$$= \frac{4g(3 - 2\tan\theta)}{\sqrt{5}}$$

$$v_y = \frac{4g}{\sqrt{5}} - g\left[\frac{8}{\sqrt{5}\cos\theta}\right]\cos\theta$$

$$= \frac{4g}{\sqrt{5}} - \frac{8g}{\sqrt{5}}$$

$$= -\frac{4g}{\sqrt{5}}$$

$$\tan\theta = \frac{v_y}{v_x}$$

$$= \frac{4g}{\sqrt{5}} \times \frac{\sqrt{5}}{4g(3 - 2\tan\theta)}$$

$$\Rightarrow \tan\theta = \frac{1}{3 - 2\tan\theta}$$

$$\Rightarrow 3\tan\theta - 2\tan^2\theta = 1$$

$$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$$

$$\Rightarrow (2\tan\theta - 1)(\tan\theta - 1) = 0$$

$$\Rightarrow \tan\theta = \frac{1}{2} \quad \textbf{OR} \quad \tan\theta = 1$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right) \quad \textbf{OR} \quad \theta = 45^\circ$$

(b) (i) Let $\tan\theta = 0.5$

$$\Rightarrow v_x = \frac{8g}{\sqrt{5}} \quad \text{and} \quad v_y = -\frac{4g}{\sqrt{5}}$$

$$\Rightarrow |v| = \sqrt{\left(\frac{8g}{\sqrt{5}}\right)^2 + -\left(\frac{4g}{\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{64g^2}{5} + \frac{16g^2}{5}}$$

$$= \sqrt{\frac{80g^2}{5}}$$

$$= \sqrt{16g^2} = 4g \text{ m/s}$$

(ii) Total Energy at $P = \frac{1}{2}mu^2$

$$= \frac{1}{2}m(4g\sqrt{2})^2$$

$$= \frac{1}{2}m(32g^2)$$

$$= 16mg^2$$

Total Energy at $Q = \frac{1}{2}mv^2 + mgh$

In order to find $h$, must firstly find range.

i.e. find $s_x$ when $t = \frac{8}{\sqrt{5}\cos\theta}$

$$= \frac{8}{\sqrt{5}\left(\frac{2}{\sqrt{5}}\right)} = 4$$

$$\text{Range} = \frac{12g(4)}{\sqrt{5}} - \frac{1}{2}g(16)\left(\frac{1}{\sqrt{5}}\right)$$

$$= \frac{48g}{\sqrt{5}} - \frac{8g}{\sqrt{5}}$$

$$= \frac{40g}{\sqrt{5}}$$

$$= 8g\sqrt{5}$$

$$\sin\theta = \frac{h}{\text{Range}}$$

$$\Rightarrow \frac{1}{\sqrt{5}} = \frac{h}{8g\sqrt{5}}$$

$$\Rightarrow h = 8g$$

$\Rightarrow$ Total Energy at $Q$

$$= \frac{1}{2}m(4g)^2 + mg(8g)$$

$$= 8mg^2 + 8mg^2$$

$$= 16mg^2$$

This is the same as the total energy at $P$.

**Q. 8.**

$$v_x = u\cos\theta - gt\sin\alpha$$

$$v_y = u\cos\theta - gt\cos\alpha$$

$$s_x = ut\cos\theta - \frac{1}{2}gt^2\sin\alpha$$

$$s_y = ut\sin\theta - \frac{1}{2}gt^2\cos\alpha$$

For landing angle, need $v_x$ and $v_y$ when $s_y = 0$

$$ut\sin\theta - \frac{1}{2}gt^2\cos\alpha = 0$$

$$\Rightarrow 2ut\sin\theta - gt^2\cos\alpha = 0$$

$$\Rightarrow t(2u\sin\theta - gt\cos\alpha) = 0$$

$$\Rightarrow t = 0 \qquad t = \frac{2u\sin\theta}{g\cos\alpha}$$

Point of Projection — Time of Flight

$$v_x = u\cos\theta - g\left[\frac{2u\sin\theta}{g\cos\alpha}\right]\sin\alpha$$

$$= u\cos\theta - 2u\sin\theta\tan\alpha$$

$$= u(\cos\theta - 2\sin\theta\tan\alpha)$$

$$v_y = u\sin\theta - g\left[\frac{2u\sin\theta}{g\cos\alpha}\right]\cos\alpha$$

$$= u\sin\theta - 2u\sin\theta$$

$$= -u\sin\theta$$

Lands horizontally

$\Rightarrow$ Landing angle $= \alpha$

$$\tan\alpha = \frac{-v_y}{v_x}$$

$$= \frac{u\sin\theta}{u(\cos\theta - 2\sin\theta\tan\alpha)}$$