

Range for 2nd hop = 2[Range for 1st hop]

$$\Rightarrow \frac{eu^2}{g} [3 + e] = 2 \left[\frac{2u^2}{g} \right]$$

... multiply by $\frac{g}{u^2}$

$$\Rightarrow e(3 + e) = 4$$

$$\Rightarrow 3e + e^2 = 4$$

$$\Rightarrow e^2 + 3e - 4 = 0$$

$$\Rightarrow (e + 4)(e - 1) = 0$$

$$\Rightarrow e = \cancel{4} \quad e = 1 \quad \dots 0 \leq e \leq 1$$

Q. 6. Fall from height h :

$$u = 0, \quad a = g, \quad s = h$$

$$v = \sqrt{u^2 + 2as} = \sqrt{2gh}$$

\Rightarrow Strikes the plane at a speed of $\sqrt{2gh}$

x-component of velocity at impact

$$= \sqrt{2gh} \cos 60^\circ = \frac{\sqrt{2gh}}{2} = \frac{\sqrt{gh}}{2}$$

y-component of velocity at impact

$$= -\sqrt{2gh} \sin 60^\circ = -\sqrt{2gh} \left(\frac{\sqrt{3}}{2} \right) = -\sqrt{\frac{3gh}{2}}$$

x-component is unchanged during impact

$$\text{y-component after impact} = \frac{1}{2} \sqrt{\frac{3gh}{2}}$$

Magnitude of velocity after impact

$$= \sqrt{\frac{gh}{2} + \frac{3gh}{8}} = \sqrt{\frac{4gh + 3gh}{8}}$$

$$= \sqrt{\frac{7gh}{8}}$$

Let θ be the angle at which the particle leaves the plane

$$\begin{aligned} \tan \theta &= \frac{1}{2} \sqrt{\frac{3gh}{2}} \times \sqrt{\frac{2}{gh}} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{3}}{\sqrt{7}} \text{ and } \cos \theta = \frac{2}{\sqrt{7}}$$

Projectile equations are:

$$\begin{aligned} v_x &= \left[\sqrt{\frac{7gh}{8}} \right] \left[\frac{2}{\sqrt{7}} \right] + gt \sin 30^\circ \\ &= \sqrt{\frac{gh}{2}} + \frac{gt}{2} \end{aligned}$$

$$\begin{aligned} v_y &= \left[\sqrt{\frac{7gh}{8}} \right] \left[\frac{\sqrt{3}}{\sqrt{7}} \right] - gt \cos 30^\circ \\ &= \sqrt{\frac{3gh}{8}} - \frac{gt\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} s_x &= \left[\sqrt{\frac{7gh}{8}} \right] [t] \left[\frac{2}{\sqrt{7}} \right] + \frac{1}{2} gt^2 \sin 30^\circ \\ &= t \sqrt{\frac{gh}{2}} + \frac{gt^2}{4} \end{aligned}$$

$$\begin{aligned} s_y &= \left[\sqrt{\frac{7gh}{8}} \right] [t] \left[\frac{\sqrt{3}}{\sqrt{7}} \right] - \frac{1}{2} gt^2 \cos 30^\circ \\ &= t \sqrt{\frac{3gh}{8}} - \frac{gt^2\sqrt{3}}{4} \end{aligned}$$

Range: s_x when $s_y = 0$

$$t \sqrt{\frac{3gh}{8}} - \frac{gt^2\sqrt{3}}{4} = 0 \quad \dots \text{multiply by } \frac{4\sqrt{8}}{\sqrt{3}}$$

$$\Rightarrow 4t\sqrt{gh} - gt^2\sqrt{8} = 0$$

$$\Rightarrow 4t\sqrt{gh} - 2gt^2\sqrt{2} = 0$$

$$\Rightarrow 2t\sqrt{gh} - gt^2\sqrt{2} = 0$$

$$\Rightarrow t\sqrt{2gh} - gt^2 = 0$$

$$\Rightarrow t(\sqrt{2gh} - gt) = 0$$

$$\Rightarrow t = 0 \quad \underbrace{t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}}}_{\text{Time of Flight}}$$

Point of Projection

$$\text{Range} = \left[\sqrt{\frac{2h}{g}} \right] \sqrt{\frac{gh}{2}} + \frac{g}{4} \left[\frac{2h}{g} \right]$$

$$\begin{aligned} \Rightarrow \text{Range} &= h + \frac{h}{2} \\ &= \frac{3h}{2} \end{aligned}$$