrac{\sqrt{5}u^2}{4g} =$ Range

Q. 2. $v_x = u\cos 30° - gt\sin 30° = \dfrac{u\sqrt{3}}{2} - \dfrac{gt}{2}$

$v_y = u\cos 30° - gt\cos 30° = \dfrac{u}{2} - \dfrac{gt\sqrt{3}}{2}$

$s_x = ut\cos 30° - \dfrac{1}{2}gt^2\sin 30° = \dfrac{ut\sqrt{3}}{2} - \dfrac{gt^2}{4}$

$s_y = ut\sin 30° - \dfrac{1}{2}gt^2\cos 30° = \dfrac{ut}{2} - \dfrac{gt^2\sqrt{3}}{4}$

For landing angle, need v_x and v_y when

$s_y = 0$

$\dfrac{ut}{2} - \dfrac{gt^2\sqrt{3}}{4} = 0$ … multiply by 4

$\Rightarrow 2ut - gt^2\sqrt{3} = 0$

$\Rightarrow t(2u - gt\sqrt{3}) = 0$

$\Rightarrow t = 0$ (Point of Projection) $\quad t = \dfrac{2u}{g\sqrt{3}}$ (Time of Flight)

$v_x = \dfrac{u\sqrt{3}}{2} - \dfrac{g}{2}\left[\dfrac{2u}{g\sqrt{3}}\right] = \dfrac{u\sqrt{3}}{2} - \dfrac{u}{\sqrt{3}}$

$= \dfrac{3u - 2u}{2\sqrt{3}}$

$= \dfrac{u}{2\sqrt{3}}$

$v_y = \dfrac{u}{2} - \dfrac{g\sqrt{3}}{2}\left[\dfrac{2u}{g\sqrt{3}}\right] = \dfrac{u}{2} - u = -\dfrac{u}{2}$

Let landing angle $= \theta$

$\tan\theta = -\dfrac{v_y}{v_x} = \dfrac{u}{2} \times \dfrac{2\sqrt{3}}{u} = \sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\sqrt{3} = 60°$