$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}}$ $\quad \underbrace{t = \frac{u\sqrt{5}\sin\theta}{g}}_{\text{Time of Flight}}$

For landing angle, find $v_x$ and $v_y$

when $t = \frac{u\sqrt{5}\sin\theta}{g}$

$v_x = u\cos\theta - \frac{g}{\sqrt{5}}\left[\frac{u\sqrt{5}\sin\theta}{g}\right]$

$= u(\cos\theta - \sin\theta)$

$v_y = u\sin\theta - \frac{2g}{\sqrt{5}}\left[\frac{u\sqrt{5}\sin\theta}{g}\right]$

$= u\sin\theta - 2u\sin\theta = -u\sin\theta$

We know that the landing angle is $\alpha$ where $\tan\alpha = \frac{1}{2}$

We also know that $\tan\alpha = \frac{-v_y}{v_x}$

$\Rightarrow \frac{-v_y}{v_x} = \frac{1}{2}$

$\Rightarrow \frac{u\sin\theta}{u(\cos\theta - \sin\theta)} = \frac{1}{2}$

$\Rightarrow 2\sin\theta = \cos\theta - \sin\theta$

$\Rightarrow 3\sin\theta = \cos\theta$ ... divide by $\cos\theta$

$\Rightarrow 3\tan\theta = 1$

$\Rightarrow \tan\theta = \frac{1}{3}$ ... as required

$\Rightarrow$ Range: Find $s_x$ when $t = \frac{u\sqrt{5}\sin\theta}{g}$

$\Rightarrow$ Range $= ut\cos\theta - \frac{gt^2}{2\sqrt{5}}$

... $\tan\theta = \frac{1}{3} \Rightarrow \cos\theta = \frac{3}{\sqrt{10}}$ and $\sin\theta = \frac{1}{\sqrt{10}}$

$\Rightarrow$ Range

$$= u\left[\frac{u\sqrt{5}}{g}\left(\frac{1}{\sqrt{10}}\right)\right]\left[\frac{3}{\sqrt{10}}\right] - \frac{g}{2\sqrt{5}}\left[\frac{5u^2}{g^2}\left(\frac{1}{10}\right)\right]$$

$\Rightarrow$ Range $= \frac{3u^2\sqrt{5}}{10g} - \frac{u^2}{4g\sqrt{5}}$

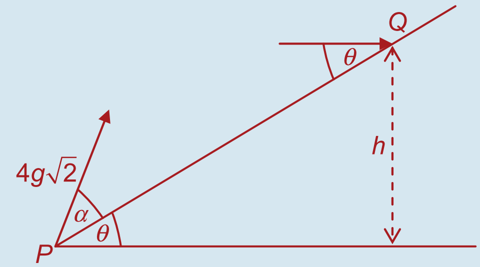
$\Rightarrow$ Range $= \frac{30u^2 - 5u^2}{20g\sqrt{5}}$

$= \frac{25u^2}{20g\sqrt{5}}$

$= \frac{5u^2}{4g\sqrt{5}}$

$= \frac{u^2\sqrt{5}}{4g}$

**Q. 7.**




$\tan\alpha = \frac{1}{3} \Rightarrow \cos\alpha = \frac{3}{\sqrt{10}}$ and $\sin\alpha = \frac{1}{\sqrt{10}}$

(a) $v_x = u\cos\alpha - gt\sin\theta$

$= 4g\sqrt{2}\left(\frac{3}{\sqrt{10}}\right) - gt\sin\theta$

$= \frac{12g}{\sqrt{5}} - gt\sin\theta$

$v_y = u\sin\alpha - gt\cos\theta$

$= 4g\sqrt{2}\left(\frac{1}{\sqrt{10}}\right) - gt\cos\theta$

$= \frac{4g}{\sqrt{5}} - gt\cos\theta$

$s_x = ut\cos\alpha - \frac{1}{2}gt^2\sin\theta$

$= 4g\sqrt{2}t\left(\frac{3}{\sqrt{10}}\right) - \frac{1}{2}gt^2\sin\theta$

$= \frac{12gt}{\sqrt{5}} - \frac{1}{2}gt^2\sin\theta$

$s_y = ut\sin\alpha - \frac{1}{2}gt^2\cos\theta$

$= 4g\sqrt{2}\left(\frac{1}{\sqrt{10}}\right) - \frac{1}{2}gt^2\cos\theta$

$= \frac{4gt}{\sqrt{5}} - \frac{1}{2}gt^2\cos\theta$

For landing angle, need $v_x$ and $v_y$ when $s_y = 0$

$\frac{4gt}{\sqrt{5}} - \frac{1}{2}gt^2\cos\theta = 0$

... multiply by $2\sqrt{5}$

$\Rightarrow 8gt - gt^2\sqrt{5}\cos\theta = 0$

... divide by $g$

$\Rightarrow 8t - t^2\sqrt{5}\cos\theta = 0$

$t(8 - t\sqrt{5}\cos\theta) = 0$

$\Rightarrow \underbrace{t = 0}_{\text{Point of Projection}}$ $\quad \underbrace{t = \frac{8}{\sqrt{5}\cos\theta}}_{\text{Time of Flight}}$

$v_x = \frac{12g}{\sqrt{5}} - g\left[\frac{8}{\sqrt{5}\cos\theta}\right]\sin\theta$

$= \frac{12g}{\sqrt{5}} - \frac{8g\tan\theta}{\sqrt{5}}$