Q. 3. $s_y = 0 \Rightarrow u\sin(\alpha - 45^\circ)t - \frac{1}{2}g\cos 45^\circ\, t^2 = 0$

$\Rightarrow u\sin(\alpha - 45^\circ)t - \frac{1}{2\sqrt{2}}g\,t^2 = 0$

$\Rightarrow t = 0$ **OR** $t = \dfrac{2\sqrt{2}u\sin(\alpha - 45^\circ)}{g}$

$= \text{time of flight}$

$v_y = u\sin(\alpha - 45^\circ) - \frac{g}{\sqrt{2}}t$

At time of flight,

$v_y = u\sin(\alpha - 45^\circ) - \frac{g}{2}\left(\frac{2\sqrt{2}\,u\sin(\alpha - 45^\circ)}{g}\right)$

$= -u\sin(\alpha - 45^\circ)$

$v_x = u\cos(\alpha - 45^\circ) - \frac{g}{\sqrt{2}}t$

At time of flight,

$v_x = u\cos(\alpha - 45^\circ) - \frac{g}{\sqrt{2}}\left(\frac{2\sqrt{2}\,u\sin(\alpha - 45^\circ)}{g}\right)$

$= u\cos(\alpha - 45^\circ) - 2u\sin(\alpha - 45^\circ)$

$\tan\beta = \dfrac{-v_y}{v_x}$

$= \dfrac{u\sin(\alpha - 45^\circ)}{u\cos(\alpha - 45^\circ) - 2u\sin(\alpha - 45^\circ)}$

$= \dfrac{\tan(\alpha - 45^\circ)}{1 - 2\tan(\alpha - 45^\circ)}$

But $\tan(\alpha - 45^\circ) = \dfrac{\tan\alpha - \tan 45^\circ}{1 + \tan\alpha\tan 45^\circ}$

$= \dfrac{\tan\alpha - 1}{1 + \tan\alpha}$

$\therefore \tan\beta = \dfrac{\frac{\tan\alpha - 1}{1 + \tan\alpha}}{1 - 2\left(\frac{\tan\alpha - 1}{1 + \tan\alpha}\right)}$

$= \dfrac{\tan\alpha - 1}{1 + \tan\alpha - 2\tan\alpha + 2}$

$= \dfrac{\tan\alpha - 1}{3 - \tan\alpha}$

(i) If it lands horizontally, then $\beta = 45^\circ$

$\Rightarrow 1 = \dfrac{\tan\alpha - 1}{3 - \tan\alpha}$

$\Rightarrow 3 - \tan\alpha = \tan\alpha - 1$

$\Rightarrow \tan\alpha = 2$

(ii) If it lands vertically, then $\beta = 90^\circ$

$\Rightarrow \dfrac{\tan\alpha - 1}{3 - \tan\alpha} = \text{undefined}$

$\therefore 3 - \tan\alpha = 0$

$\therefore \tan\alpha = 3$

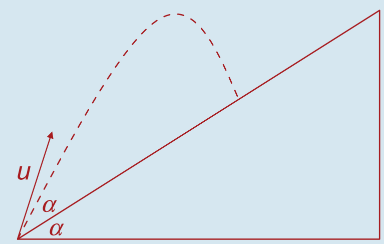
(iii) $\dfrac{\tan\alpha - 1}{3 - \tan\alpha} = \dfrac{1}{3}$

$\therefore 3\tan\alpha - 3 = 3 - \tan\alpha$

$\therefore 4\tan\alpha = 6$

$\therefore \tan\alpha = \dfrac{6}{4} = \dfrac{3}{2}$

Q. 4.



$\tan\alpha = \frac{1}{\sqrt{2}} \Rightarrow \cos\alpha = \frac{\sqrt{2}}{\sqrt{3}}$ and $\sin a = \frac{1}{\sqrt{3}}$

$v_x = u\cos\alpha - gt\sin\alpha = \frac{u\sqrt{2}}{\sqrt{3}} - \frac{gt}{\sqrt{3}}$

$v_y = u\sin\alpha - gt\cos\alpha = \frac{u}{\sqrt{3}} - \frac{gt\sqrt{2}}{\sqrt{3}}$

$s_x = ut\cos\alpha - \frac{1}{2}gt^2\sin\alpha = \frac{ut\sqrt{2}}{\sqrt{3}} - \frac{gt^2}{2\sqrt{3}}$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2\cos\alpha = \frac{ut}{\sqrt{3}} - \frac{gt^2\sqrt{2}}{2\sqrt{3}}$

(i) Lands at right angles if $v_x = 0$ when $s_y = 0$

$\Rightarrow \frac{ut}{\sqrt{3}} - \frac{gt^2\sqrt{2}}{2\sqrt{3}} = 0$

$\Rightarrow 2ut - gt^2\sqrt{2} = 0$

$\Rightarrow t\left(2u - gt\sqrt{2}\right) = 0$

$\Rightarrow \underset{\text{Point of Projection}}{t = 0} \qquad \underbrace{t = \frac{2u}{g\sqrt{2}} = \frac{u\sqrt{2}}{g}}_{\text{Time of Flight}}$

$v_x = \frac{u\sqrt{2}}{\sqrt{3}} - \frac{g}{\sqrt{3}}\left[\frac{u\sqrt{2}}{\cancel{g}}\right]$

$\Rightarrow v_x = \frac{u\sqrt{2}}{\sqrt{3}} - \frac{u\sqrt{2}}{\sqrt{3}} = 0$

$\Rightarrow$ Strikes plane at right angles

(ii) Range: Find s_x when $t = \frac{u\sqrt{2}}{g}$

$\Rightarrow \text{Range} = \frac{u\sqrt{2}}{\sqrt{3}}\left[\frac{u\sqrt{2}}{g}\right] - \frac{g}{2\sqrt{3}}\left[\frac{2u^2}{g^2}\right]$

$\Rightarrow \text{Range} = \frac{2u^2}{g\sqrt{3}} - \frac{u^2}{g\sqrt{3}} = \frac{u^2}{g\sqrt{3}}$

(iii) Total Energy in the beginning $= \frac{1}{2}mu^2$

Total Energy at point of landing

$= \frac{1}{2}mv^2 + mgh$

… where h = height above take-off position