$\Rightarrow$ Velocity after impact $= 2u\vec{i} + 2u\vec{j}$

$\Rightarrow$ Particle rebounds at an angle of 45° to the inclined plane. Given that the plane is inclined at 45° to the horizontal, the particle rises vertically (at 90° to horizontal) from P.

Q. 5. (i) $v_x = u\cos 60° + gt\sin 30°$

$$= \frac{u}{2} + \frac{gt}{2}$$

$$v_y = u\sin 60° - gt\cos 30°$$

$$= \frac{u\sqrt{3}}{2} - \frac{gt\sqrt{3}}{2}$$

$$s_x = ut\cos 60° + \frac{1}{2}gt^2\sin 30°$$

$$= \frac{ut}{2} - \frac{gt^2}{4}$$

$$s_y = ut\sin 60° - \frac{1}{2}gt^2\cos 30°$$

$$= \frac{ut\sqrt{3}}{2} - \frac{gt^2\sqrt{3}}{4}$$

Range: s_x when $s_y = 0$

$$\frac{ut\sqrt{3}}{2} - \frac{gt^2\sqrt{3}}{4} = 0$$

$$\Rightarrow 2ut - gt^2 = 0$$

$$\Rightarrow t(2u - gt) = 0$$

$\Rightarrow t = 0$ (Point of Projection) $\quad t = \frac{2u}{g}$ (Time of Flight)

$$\text{Range} = \frac{u}{2}\left[\frac{2u}{g}\right] + \frac{g}{4}\left[\frac{4u^2}{g^2}\right]$$

$$= \frac{u^2}{g} + \frac{u^2}{g} = \frac{2u^2}{g}$$

(ii) Need to firstly find landing velocity, i.e. v_x and v_y when $t = \frac{2u}{g}$

$$v_x = \frac{u}{2} + \frac{g}{2}\left[\frac{2u}{g}\right] = \frac{u}{2} + u = \frac{3u}{2}$$

$$v_y = \frac{u\sqrt{3}}{2} - \frac{g\sqrt{3}}{2}\left[\frac{2u}{g}\right] = \frac{u\sqrt{3}}{2} - u\sqrt{3}$$

$$= -\frac{u\sqrt{3}}{2}$$

x–velocity is unchanged after impact

New y–velocity $= \frac{eu\sqrt{3}}{2}$

$\Rightarrow$ velocity at start of 2nd hop

$$= \frac{3u}{2}\vec{i} + \frac{eu\sqrt{3}}{2}\vec{j}$$

$$\text{Magnitude} = \sqrt{\frac{9u^2}{4} + \frac{3e^2u^2}{4}}$$

$$= \frac{u\sqrt{3}}{2}\sqrt{e^2 + 3}$$

Let θ be the angle at which the particle leaves the slope

$$\tan\theta = \frac{eu\sqrt{3}}{2} \times \frac{2}{3u} = \frac{e\sqrt{3}}{3} = \frac{e}{\sqrt{3}}$$

$\Rightarrow \sin\theta = \frac{e}{\sqrt{e^2+3}}$ and $\cos\theta = \frac{\sqrt{3}}{\sqrt{e^2+3}}$

Projectile equations are:

$$v_x = \left[\frac{u\sqrt{3}}{2}\sqrt{e^2+3}\right]\left[\frac{\sqrt{3}}{\sqrt{e^2+3}}\right] + gt\sin 30°$$

$$= \frac{3u}{2} + \frac{gt}{2}$$

$$v_y = \left[\frac{u\sqrt{3}}{2}\sqrt{e^2+3}\right]\left[\frac{e}{\sqrt{e^2+3}}\right] - gt\cos 30°$$

$$= \frac{eu\sqrt{3}}{2} - \frac{gt\sqrt{3}}{2}$$

$$s_x = \left[\frac{u\sqrt{3}}{2}\sqrt{e^2+3}\right][t]\left[\frac{\sqrt{3}}{\sqrt{e^2+3}}\right] + \frac{1}{2}gt^2\sin 30°$$

$$= \frac{3ut}{2} + \frac{gt^2}{4}$$

$$s_y = \left[\frac{u\sqrt{3}}{2}\sqrt{e^2+3}\right][t]\left[\frac{e}{\sqrt{e^2+3}}\right] - \frac{1}{2}gt^2\cos 30°$$

$$= \frac{eut\sqrt{3}}{2} - \frac{gt^2\sqrt{3}}{4}$$

Range: s_x when $s_y = 0$

$$\frac{eut\sqrt{3}}{2} - \frac{gt^2\sqrt{3}}{4} = 0$$

$$\Rightarrow 2eut - gt^2 = 0$$

$$\Rightarrow t(2eu - gt) = 0$$

$t = 0$ (Point of Projection) $\quad t = \frac{2eu}{g}$ (Time of Flight)

$$\text{Range} = \frac{3u}{2}\left[\frac{2eu}{g}\right] + \frac{g}{4}\left[\frac{4e^2u^2}{g^2}\right]$$

$$= \frac{3eu^2}{g} + \frac{e^2u^2}{g}$$

$$= \frac{eu^2}{g}[3 + e]$$