$\Rightarrow \tan\alpha = \dfrac{\sin\theta}{\cos\theta - 2\sin\theta\tan\alpha}$

$\Rightarrow \sin\theta = \tan\alpha\cos\theta - 2\sin\theta\tan^2\alpha$ ... divide by $\cos\theta$

$\Rightarrow \tan\theta = \tan\alpha - 2\tan\theta\tan^2\alpha$

$\Rightarrow \tan\theta(1 + 2\tan^2\alpha) = \tan\alpha$

$\Rightarrow \tan\theta = \dfrac{\tan\alpha}{1 + 2\tan^2\alpha}$

$\Rightarrow \tan\theta = \dfrac{\frac{\sin\alpha}{\cos\alpha}}{1 + \frac{2\sin^2\alpha}{\cos^2\alpha}}$

... multiply top and bottom by $\cos^2\alpha$

$\Rightarrow \tan\theta = \dfrac{\sin\alpha\cos\alpha}{\cos^2\alpha + 2\sin^2\alpha}$

$\Rightarrow \tan\theta = \dfrac{\sin\alpha\cos\alpha}{\cos^2\alpha + 2(1 - \cos^2\alpha)}$

$\Rightarrow \tan\theta = \dfrac{\sin\alpha\cos\alpha}{\cos^2\alpha + 2 - 2\cos^2\alpha}$

$\Rightarrow \tan\theta = \dfrac{\sin\alpha\cos\alpha}{2 - \cos^2\alpha}$

Q. 9. $v_x = u\cos\theta - gt\sin 60°$

$= u\cos\theta - \dfrac{gt\sqrt{3}}{2}$

$v_y = u\sin\theta - gt\cos 60°$

$= u\sin(\theta - 60°) - \dfrac{gt}{2}$

Note: θ is the angle between the projection and the plane.

$s_x = ut\cos\theta - \frac{1}{2}gt^2\sin 60°$

$= ut\cos\theta - \dfrac{gt^2\sqrt{3}}{4}$

$s_y = ut\sin\theta - \frac{1}{2}gt^2\cos 60°$

$= ut\sin\theta - \dfrac{gt^2}{4}$

Maximum Perpendicular Height: Find s_y when $v_y = 0$

$u\sin\theta - \dfrac{gt}{2} = 0$

$\Rightarrow 2u\sin\theta - gt = 0$

$\Rightarrow t = \dfrac{2u\sin\theta}{g}$

$\Rightarrow H = u\left[\dfrac{2u\sin\theta}{g}\right]\sin\theta - \dfrac{g}{4}\left[\dfrac{4u^2\sin^2\theta}{g^2}\right]$

$\Rightarrow H = \dfrac{2u^2\sin^2\theta}{g} - \dfrac{u^2\sin^2\theta}{g}$

$\Rightarrow H = \dfrac{u^2\sin^2\theta}{g}$

For the second part of the question, we let $s_y = H\sin^2\theta$

$\Rightarrow ut\sin\theta - \dfrac{gt^2}{4}$

$= \left[\dfrac{u^2\sin^2\theta}{g}\right]\sin^2\theta$ ... multiply by $4g$

$\Rightarrow 4gut\sin\theta - g^2t^2 = 4u^2\sin^4\theta$

$\Rightarrow g^2t^2 - 4gut\sin\theta + 4u^2\sin^4\theta = 0$

This is a quadratic equation in t

$\Rightarrow t = \dfrac{4gu\sin\theta \pm \sqrt{16g^2u^2\sin^2\theta - 16g^2u^2\sin^4\theta}}{2g^2}$

$\Rightarrow t = \dfrac{4gu\sin\theta \pm \sqrt{16g^2u^2\sin^2\theta(1 - \sin^2\theta)}}{2g^2}$

$\Rightarrow t = \dfrac{4gu\sin\theta \pm \sqrt{16g^2u^2\sin^2\theta\cos^2\theta}}{2g^2}$

$\Rightarrow t = \dfrac{4gu\sin\theta \pm 4gu\sin\theta\cos\theta}{2g^2}$

... divide top and bottom by $2g$

$\Rightarrow t = \dfrac{2u\sin\theta \pm 2u\sin\theta\cos\theta}{g}$

$\Rightarrow t_1 = \dfrac{2u\sin\theta + 2u\sin\theta\cos\theta}{g}$

$= \dfrac{2u\sin\theta(1 + \cos\theta)}{g}$

and $t_2 = \dfrac{2u\sin\theta - 2u\sin\theta\cos\theta}{g}$

$= \dfrac{2u\sin\theta(1 - \cos\theta)}{g}$

$t_1 - t_2 = \dfrac{2u\sin\theta(1 + \cos\theta)}{g} - \dfrac{2u\sin\theta(1 - \cos\theta)}{g}$

$= \dfrac{2u\sin\theta}{g}[1 + \cos\theta - (1 - \cos\theta)]$

$= \dfrac{2u\sin\theta}{g}(2\cos\theta)$

$= \dfrac{2u[2\sin\theta\cos\theta]}{g}$

... $2\sin\theta\cos\theta = \sin 2\theta$

$= \dfrac{2u\sin 2\theta}{g}$