

Q. 3. $\frac{dy}{dx} = \cos x$

$$\Rightarrow \int dy = \int \cos x \, dx$$

$$\Rightarrow y = \sin x + c$$

Q. 4. $\frac{dy}{dx} = \frac{4x^3}{y}$

$$\Rightarrow \int y \, dy = 4 \int x^3 \, dx$$

$$\Rightarrow \frac{y^2}{2} = x^4 + c$$

$$\Rightarrow y^2 = 2(x^4 + c)$$

$$\Rightarrow y = \pm \sqrt{2(x^4 + c)}$$

Q. 5. $\frac{dy}{dx} = 3x^2y$

$$\Rightarrow \int \frac{dy}{y} = 3 \int x^2 \, dx$$

$$\Rightarrow \ln y = x^3 + c$$

$$\Rightarrow y = e^{(x^3 + c)}$$

Q. 6. $\frac{dy}{dx} + \frac{\sin 2x}{y} = 0$

$$\Rightarrow \int \frac{dy}{y} = -\int \frac{\sin 2x}{y} \, dx$$

$$\Rightarrow \int y \, dy = -\int \sin 2x \, dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{\cos 2x}{2} + c$$

$$\Rightarrow y^2 = \cos 2x + 2c$$

$$\Rightarrow y = \pm \sqrt{\cos 2x + 2c}$$

Q. 7. $\frac{dy}{dx} = 2x(y^2 + 1)$

$$\Rightarrow \int \frac{dy}{y^2 + 1} = 2 \int x \, dx$$

$$\Rightarrow \tan^{-1} y = x^2 + c$$

$$\Rightarrow y = \tan(x^2 + c)$$

Q. 8. $\frac{dy}{dx} = 2\sqrt{1 - y^2}$

$$\Rightarrow \int \frac{dy}{\sqrt{1 - y^2}} = 2 \int dx$$

$$\Rightarrow \sin^{-1} y = 2x + c$$

$$\Rightarrow y = \sin(2x + c)$$

Exercise 12E

Q. 1. $\frac{dy}{dx} = 3y$

$$\Rightarrow \frac{1}{y} \, dy = 3 \, dx$$

$$\Rightarrow \int_1^y \frac{1}{y} \, dy = \int_0^x 3 \, dx$$

$$\Rightarrow \log_e y \Big|_1^y = 3x \Big|_0^x$$

$$\Rightarrow \log_e y - \log_e 1 = 3x - 3(0)$$

$$\Rightarrow \log_e y = 3x$$

$$\Rightarrow y = e^{3x}$$

Q. 2. $\frac{dy}{dx} = 5y$

$$\Rightarrow \frac{1}{y} \, dy = 5 \, dx$$

$$\Rightarrow \int_2^y \frac{1}{y} \, dy = \int_0^x 5 \, dx$$

$$\Rightarrow \log_e y \Big|_2^y = 5x \Big|_0^x$$

$$\Rightarrow \log_e y - \log_e 2 = 5x - 5(0)$$

$$\Rightarrow \log_e \frac{y}{2} = 5x$$

$$\Rightarrow \frac{y}{2} = e^{5x}$$

$$\Rightarrow y = 2e^{5x}$$

Q. 3. $\frac{dy}{dx} = 2x\sqrt{1 - y^2}$

$$\Rightarrow \frac{1}{\sqrt{1 - y^2}} \, dy = 2x \, dx$$

$$\Rightarrow \int_1^y \frac{1}{\sqrt{1 - y^2}} \, dy = \int_0^x 2x \, dx$$

$$\Rightarrow \sin^{-1} y \Big|_1^y = x^2 \Big|_0^x$$

$$\Rightarrow \sin^{-1} y - \sin^{-1} 1 = x^2 - 0^2$$

$$\Rightarrow \sin^{-1} y - \frac{\pi}{2} = x^2$$

$$\Rightarrow \sin^{-1} y = x^2 + \frac{\pi}{2}$$

$$\Rightarrow y = \sin \left(x^2 + \frac{\pi}{2} \right)$$

Q. 4. $\frac{dx}{dt} = tx$

$$\Rightarrow \frac{1}{x} \, dx = t \, dt$$

$$\Rightarrow \int_{\sqrt{e}}^x \frac{1}{x} \, dx = \int_1^t t \, dt$$

$$\Rightarrow \log_e x \Big|_{\sqrt{e}}^x = \frac{t^2}{2} \Big|_1^t$$

$$\Rightarrow \log_e x - \log_e \sqrt{e} = \frac{t^2}{2} - \frac{1}{2}$$

$$\Rightarrow \log_e x - \frac{1}{2} \log_e e = \frac{t^2 - 1}{2}$$