$$\Rightarrow 2\log_e x - 1 = t^2 - 1$$

$$\Rightarrow \log_e x = \frac{t^2}{2}$$

$$\Rightarrow x = e^{\frac{t^2}{2}}$$

Q. 5. $\dfrac{dy}{dx} = \dfrac{y}{x}$

$$\Rightarrow \frac{1}{y}\,dy = \frac{1}{x}\,dx$$

$$\Rightarrow \int_3^y \frac{1}{y}\,dy = \int_1^x \frac{1}{x}\,dx$$

$$\Rightarrow \log_e y\big|_3^y = \log_e x\big|_1^x$$

$$\Rightarrow \log_e y - \log_e 3 = \log_e x - \log_e 1$$

$$\Rightarrow \log_e \frac{y}{3} = \log_e x$$

$$\Rightarrow \frac{y}{3} = x \quad \text{...let } y = 21$$

$$\Rightarrow x = 7$$

Q. 6. $\dfrac{ds}{dt} + \dfrac{\sin t}{s} = 0$

$$\Rightarrow \frac{ds}{dt} = \frac{-\sin t}{s}$$

$$\Rightarrow s\,ds = -\sin t\,dt$$

$$\Rightarrow \int_{\sqrt{2}}^s s\,ds = \int_{\frac{\pi}{3}}^t (-\sin t)\,dt$$

$$\Rightarrow \frac{s^2}{2}\Big|_{\sqrt{2}}^s = \cos t\big|_{\frac{\pi}{3}}^t$$

$$\Rightarrow \frac{s^2}{2} - 1 = \cos t - \cos\frac{\pi}{3}$$

$$\Rightarrow \frac{s^2}{2} - 1 = \cos t - \frac{1}{2}$$

$$\Rightarrow s^2 - 2 = 2\cos t - 1$$

$$\Rightarrow s^2 = 2\cos t + 1$$

$$\Rightarrow s = \sqrt{2\cos t + 1}$$

Q. 7. $\dfrac{dy}{dx} - 4x^3y = 0$

$$\Rightarrow \frac{dy}{dx} = 4x^3y$$

$$\Rightarrow \frac{1}{y}\,dy = 4x^3\,dx$$

$$\Rightarrow \int_3^y \frac{1}{y}\,dy = \int_0^x 4x^3\,dx$$

$$\Rightarrow \log_e y\big|_3^y = x^4\big|_0^x$$

$$\Rightarrow \log_e y - \log_e 3 = x^4 - 0^4$$

$$\Rightarrow \log_e \frac{y}{3} = x^4$$

$$\Rightarrow \frac{y}{3} = e^{x^4}$$

$$\Rightarrow y = 3e^{x^4}$$

Q. 8. $\dfrac{y}{x^2}\dfrac{dy}{dx} = 1$

$$\Rightarrow y\,dy = x^2\,dx$$

$$\Rightarrow \int_0^y y\,dy = \int_1^x x^2\,dx$$

$$\Rightarrow \frac{y^2}{2}\Big|_0^y = \frac{x^3}{3}\Big|_1^x$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^3}{3} - \frac{1}{3}$$

$$\Rightarrow 3y^2 = 2x^3 - 2$$

$$\Rightarrow y^2 = \frac{2}{3}(x^3 - 1)$$

$$\Rightarrow y = \sqrt{\frac{2}{3}(x^3 - 1)}$$

Q. 9. $\dfrac{dy}{dx} - \dfrac{\cos x}{y} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos x}{y}$$

$$\Rightarrow y\,dy = \cos x\,dx$$

$$\Rightarrow \int_1^y y\,dy = \int_{\frac{\pi}{2}}^x \cos x\,dx$$

$$\Rightarrow \frac{y^2}{2}\Big|_1^y = \sin x\Big|_{\frac{\pi}{2}}^x$$

$$\Rightarrow \frac{y^2}{2} - \frac{1}{2} = \sin x - \sin\frac{\pi}{2}$$

$$\Rightarrow \frac{y^2}{2} - \frac{1}{2} = \sin x - 1$$

$$\Rightarrow y^2 - 1 = 2\sin x - 2$$

$$\Rightarrow y^2 = 2\sin x - 1$$

$$\Rightarrow y = \sqrt{2\sin x - 1}$$

Q. 10. $\dfrac{dy}{dx} - \dfrac{y}{x} = \dfrac{1}{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{y+1}{x}$$

$$\Rightarrow \frac{1}{y+1}\,dy = \frac{1}{x}\,dx$$

$$\Rightarrow \int_0^y \frac{1}{y+1}\,dy = \int_4^x \frac{1}{x}\,dx$$

$$\Rightarrow \log_e |y+1|\big|_0^y = \log_e x\big|_4^x$$

$$\Rightarrow \log_e |y+1| - \log_e 1 = \log_e x - \log_e 4$$

$$\Rightarrow \log_e |y+1| = \log_e \frac{x}{4}$$

$$\Rightarrow y + 1 = \frac{x}{4}$$

$$\Rightarrow y = \frac{x}{4} - 1$$