Q. 17. (i) $\int_1^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}}$

$= \left[\sin^{-1}\frac{x}{2}\right]_1^{\sqrt{3}}$

$= \left[\sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1}\frac{1}{2}\right]$

$= \frac{\pi}{3} - \frac{\pi}{6}$

$= \frac{\pi}{6}$

(ii) $\int_0^{\frac{\pi}{3}} \sin^2 x\, dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{3}}(1 - \cos 2x)dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{3}}$

$= \frac{1}{2}\left[\frac{\pi}{3} - \frac{\sin\frac{2\pi}{3}}{2}\right]$

$= \frac{1}{2}\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$

$= \frac{\pi}{6} - \frac{\sqrt{3}}{8}$

Q. 18. (i) $\int_0^9 \frac{dx}{x^2+81}$

$= \frac{1}{9}\left[\tan^{-1}\frac{x}{9}\right]_0^9$

$= \frac{1}{9}[\tan^{-1} 1 - \tan^{-1} 0]$

$= \frac{\pi}{36}$

(ii) $\int_0^{\frac{\pi}{4}} \cos^2 2x\, dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{4}}(1 + \cos 4x)dx$

$= \frac{1}{2}\left[x + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{2}\left[\frac{\pi}{4} + 0\right]$

$= \frac{\pi}{8}$

Q. 19. (i) $\int_0^1 \frac{dx}{\sqrt{2-x^2}}$

$= \int_0^1 \frac{dx}{\sqrt{(\sqrt{2})^2 - x^2}}$

$= \left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1$

$= \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\pi}{4}$

(ii) $\int_0^{\frac{\pi}{8}} \sin^2(2x)\, dx$

$= \frac{1}{2}\int_0^{\frac{\pi}{8}}(1 - \cos 4x)\, dx$

$= \frac{1}{2}\left[x - \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{8}}$

$= \frac{1}{2}\left[\frac{\pi}{8} - \frac{1}{4}\right]$

$= \frac{1}{8}\left[\frac{\pi}{2} - 1\right]$

Q. 20. (i) $\int_1^{\sqrt{3}} \frac{dx}{x^2+3}$

$= \int_1^{\sqrt{3}} \frac{dx}{x^2 + (\sqrt{3})^2}$

$= \frac{1}{\sqrt{3}}\left[\tan^{-1}\frac{x}{\sqrt{3}}\right]_1^{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}\left[\tan^{-1} 1 - \tan^{-1}\frac{1}{\sqrt{3}}\right]$

$= \frac{1}{\sqrt{3}}\left[\frac{\pi}{4} - \frac{\pi}{6}\right]$

$= \frac{\pi}{12\sqrt{3}}$

(ii) $\int_0^{\sqrt{2}} \frac{dx}{\sqrt{8-x^2}}$

$= \int_0^{\sqrt{2}} \frac{dx}{\sqrt{(\sqrt{8})^2 - x^2}}$

$= \left[\sin^{-1}\frac{x}{\sqrt{8}}\right]_0^{\sqrt{2}}$

$= \sin^{-1}\frac{\sqrt{2}}{8}$

$= \sin^{-1}\frac{1}{2}$

$= \frac{\pi}{6}$

Exercise 12D

Q. 1. $\frac{dy}{dx} = 6y$

$\Rightarrow \int\frac{dy}{y} = 6\int dx$

$\Rightarrow \ln y = 6x + c$

$\Rightarrow y = e^{6x+c}$

Q. 2. $\frac{dy}{dx} = 2xy$

$\Rightarrow \int\frac{dy}{y} = 2\int x\, dx$

$\Rightarrow \ln y = x^2 + c$

$\Rightarrow y = e^{x^2+c}$